

Limite de siruri

1. a) Utilizand definitia limitei cu " ε " sa se arate ca sirul numeric a_n este convergent si are limita a .
 b) Sa se determine rangul, incepand de la care termenul sirului difera de a cu mai putin de 0.001.

$$1.1. \quad a_n = \frac{4n+2}{5n+1}, \quad a = \frac{4}{5}.$$

$$1.2. \quad a_n = \frac{2n+1}{3n-2}, \quad a = \frac{2}{3}.$$

$$1.3. \quad a_n = \frac{1-2n}{n+2}, \quad a = -2.$$

$$1.4. \quad a_n = \frac{3n}{2n-1}, \quad a = \frac{3}{2}.$$

$$1.5. \quad a_n = \frac{7n+1}{1-3n}, \quad a = -\frac{7}{3}.$$

$$1.6. \quad a_n = \frac{2n-1}{7n-3}, \quad a = \frac{2}{7}.$$

$$1.7. \quad a_n = \frac{2-n}{1-2n}, \quad a = \frac{1}{2}.$$

$$1.8. \quad a_n = \frac{6-3n}{2n-1}, \quad a = -\frac{3}{2}.$$

$$1.9. \quad a_n = \frac{2n^2+1}{8n^2-1}, \quad a = \frac{1}{4}.$$

$$1.10. \quad a_n = \frac{2n^2}{3-n^2}, \quad a = -2.$$

$$1.11. \quad a_n = \frac{3n^2+2}{1-4n^2}, \quad a = -\frac{3}{4}.$$

$$1.12. \quad a_n = \frac{1-5n^2}{2-4n^2}, \quad a = \frac{5}{4}.$$

$$1.13. \quad a_n = \frac{4n^3}{2n^3-1}, \quad a = 2.$$

$$1.14. \quad a_n = \frac{8-n^3}{1+2n^3}, \quad a = -\frac{1}{2}.$$

$$1.15. \quad a_n = \frac{2+n^3}{2n^3-1}, \quad a = \frac{1}{2}.$$

$$1.16. \quad a_n = \frac{1-n^3}{1+n^3}, \quad a = -1.$$

2. a) Sa se arate ca sirurile date sunt convergente:

$$2.1. \quad a_n = \frac{2n+3}{3n-2}.$$

$$2.2. \quad a_n = \frac{n-1}{n+1}.$$

$$2.3. \quad a_n = \frac{2n+1}{n+3}.$$

$$2.4. \quad a_n = \frac{1-2n}{n+1}.$$

$$2.5. \quad a_n = \frac{1-3n}{1-4n}.$$

$$2.6. \quad a_n = \frac{2n-1}{n+2}.$$

$$2.7. \quad a_n = \sum_{k=1}^n \frac{1}{k(k+1)}.$$

$$2.8. \quad a_n = \sum_{k=2}^n \frac{1}{k(k-1)}.$$

$$2.9. \quad a_n = \sum_{k=1}^n \frac{2}{(2k-1)(2k+1)}.$$

$$2.10. \quad a_n = \sum_{k=1}^n \frac{1}{k^2}.$$

$$2.11. \quad a_n = \underbrace{\sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}}}_{n \text{ rădăcini}}.$$

$$2.12. \quad a_n = \underbrace{\sqrt[3]{6 + \sqrt[3]{6 + \dots + \sqrt[3]{6}}}}_{n \text{ rădăcini}}.$$

$$2.13. \quad a_n = \sum_{k=1}^n \frac{\sin k}{3^k}.$$

$$2.14. \quad a_n = \sum_{k=1}^n \frac{\sin k!}{k(k+1)}.$$

$$2.15. \quad a_n = 1 + \frac{1}{2!} + \dots + \frac{1}{n!}.$$

$$2.16. \quad a_n = \frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \dots + \frac{(-1)^{n-1}}{n(n+1)}.$$

b) Sa se arate ca:

$$2.17. \lim_{n \rightarrow \infty} \sin \frac{\pi}{2} n \neq 1.$$

$$2.18. \lim_{n \rightarrow \infty} \cos \pi n \neq 1.$$

$$2.19. \lim_{n \rightarrow \infty} \frac{2n-1}{n+1} \neq 1.$$

$$2.20. \lim_{n \rightarrow \infty} \frac{3n-1}{2n+1} \neq 2.$$

$$2.21. \lim_{n \rightarrow \infty} n^2 \sin \frac{\pi n}{4} \neq 0.$$

$$2.22. \lim_{n \rightarrow \infty} \frac{n}{n+1} \cos \frac{2\pi n}{3} \neq 1.$$

3. Sa se calculeze urmatoarele limite:

$$3.1. \lim_{n \rightarrow \infty} \frac{(n+2)^2 + (n-1)^2}{(2n-1)^2 + (n+1)^2}.$$

$$3.2. \lim_{n \rightarrow \infty} \frac{(n+1)^2 - (n-4)^2}{(3n+1)^2 + (n-1)^2}.$$

$$3.3. \lim_{n \rightarrow \infty} \frac{(2-n)^2 - (1+n)^2}{(n-3)^2 - (n+2)^2}.$$

$$3.4. \lim_{n \rightarrow \infty} \frac{(2n-1)^2 - (n-1)^2}{(n+1)^2 + (n-1)^2}.$$

$$3.5. \lim_{n \rightarrow \infty} \frac{(1+2n)^3 - 8n^3}{(1-3n)^2 - 3n^2}.$$

$$3.6. \lim_{n \rightarrow \infty} \frac{(n+3)^3 + (n-1)^3}{2n^3 + 3n}.$$

$$3.7. \lim_{n \rightarrow \infty} \frac{(n+5)^2 + (n+2)^2}{(n+2)^3 - (n+1)^3}.$$

$$3.8. \lim_{n \rightarrow \infty} \frac{(2n+1)^2 + (1-3n)^2}{(n-2)^3 - (n-1)^3}.$$

$$3.9. \lim_{n \rightarrow \infty} \frac{(n+2)^4 - (n-2)^4}{(n+3)^2 + (n-3)^2}.$$

$$3.10. \lim_{n \rightarrow \infty} \frac{(n+1)^4 - (n-1)^4}{(n+1)^3 + (n-1)^3}.$$

$$3.11. \lim_{n \rightarrow \infty} \frac{(2n+1)! + (2n+2)!}{(2n+3)!}.$$

$$3.12. \lim_{n \rightarrow \infty} \frac{n! + (n+2)!}{(n-1)! + (n+2)!}.$$

$$3.13. \lim_{n \rightarrow \infty} \frac{(n+3)! - (n+1)!}{(n+2)!}.$$

$$3.14. \lim_{n \rightarrow \infty} \frac{(2n-1)! + (2n+1)!}{(2n)!(n+1)}.$$

$$3.15. \lim_{n \rightarrow \infty} \frac{(3n)! + (3n-2)!}{(3n-1)!(2n+1)}.$$

$$3.16. \lim_{n \rightarrow \infty} \frac{(n-1)! + (n-2)!}{(n-3)!(3n^2-1)}.$$

4. Sa se calculeze limitele:

$$4.1. \lim_{n \rightarrow \infty} \sqrt{n+1} \left(\sqrt{n+3} - \sqrt{n+2} \right).$$

$$4.2. \lim_{n \rightarrow \infty} \left(\sqrt{(n-1)(n+4)} - n \right).$$

$$4.3. \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 3n + 2} - n \right).$$

$$4.4. \lim_{n \rightarrow \infty} \left(n + \sqrt[3]{n^2 - n^3} \right).$$

$$4.5. \lim_{n \rightarrow \infty} \left(\sqrt{n^2 + 4n - 2} - \sqrt{n^2 - 2} \right).$$

$$4.6. \lim_{n \rightarrow \infty} \sqrt{n-1} \left(\sqrt{n+1} - \sqrt{n-3} \right).$$

$$4.7. \lim_{n \rightarrow \infty} \left(n\sqrt{n} - \sqrt{n(n^2-1)} \right).$$

$$4.8. \lim_{n \rightarrow \infty} n \left(\sqrt[3]{2+8n^3} - 2n \right).$$

$$4.9. \lim_{n \rightarrow \infty} \sqrt[3]{n} \left(\sqrt[3]{n^2} - \sqrt[3]{n(n+1)} \right).$$

$$4.10. \lim_{n \rightarrow \infty} n^2 \left(\sqrt[3]{n^3+7} - \sqrt[3]{n^3+1} \right).$$

$$4.11. \lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n-1}{n^2} \right).$$

$$4.12. \lim_{n \rightarrow \infty} \left(\frac{2+4+\dots+2n}{n+2} - n \right).$$

$$\begin{array}{ll}
4.13. \quad \lim_{n \rightarrow \infty} \left(\frac{n+2}{1+2+\dots+n} - \frac{3}{2} \right). & 4.14. \quad \lim_{n \rightarrow \infty} \frac{1+3+5+\dots+2n-1}{2+4+6+\dots+2n}. \\
4.15. \quad \lim_{n \rightarrow \infty} \frac{5+10+\dots+5n}{n^2+1}. & 4.16. \quad \lim_{n \rightarrow \infty} \frac{1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1)}{n^3}. \\
4.17. \quad \lim_{n \rightarrow \infty} \frac{1^2+3^2+\dots+(2n-1)^2}{n^3}. & 4.18. \quad \lim_{n \rightarrow \infty} \left(\frac{1^2+2^2+\dots+n^2}{n^2} - \frac{n}{3} \right). \\
4.19. \quad \lim_{n \rightarrow \infty} \frac{3^n - 5^{n+1}}{3^{n+1} + 5^{n+2}}. & 4.20. \quad \lim_{n \rightarrow \infty} \left(\frac{7}{10} + \frac{29}{100} + \dots + \frac{2^n + 5^n}{10^n} \right). \\
4.21. \quad \lim_{n \rightarrow \infty} \frac{4^n + 7^n}{4^n - 7^{n-1}}. & 4.22. \quad \lim_{n \rightarrow \infty} \left(\frac{3}{4} + \frac{5}{16} + \dots + \frac{1+2^n}{4^n} \right). \\
4.23. \quad \lim_{n \rightarrow \infty} \frac{3^n + 5^{-n}}{3^{-n} + 5^n}. & 4.24. \quad \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{5} + \dots + \frac{1}{5^n}}{1 + \frac{1}{7} + \dots + \frac{1}{7^n}}.
\end{array}$$

5. Sa se calculeze limitele:

$$\begin{array}{ll}
5.1. \quad \lim_{n \rightarrow \infty} \left(\frac{2n+3}{2n-1} \right)^n. & 5.2. \quad \lim_{n \rightarrow \infty} \left(\frac{n+2}{n+1} \right)^{1-n}. \\
5.3. \quad \lim_{n \rightarrow \infty} \left(\frac{3n-1}{3n+2} \right)^{2n+1}. & 5.4. \quad \lim_{n \rightarrow \infty} \left(\frac{2n+1}{2n-5} \right)^{\frac{n}{6}}. \\
5.5. \quad \lim_{n \rightarrow \infty} \left(\frac{n^2-1}{n^2+1} \right)^{2n-1}. & 5.6. \quad \lim_{n \rightarrow \infty} \left(\frac{3n^2+2}{3n^2-1} \right)^{\frac{n}{3}}. \\
5.7. \quad \lim_{n \rightarrow \infty} \left(\frac{3n+1}{3n} \right)^{1-n^2}. & 5.8. \quad \lim_{n \rightarrow \infty} \left(\frac{2n^2+1}{2n^2-3} \right)^{1-n^3}. \\
5.9. \quad \lim_{n \rightarrow \infty} \left(\frac{2n^2+2}{2n^2+1} \right)^{n^2}. & 5.10. \quad \lim_{n \rightarrow \infty} \left(\frac{n^2+n+1}{n^2+n-1} \right)^{n^2-1}. \\
5.11. \quad \lim_{n \rightarrow \infty} \left(\frac{3n^2+2}{3n^2-1} \right)^{n^2+1}. & 5.12. \quad \lim_{n \rightarrow \infty} \left(\frac{2n^2+n+2}{2n^2-2n+3} \right)^{2n}. \\
5.13. \quad \lim_{n \rightarrow \infty} \left(\frac{3n^2+6n+7}{3n^2+6n+4} \right)^{6n^2-5n+4}. & 5.14. \quad \lim_{n \rightarrow \infty} \left(\frac{n^2+1}{n^2-1} \right)^{2n^2}. \\
5.15. \quad \lim_{n \rightarrow \infty} \left(\frac{n^2+2n+3}{n^2+3n+4} \right)^{2n-1}. & 5.16. \quad \lim_{n \rightarrow \infty} \left(\frac{4n^2+2}{4n^2-2} \right)^{n^2}. \\
5.17. \quad \lim_{n \rightarrow \infty} \left(\frac{n^2+n-1}{3n^2-n+1} \right)^{\frac{1}{n}}. & 5.18. \quad \lim_{n \rightarrow \infty} \left(\frac{1+2n}{3n-1} \right)^n. \\
5.19. \quad \lim_{n \rightarrow \infty} \left(\frac{n+1}{n-1} \right)^{\frac{2n^2+1}{2n^2-1}}. & 5.20. \quad \lim_{n \rightarrow \infty} \left(\frac{n^2-1}{2n^2+1} \right)^{\frac{2n}{n+1}}.
\end{array}$$