

Limite de functii

1. Sa se calculeze urmatoarele limite:

$$1.1. \quad \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + x - 6}.$$

$$1.2. \quad \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 + 6x - 7}.$$

$$1.3. \quad \lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - x - 6}.$$

$$1.4. \quad \lim_{x \rightarrow 4} \frac{x^2 - 7x + 12}{x^2 - 6x + 8}.$$

$$1.5. \quad \lim_{x \rightarrow 0} \frac{x^3 - 6x^2 + 7x}{x^2 + x}.$$

$$1.6. \quad \lim_{x \rightarrow -2} \frac{x^2 + x - 2}{x^2 - x - 6}.$$

$$1.7. \quad \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}.$$

$$1.8. \quad \lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1}, \quad m, n \in \mathbb{N}.$$

$$1.9. \quad \lim_{x \rightarrow 0} \frac{(x+2)(1-x)(2x+1) - 2}{x^2 + x}.$$

$$1.10. \quad \lim_{x \rightarrow 2} \frac{x^4 - 5x^2 + 4}{x^4 - 3x^2 - 4}.$$

$$1.11. \quad \lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - x - 2}{x^3 - 7x - 6}.$$

$$1.12. \quad \lim_{x \rightarrow -1} \frac{x^4 + x^2 - 2}{x^4 - 1}.$$

$$1.13. \quad \lim_{x \rightarrow -1} \frac{x^3 + 2x + 3}{x^3 + 1}.$$

$$1.14. \quad \lim_{x \rightarrow 2} \frac{x^4 - 2x^3 - 3x^2 + 4x + 4}{x^4 - 6x^3 + 13x^2 - 12x + 4}.$$

$$1.15. \quad \lim_{x \rightarrow 1} \frac{2x^4 - x^2 - 1}{x^4 - 1}.$$

$$1.16. \quad \lim_{x \rightarrow 0} \frac{(x+1)^3 - (3x+1)}{2x^4 + x^2}.$$

$$1.17. \quad \lim_{x \rightarrow -1} \frac{x^3 + 3x^2 + 7x + 5}{x^3 - x^2 - x + 1}.$$

$$1.18. \quad \lim_{x \rightarrow 2} \frac{x^3 - 3x - 2}{x^2 - 4}.$$

$$1.19. \quad \lim_{x \rightarrow -1} \frac{(x^3 - 2x - 1)^2}{x^4 - 2x^2 + 1}.$$

$$1.20. \quad \lim_{x \rightarrow 0} \frac{(x+2)^3 - 8}{(x+1)^4 - (1+2x)}.$$

2. Sa se calculeze urmatoarele limite:

$$2.1. \quad \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^2 + x - 2}.$$

$$2.2. \quad \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{\sqrt{x+2} - 2}.$$

$$2.3. \quad \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + x + 4} - 2}{\sqrt{1-x} + x^2 - 1}.$$

$$2.4. \quad \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{\sqrt{4+3x} - 4}.$$

$$2.5. \quad \lim_{x \rightarrow 0} \frac{\sqrt{4-x+x^2} - (2+x)}{x^2 + x}.$$

$$2.6. \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{2+x} - \sqrt{2-x}}.$$

$$2.7. \quad \lim_{x \rightarrow 5} \frac{\sqrt{x+4} - \sqrt{2x-1}}{x^2 - 25}.$$

$$2.8. \quad \lim_{x \rightarrow 0} \frac{3\sqrt{x^2 + x + 1} - (3+x)}{x^2 + 3x}.$$

$$2.9. \quad \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}.$$

$$2.10. \quad \lim_{x \rightarrow 4} \frac{\sqrt[3]{16x} - 4}{\sqrt{x+4} - \sqrt{2x}}.$$

$$\begin{array}{ll}
2.11. \lim_{x \rightarrow 2} \frac{\sqrt[3]{x-1} - 1}{x^3 - 8}. & 2.12. \lim_{x \rightarrow -2} \frac{x^3 + 8}{\sqrt[3]{x-6} + 2}. \\
2.13. \lim_{x \rightarrow 1} \frac{\sqrt{x+2} - \sqrt{3x}}{\sqrt[3]{x} - 1}. & 2.14. \lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - 2}{\sqrt{x+1} - 3}. \\
2.15. \lim_{x \rightarrow 0} \frac{\sqrt[3]{2+x} - \sqrt[3]{2-x}}{\sqrt{2+x} - \sqrt{2-x}}. & 2.16. \lim_{x \rightarrow 1} \frac{\sqrt{x} + \sqrt{x-1} - 1}{\sqrt{x^2-1}}. \\
2.17. \lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - 2}{x - 8}. & 2.18. \lim_{x \rightarrow -8} \frac{\sqrt[3]{15+2x} + 1}{\sqrt[3]{9+x} + x + 7}. \\
2.19. \lim_{x \rightarrow 7} \frac{\sqrt{x+2} - \sqrt[3]{x+20}}{\sqrt[4]{x+9} - 2}. & 2.20. \lim_{x \rightarrow 0} \frac{\sqrt[5]{2x^2+10x+1} - \sqrt[7]{x^2+10x+1}}{x}.
\end{array}$$

3. Sa se calculeze urmatoarele limite:

$$\begin{array}{ll}
3.1. \lim_{x \rightarrow \infty} (\sqrt{x^2+1} - \sqrt{x^2-1}). & 3.2. \lim_{x \rightarrow \infty} (\sqrt{9x^4+3x^2-7} - 3x^2). \\
3.3. \lim_{x \rightarrow \infty} (\sqrt{x^2+2x-1} - \sqrt{x^2-2x-1}). & 3.4. \lim_{x \rightarrow \infty} (\sqrt{x^4+x^2} - \sqrt{x^4+8x^2+3}). \\
3.5. \lim_{x \rightarrow \infty} (\sqrt{x^2-3x+2} - x). & 3.6. \lim_{x \rightarrow \infty} (\sqrt{x^2+2x} - \sqrt{x^2+2x+3}). \\
3.7. \lim_{x \rightarrow \infty} (x\sqrt[3]{8x^3+5} - 2x). & 3.8. \lim_{x \rightarrow \infty} \sqrt{x^3+8} (\sqrt{x^3+2} - \sqrt[3]{x^3-1}). \\
3.9. \lim_{x \rightarrow \infty} x\sqrt{x} (x - \sqrt[3]{x^3-5}). & 3.10. \lim_{x \rightarrow \infty} x\sqrt{x} (\sqrt{x^4+3} - \sqrt{x^4+2}). \\
3.11. \lim_{x \rightarrow \infty} \sqrt{x} (\sqrt{x+2} - \sqrt{x+3}). & 3.12. \lim_{x \rightarrow \infty} (x - \sqrt{x^2-x}). \\
3.13. \lim_{x \rightarrow \infty} \left(\frac{x^3}{2x^2-1} - \frac{x^2}{2x+1} \right). & 3.14. \lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{2}{1-x^2} \right). \\
3.15. \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{3}{x^3-1} \right). & 3.16. \lim_{x \rightarrow 2} \left(\frac{1}{(x-2)(x-1)} - \frac{2}{x^2-2x} \right). \\
3.17. \lim_{x \rightarrow -1} \left(\frac{2}{x+1} - \frac{x-3}{x^2-1} \right). & 3.18. \lim_{x \rightarrow 2} \left(\frac{1}{x(x-2)^2} - \frac{1}{x^2-3x+2} \right).
\end{array}$$

4. Sa se calculeze urmatoarele limite:

$$\begin{array}{ll}
4.1. \lim_{x \rightarrow 0} \frac{\sin 5x}{x}. & 4.2. \lim_{x \rightarrow 0} \frac{\sin 8x + \sin 6x}{2x}. \\
4.3. \lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x}. & 4.4. \lim_{x \rightarrow 0} \frac{\cos 5x - \cos 3x}{4x^2}. \\
4.5. \lim_{x \rightarrow 0} \frac{\sin^2 2x}{\sin^2 3x}. & 4.6. \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 8x}. \\
4.7. \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}. & 4.8. \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{2x \sin x}.
\end{array}$$

$$4.9. \quad \lim_{x \rightarrow \pi} \frac{\sin 2x}{\sin 3x}.$$

$$4.11. \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{tg} 5x}{\operatorname{tg} 3x}.$$

$$4.13. \quad \lim_{x \rightarrow \pi} \frac{\sin 2x}{\operatorname{tg} 3x}.$$

$$4.15. \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - 2 \cos x}{\pi - 4x}.$$

$$4.17. \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{2x^3}.$$

$$4.19. \quad \lim_{x \rightarrow 0} \frac{\sqrt{1 + x \sin x} - 1}{x^2}.$$

$$4.21. \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg}(\sin x) - \sin(\operatorname{tg} x)}{x^3}.$$

$$4.10. \quad \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{\pi - 6x}.$$

$$4.12. \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \operatorname{tg}^2 x}{\sqrt{2} \cos x - 1}.$$

$$4.14. \quad \lim_{x \rightarrow -\frac{\pi}{4}} \frac{1 + \sin 2x}{1 + \cos 4x}.$$

$$4.16. \quad \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \operatorname{ctg} x \right).$$

$$4.18. \quad \lim_{x \rightarrow \frac{\pi}{6}} \frac{6 \sin^2 x - 5 \sin x + 1}{4 \sin^2 x - 1}.$$

$$4.20. \quad \lim_{x \rightarrow \pi} \frac{\sqrt{1 - \operatorname{tg} x} - \sqrt{1 + \operatorname{tg} x}}{\sin 2x}.$$

$$4.22. \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg}(\operatorname{tg} x) - \sin(\sin x)}{\operatorname{tg} x - \sin x}.$$

5. Sa se calculeze urmatoarele limite:

$$5.1. \quad \lim_{x \rightarrow \infty} \left(\frac{x+2}{x-3} \right)^{2x-1}.$$

$$5.2. \quad \lim_{x \rightarrow \infty} \left(\frac{x^2+4}{x^2-4} \right)^{x^2}.$$

$$5.3. \quad \lim_{x \rightarrow \infty} \left(\frac{2x+1}{2x+3} \right)^{\frac{x}{2}}.$$

$$5.4. \quad \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x}+3}{\sqrt{x}+2} \right)^{\frac{1-x}{1-\sqrt{x}}}.$$

$$5.5. \quad \lim_{x \rightarrow 0} (1 + 5x)^{\frac{1}{x}}.$$

$$5.6. \quad \lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{\sin 2x}}.$$

$$5.7. \quad \lim_{x \rightarrow 0} (1 + 2 \operatorname{tg}^2 x)^{\operatorname{ctg}^2 x}.$$

$$5.8. \quad \lim_{x \rightarrow 0} (\cos 2x)^{\frac{1}{x^2}}.$$

$$5.9. \quad \lim_{x \rightarrow 0} (\cos x + \sin x)^{\frac{1}{x}}.$$

$$5.10. \quad \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}}.$$

$$5.11. \quad \lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\operatorname{tg}^2 x}.$$

$$5.12. \quad \lim_{x \rightarrow \frac{\pi}{2}} (1 + \operatorname{ctg} x)^{\operatorname{tg} x}.$$

$$5.13. \quad \lim_{x \rightarrow \frac{\pi}{2}} \left(\operatorname{ctg} \frac{x}{2} \right)^{\frac{1}{\cos x}}.$$

$$5.14. \quad \lim_{x \rightarrow 1} (2 - x)^{\operatorname{tg} \frac{\pi x}{2}}.$$

$$5.15. \quad \lim_{x \rightarrow 0} \left(4 - \frac{3}{\cos x} \right)^{\operatorname{tg}^2 x}.$$

$$5.16. \quad \lim_{x \rightarrow 0} \left[\operatorname{tg} \left(\frac{\pi}{4} - x \right) \right]^{\operatorname{ctg} x}.$$

6. Sa se calculeze limitele:

$$6.1. \quad \lim_{x \rightarrow 0} \frac{\ln(1 + 2x^2)}{\sqrt{1 + x^2} - 1}.$$

$$6.2. \quad \lim_{x \rightarrow 0} \frac{\ln(1 + \sin 2x)}{\sin 4x - \sin 2x}.$$

$$6.3. \quad \lim_{x \rightarrow 0} \frac{3^x - 1}{\ln(1 + 2x)}.$$

$$6.4. \quad \lim_{x \rightarrow 0} \frac{\arcsin 2x}{\operatorname{arctg} 4x}.$$

$$6.5. \quad \lim_{x \rightarrow 0} \frac{\ln(1+2x)}{\operatorname{arctg} 3x}.$$

$$6.6. \quad \lim_{x \rightarrow 0} \frac{3^x - 2^x}{2x - \operatorname{arctg} x}.$$

$$6.7. \quad \lim_{x \rightarrow 0} \frac{2^{3x} - 3^{2x}}{2 \arcsin x - \sin x}.$$

$$6.8. \quad \lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x + \sin x^2}.$$

$$6.9. \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+x \sin x} - 1}{e^{x^2} - 1}.$$

$$6.10. \quad \lim_{x \rightarrow 2} \frac{x^2 - 4}{\ln(x-1)}.$$

$$6.11. \quad \lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt[4]{x} - 1}.$$

$$6.12. \quad \lim_{x \rightarrow -1} \frac{3 - \sqrt{10+x}}{\sin 3\pi x}.$$

$$6.13. \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{2^{\cos^2 x} - 1}{\ln \sin x}.$$

$$6.14. \quad \lim_{x \rightarrow \frac{\pi}{6}} \frac{\ln \sin 3x}{(6x - \pi)^2}.$$

$$6.15. \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x - 3 \arcsin x}{\sin 6x - 6 \operatorname{arctg} 2x}.$$

$$6.16. \quad \lim_{x \rightarrow \infty} x \left(2^{\frac{1}{x}} - 1 \right).$$

$$6.17. \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln \sin 5x}{\ln \sin 3x}.$$

$$6.18. \quad \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - 1 - \sin x}{\ln(1+x)}.$$

$$6.19. \quad \lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos x} - \sqrt[4]{\cos 2x}}{1 - \cos 12x}.$$

$$6.20. \quad \lim_{x \rightarrow \frac{1}{4}} \frac{1 - \operatorname{ctg} \pi x}{\ln \operatorname{tg} \pi x}.$$

$$6.21. \quad \lim_{x \rightarrow 0} (\cos 2x)^{-\frac{1}{x^2}}.$$

$$6.22. \quad \lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{\sin^2 x}.$$