

Integrale Riemann

1. Sa se calculeze:

1.1. $\int_{-1}^2 x^2 dx.$

1.2. $\int_{-1}^2 \sqrt[3]{x} dx.$

1.3. $\int_{-1}^1 (4x^3 - 3x^2 + 2x - 1) dx.$

1.4. $\int_1^3 (x^2 + x - 2) dx.$

1.5. $\int_0^{\frac{\pi}{2}} \sin x dx.$

1.6. $\int_0^{\pi} \cos x dx.$

1.7. $\int_0^{\frac{\pi}{4}} \frac{x^2}{1+x^2} dx.$

1.8. $\int_0^1 \frac{dx}{\sqrt{x^2+1}}.$

1.9. $\int_0^1 e^x dx.$

1.10. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}.$

1.11. $\int_{-\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}.$

1.12. $\int_{\frac{1}{e}}^{e^2} \frac{dx}{x}.$

1.13. $\int_0^1 \frac{dx}{1+x^2}.$

1.14. $\int_0^{\frac{\pi}{2}} x \sin x dx.$

1.15. $\int_1^e x \ln x dx.$

1.16. $\int_2^3 \frac{x+1}{x^2(x-1)} dx.$

1.17. $\int_1^2 \frac{e^{\frac{1}{x^2}}}{x^3} dx.$

1.18. $\int_0^1 x e^x dx.$

1.19. $\int_{-\pi}^{\pi} \sin^2 x dx.$

1.20. $\int_{-\pi}^{\pi} \cos^2 x dx.$

1.21. $\int_e^{e^3} \frac{dx}{x \ln x}.$

1.22. $\int_0^{\frac{\pi}{4}} \operatorname{tg}^3 x dx.$

2. Sa se calculeze ariile plane limitate de curbele:

2.1. $f(x) = 3x - x^2, \quad g(x) = 0.$

2.2. $f(x) = 4x - x^2, \quad g(x) = 0.$

2.3. $f(x) = x^2 + 1, \quad g(x) = 2.$

2.4. $f(x) = x^2, \quad g(x) = 4.$

2.5. $f(x) = x^2, \quad g(x) = x + 2.$

2.6. $f(x) = x^2 - x, \quad g(x) = 3x.$

2.7. $f(x) = 2x - x^2, \quad g(x) = x.$

2.8. $f(x) = (x - 1)^2 + 2, \quad g(x) = 3x - 1.$

2.9. $f(x) = x^2, \quad g(x) = 2x - x^2.$

2.10. $f(x) = x^2, \quad g(x) = 3x + 4.$

2.11. $f(x) = x^3, \quad g(x) = \sqrt{x}.$

2.12. $f(x) = \frac{5}{x}, \quad g(x) = 6 - x.$

2.13. $f(x) = x^2, \quad g(x) = \sqrt[3]{x}.$

2.14. $f(x) = x^2, \quad g(x) = 2\sqrt{2x}.$

2.15. $f(x) = -\sqrt{x}, \quad g(x) = \sqrt{x}, \quad x \in [0, 4].$

2.16. $f(x) = e^x, \quad g(x) = e^{-x}, \quad x \in [0, 1].$

2.17. $f(x) = \ln x, \quad g(x) = \ln^2 x.$

2.18. $f(x) = \frac{1}{4}|4 - x^2|, \quad g(x) = 7 - |x|.$

2.19. $f(x) = 0, \quad g(x) = -x + 2, \quad h(x) = \sqrt{x}.$

2.20. $f(x) = \frac{1}{x}, \quad g(x) = x, \quad x = 2.$

2.21. $f(x) = \sin x, \quad g(x) = \cos x, \quad x \in \left[0, \frac{\pi}{4}\right].$

2.22. $f(x) = x - \frac{\pi}{2}, \quad g(x) = \cos x, \quad x = 0.$

2.23. $f(x) = \sin^2 x, \quad g(x) = x \sin x, \quad x \in [0, \pi].$

2.24. $f(x) = \sin 2x, \quad g(x) = \sin x, \quad x \in \left[\frac{\pi}{3}, \pi\right].$

$$2.25. \quad f(x) = \operatorname{tg} x, \quad g(x) = \frac{2}{3} \cos x, \quad x = 0.$$

$$2.26. \quad f(x) = \arcsin x, \quad g(x) = \arccos x, \quad h(x) = 0.$$

$$2.27. \quad f(x) = 2^{x-2} + 1, \quad g(x) = 2^{2-x} + 1, \quad h(x) = \frac{3}{2}.$$

$$2.28. \quad f(x) = 2 - |2 - x|, \quad g(x) = \frac{6}{|x + 1|}.$$

$$2.29. \quad f(x) = |\lg x|, \quad g(x) = 0, \quad x = \frac{1}{10}, \quad x = 10.$$

$$2.30. \quad f(x) = \ln |1 + x|, \quad g(x) = -xe^{-x}, \quad x = 1.$$

3. Sa se calculeze ariile plane limitate de curbele:

$$3.1. \quad \rho^2 = a^2 \cos 2\varphi.$$

$$3.2. \quad x = a \cos t, \quad y = b \sin t.$$

$$3.3. \quad \rho = 4 \sin^2 \varphi.$$

$$3.4. \quad x = a \cos^3 t, \quad y = a \sin^3 t.$$

$$3.5. \quad \rho = a(1 + \cos \varphi).$$

$$3.6. \quad x = \frac{c^2}{a} \cos^3 t, \quad y = \frac{c^2}{b} \sin^3 t, \quad c^2 = a^2 - b^2.$$

$$3.7. \quad \rho = 2 + \cos \varphi.$$

$$3.8. \quad x = \frac{1 - t^2}{(1 + t^2)^2}, \quad y = \frac{2at}{(1 + t^2)^2}.$$

$$3.9. \quad \rho = a \sin 2\varphi.$$

$$3.10. \quad x = t - t^2, \quad y = t^2 - t^3.$$

$$3.11. \quad \rho = a \cos \varphi, \quad \rho = a(\cos \varphi + \sin \varphi).$$

$$3.12. \quad x = t^2 - 1, \quad y = t^3 - t^2.$$

$$3.13. \quad \rho = 2 - \cos \varphi, \quad \rho = \cos \varphi.$$

$$3.14. \quad x = \frac{t - t^3}{1 + 3t^2}, \quad y = \frac{4t^2}{1 + 3t^2}.$$

$$3.15. \quad \rho = 2\sqrt{3} \cos \varphi, \quad \rho = 2 \sin \varphi.$$

$$3.16. \quad x = \sin 2t, \quad y = \sin t.$$

$$3.17. \quad \rho = 1 + \sqrt{2} \cos \varphi.$$

$$3.18. \quad x = 1 + t - t^3, \quad y = 1 - 15t^2.$$

$$3.19. \quad \rho = 3 \sin \varphi, \quad \rho = 5 \sin \varphi.$$

$$3.20. \quad x = 1 + 2 \cos t, \quad y = \operatorname{tg} t + 2 \sin t.$$

4. Sa se calculeze lungimile arcelor:

$$4.1. \quad f(x) = \frac{(x+1)^2}{4} - \frac{\ln(x+1)}{2}, \quad x \in [0, 1].$$

$$4.2. \quad f(x) = -\ln \cos x, \quad x \in \left[0, \frac{\pi}{6}\right].$$

$$4.3. \quad f(x) = \ln x, \quad x \in [\sqrt{3}, \sqrt{8}].$$

$$4.4. \quad f(x) = \ln(x^2 - 1), \quad x \in [2, 3].$$

$$4.5. \quad f(x) = \sqrt{2x - x^2} - 1, \quad x \in \left[\frac{1}{4}, 1\right].$$

$$4.6. \quad f(x) = x^2, \quad x \in [0, 1].$$

$$4.7. \quad f(x) = 4\sqrt{x-1}, \quad x \in [1, 2].$$

$$4.8. \quad f(x) = x^2 - \ln \sqrt{x}, \quad x \in [1, 2].$$

$$4.9. \quad f(x) = x\sqrt{x}, \quad x \in [0, 9].$$

$$4.10. \quad f(x) = \ln \sin x, \quad x \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right].$$

$$4.11. \quad x = a \cos^3 t, \quad y = a \sin^3 t, \quad t \in [0, 2\pi].$$

$$4.12. \quad \rho = 2 \sin \varphi.$$

$$4.13. \quad x = 3(2 - t^2), \quad y = 4t^3, \quad x \geq 0.$$

$$4.14. \quad \rho = \cos^3 \frac{\varphi}{3}.$$

$$4.15. \quad x = \cos^4 t, \quad y = \sin^4 t, \quad t \in \left[0, \frac{\pi}{2}\right].$$

$$4.16. \quad \rho = a(1 - \cos \varphi).$$

$$4.17. \quad x = 6 \cos^3 t, \quad y = 6 \sin^3 t, \quad t \in \left[0, \frac{\pi}{3}\right].$$

$$4.18. \quad \rho = \sin 3\varphi.$$

$$4.19. \quad x = 2(t - \sin t), \quad y = 2(1 - \cos t), \quad t \in \left[0, \frac{\pi}{2}\right].$$

$$4.20. \quad \rho = \frac{1}{2} + \sin \varphi.$$

$$4.21. \quad x = e^t(\cos t + \sin t), \quad y = e^t(\cos t - \sin t), \quad t \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right].$$

$$4.22. \quad \rho = \cos \varphi - \sin \varphi.$$

$$4.23. \quad x = 2(\cos t + t \sin t), \quad y = 2(\sin t - t \cos t), \quad t \in [0, \pi].$$

$$4.24. \quad \rho = 2 \sin 4\varphi.$$

5. Sa se calculeze volumul corpului obtinut prin rotatia in jurul axei OX a suprafetei marginite de curbele:

$$5.1. \quad f(x) = -x^2 + 7x - 12, \quad g(x) = 0.$$

$$5.2. \quad f(x) = \frac{4}{x}, \quad g(x) = 0, \quad x = 1, \quad x = 4.$$

$$5.3. \quad f(x) = 2x + \sqrt{2x}, \quad g(x) = 0, \quad x = 2 \quad x = \frac{9}{2}.$$

$$5.4. \quad f(x) = 2x - x^2, \quad g(x) = 2 - x.$$

$$5.5. \quad f(x) = \arcsin x, \quad x = 0, \quad x = 1.$$

$$5.6. \quad f(x) = xe^x, \quad g(x) = 0, \quad x = 1.$$

$$5.7. \quad f(x) = x^2, \quad g(x) = 0, \quad x = 3.$$

$$5.8. \quad f(x) = (x - 2)^2, \quad g(x) = 4.$$

$$5.9. \quad f(x) = e^{2-x}, \quad g(x) = 0, \quad x = 1, \quad x = 2.$$

$$5.10. \quad f(x) = e^x, \quad g(x) = 0, \quad x = 0, \quad x = 1.$$

$$5.11. \quad f(x) = 3 \sin x, \quad g(x) = \sin x, \quad x = 0, \quad x = \pi.$$

$$5.12. \quad f(x) = \sin x, \quad g(x) = 0, \quad x = \frac{\pi}{6}, \quad x = \frac{\pi}{2}.$$

$$5.13. \quad f(x) = 4 - x^2, \quad g(x) = 3x, \quad x = -2, \quad x = 0.$$

$$5.14. \quad f(x) = \sqrt{x}e^{-x}, \quad g(x) = 0, \quad x = 1.$$

$$5.15. \quad f(x) = \sin^2 x, \quad g(x) = x \sin x, \quad x = 0, \quad x = \pi.$$

$$5.16. \quad f(x) = \sin 2x, \quad g(x) = 0, \quad x = 0, \quad x = \frac{\pi}{4}.$$

$$5.17. \quad f(x) = 3x - x^2, \quad g(x) = 0.$$

$$5.18. \quad x = a \cos^3 t, \quad y = a \sin^3 t.$$

$$5.19. \quad f(x) = x^2, \quad g(x) = \sqrt{x}.$$

$$5.20. \quad f(x) = x^3, \quad g(x) = x^2.$$