

## Capitolul 8. Integrale improprii.

1. Să se cerceteze care din următoarele integrale improprii sunt convergente:

$$1.1. \int_0^{+\infty} \frac{dx}{1+x^2}.$$

$$1.2. \int_0^1 \ln x \, dx.$$

$$1.3. \int_1^{+\infty} \frac{dx}{x^\alpha}, \quad \alpha \in \mathbb{R}.$$

$$1.4. \int_0^1 \frac{dx}{\sqrt{1-x^2}}.$$

$$1.5. \int_1^{+\infty} \frac{1+\ln x}{x} \, dx.$$

$$1.6. \int_0^1 \frac{dx}{x^\alpha}, \quad \alpha \in \mathbb{R}.$$

$$1.7. \int_{-\infty}^0 x e^x \, dx.$$

$$1.8. \int_0^1 \frac{dx}{\sqrt{16-x^2}}.$$

$$1.9. \int_{-\infty}^0 \operatorname{arctg} x \, dx.$$

$$1.10. \int_{-1}^1 \frac{\arccos x}{\sqrt{1-x^2}} \, dx.$$

$$1.11. \int_2^{+\infty} \frac{dx}{x^2-1}.$$

$$1.12. \int_{-2}^0 \frac{\arcsin 2x}{\sqrt{4-x^2}} \, dx.$$

$$1.13. \int_{-\infty}^{+\infty} \frac{dx}{x^2+4x+5}.$$

$$1.14. \int_{-1}^0 e^{\frac{2}{x}} \frac{dx}{x^3}.$$

$$1.15. \int_0^{+\infty} \sin 5x \, dx.$$

$$1.16. \int_0^e \frac{dx}{e^x-1}.$$

$$1.17. \int_{-\infty}^0 \frac{x+2}{x^2+1} dx.$$

$$1.18. \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x}.$$

$$1.19. \int_e^{+\infty} \frac{dx}{x \ln x}.$$

$$1.20. \int_0^5 \frac{dx}{(x-5)^3}.$$

$$1.21. \int_1^{+\infty} \frac{dx}{x^3(1+x^3)}.$$

$$1.22. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{tg} x \, dx.$$

$$1.23. \int_1^{+\infty} e^{-x^2} dx.$$

$$1.24. \int_0^{\frac{\pi}{2}} (\sin x)^p (\cos x)^q dx, \quad \{p, q\} \subset \mathbb{R}.$$

$$1.25. \int_{-\infty}^{+\infty} \frac{dx}{x^2 - 5x + 14}.$$

$$1.26. \int_{-1}^1 \frac{x \, dx}{|\sqrt{4-x} - \sqrt{4+x}|}.$$

$$1.27. \int_1^{+\infty} \frac{2x+1}{x^2(x+1)} dx.$$

$$1.28. \int_a^b \frac{dx}{(b-x)^\alpha}, \quad \alpha \in \mathbb{R}.$$

$$1.29. \int_e^{+\infty} \frac{dx}{x \sqrt[3]{\ln x}}.$$

$$1.30. \int_0^1 \frac{dx}{(1-x)\sqrt{x}}.$$

$$1.31. \int_0^{+\infty} e^{-x} \sin x \, dx.$$

$$1.32. \int_1^{+\infty} \frac{dx}{x \sqrt{x^2+x+1}}.$$

**2. Să se cerceteze natura integralelor improprii:**

$$2.1. \int_1^{+\infty} \frac{x+2}{\sqrt{x^3}} dx.$$

$$2.2. \int_2^{+\infty} \frac{dx}{\sqrt{x(x+1)(x-1)}}.$$

$$2.3. \int_0^{+\infty} \frac{x^2-1}{x^4+x^2+3} dx.$$

$$2.4. \int_0^2 \frac{dx}{\sqrt[3]{4-x^2}}.$$

$$2.5. \int_0^{+\infty} \frac{x^3-2x^2+3}{x^4+1} dx.$$

$$2.6. \int_1^2 \frac{dx}{\ln x}.$$

$$2.7. \int_1^{+\infty} \frac{\sin^2 x}{x^2} dx.$$

$$2.8. \int_0^1 \frac{\cos \frac{1}{x}}{\sqrt[3]{x}}.$$

$$2.9. \int_1^{+\infty} \frac{dx}{\sqrt{9x+\ln x}}.$$

$$2.10. \int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx.$$

$$2.11. \int_1^{+\infty} \frac{x^2 dx}{x^4+\sin^2 x}.$$

$$2.12. \int_0^1 \frac{dx}{\operatorname{tg} x - x}.$$

$$2.13. \int_2^{+\infty} \frac{dx}{x^p \ln^q x}, \quad \{p, q\} \subset \mathbb{R}.$$

$$2.14. \int_0^1 \frac{\ln(1+\sqrt[3]{x^2})}{e^x-1} dx.$$

$$2.15. \int_0^{+\infty} \frac{\ln(1+x^2)}{\sqrt{x+\sqrt{x}}} dx.$$

$$2.16. \int_0^1 \frac{\sqrt{x} dx}{e^{\sin x}-1}.$$

$$2.17. \int_e^{+\infty} \frac{dx}{x \ln^\alpha x}, \alpha \in \mathbb{R}.$$

$$2.18. \int_0^1 \frac{dx}{e^x - \cos x}.$$

$$2.19. \int_2^{+\infty} \frac{e^{\alpha x}}{(x-1)^\alpha \ln x} dx, \alpha \in \mathbb{R}.$$

$$2.20. \int_0^2 \frac{dx}{\ln x}.$$

$$2.21. \int_1^{+\infty} \frac{(x + \sqrt{x+2})}{x^2 + 3\sqrt[5]{x^4 + 2}} dx.$$

$$2.22. \int_0^1 \frac{\ln x}{1-x^2} dx.$$

$$2.23. \int_1^{+\infty} \ln \frac{e^{\frac{1}{x}} + (n-1)}{n} dx, n > 0.$$

$$2.24. \int_0^1 \frac{dx}{e^{\sqrt{x}} - 1}.$$

$$2.25. \int_1^{+\infty} \frac{dx}{x \sqrt[3]{x^2 + 1}}.$$

$$2.26. \int_0^1 \frac{dx}{\sqrt{1-x^4}}.$$

$$2.27. \int_0^{+\infty} \frac{\sin^2 x}{1+x^2} dx.$$

$$2.28. \int_0^\pi \frac{dx}{\sqrt{\sin x}}.$$

$$2.29. \int_0^{+\infty} x^{p-1} e^{-x} dx, p \in \mathbb{R}.$$

$$2.30. \int_0^1 x^{p-1} (1-x)^{q-1} dx, \{p, q\} \subset \mathbb{R}.$$

**3. Să se cerceteze la convergență absolută sau semiconvergență integralele:**

$$3.1. \int_1^{+\infty} \frac{\sin x}{x} dx.$$

$$3.2. \int_0^{+\infty} \frac{\sqrt{x} \cos x}{x+10} dx.$$

$$3.3. \int_0^1 (1-x) \sin \frac{\pi}{1-x} dx.$$

$$3.4. \int_0^1 \frac{1}{1-x} \sin \frac{\pi}{1-x} dx.$$

$$3.5. \int_0^1 \frac{x^2}{x^2+1} \sin \frac{1}{x} dx.$$

$$3.6. \int_0^1 \frac{1}{x(x^2+1)} \sin \frac{1}{x} dx.$$

$$3.7. \int_0^{\frac{1}{2}} \left( \frac{x}{1-x} \right) \cos \frac{1}{x^2} dx.$$

$$3.8. \int_0^{\frac{1}{2}} \left( \frac{1-x}{x} \right)^2 \cos \frac{1}{x^2} dx.$$

$$3.9. \int_0^1 \frac{\sin x^2}{x^2} dx.$$

$$3.10. \int_0^1 \frac{\sin \frac{1}{x^2}}{x^2} dx.$$

**4. Să se calculeze:**

$$4.1. \text{ V.P. } \int_1^6 \frac{dx}{4-x}.$$

$$4.2. \text{ V.P. } \int_{-\infty}^{+\infty} \frac{1+x}{1+x^2} dx.$$

$$4.3. \text{ V.P. } \int_{-\frac{1}{2}}^1 \frac{dx}{(x+1) \ln(x+1)}.$$

$$4.4. \text{ V.P. } \int_0^{+\infty} \frac{dx}{x^2-x-2}.$$

$$4.5. \text{ V.P. } \int_{-2}^2 \frac{dx}{x}.$$

$$4.6. \text{ V.P. } \int_0^{+\infty} \frac{dx}{x^2-4x+3}.$$

$$4.7. \text{ V.P. } \int_{-1}^3 \frac{dx}{(x-2)^3}.$$

$$4.8. \text{ V.P. } \int_{-\infty}^{+\infty} \operatorname{arctg} x dx.$$

$$4.9. \text{ V.P. } \int_0^{\frac{\pi}{2}} \frac{dx}{1-2 \sin x}.$$

$$4.10. \text{ V.P. } \int_0^{+\infty} \frac{dx}{1-x^2}.$$