Derivabilitate

1. Sa se calculeze derivata functiei:

1.1. $f(x) = x^3 + x^2 - x + 1$.

1.2. $f(x) = \frac{1}{4}\, x^4 - \frac{1}{3}\, x^3 + 2x^2 - 1$.

1.3. $f(x) = 2x^5 - x^{-2} + 3x$.

1.4. $f(x) = \frac{1}{x} - \frac{4}{x^2} - \frac{1}{x^3} + x$.

1.5. $f(x) = x^{\frac{1}{2}} + x^{\frac{2}{3}} - x^{-\frac{1}{3}}$.

1.6. $f(x) = \sqrt[3]{x} - \frac{1}{\sqrt[3]{x}} + 1$.

1.7. $f(x) = e^x \sin x$.

1.8. $f(x) = \operatorname{tg} x \ln x$.

1.9. $f(x) = 2^x \operatorname{ctg} x$.

1.10. $f(x) = x \arcsin x$.

1.11. $f(x) = (x^2 + 1) \operatorname{arctg} x$.

1.12. $f(x) = x^2 \ln x$.

1.13. $f(x) = \cos x \ln x$.

1.14. $f(x) = x \operatorname{arcctg} x$.

1.15. $f(x) = \frac{x}{x^2 - 1}$.

1.16. $f(x) = \frac{x^2 - 1}{x^2 + 1}$.

1.17. $f(x) = \frac{\sin x}{\ln x}$.

1.18. $f(x) = \frac{\operatorname{arctg} x}{e^x}$.

1.19. $f(x) = \frac{\sin x - \cos x}{\sin x + \cos x}$.

1.20. $f(x) = \frac{1 - \sin x}{1 + \sin x}$.

1.21. $f(x) = \ln 3 - \cos 2$.

1.22. $f(x) = \arcsin x + \arccos x$.

2. Sa se calculeze derivata functiei:

2.1. $f(x) = (x^2 + 1)^{10}$.

2.2. $f(x) = \frac{1}{(x^2 + 2x + 3)^3}$.

2.3. $f(x) = \sqrt{x^2 - x + 7}$.

2.4. $f(x) = \frac{1}{\sqrt[3]{x^3 + x^2 + 1}}$.

2.5. $f(x) = \sin^2 x$.

2.6. $f(x) = \ln^2 x$.

2.7. $f(x) = \sin 3x$.

2.8. $f(x) = \sin(\ln x)$.

2.9. $f(x) = \cos 2x$.

2.10. $f(x) = \cos(e^x)$.

2.11. $f(x) = \operatorname{tg} 3x$.

2.12. $f(x) = \operatorname{tg} 2^x$.

2.13. $f(x) = \operatorname{ctg} \frac{x}{2}$.

2.14. $f(x) = \operatorname{ctg}(x^2 + x + 1)$.

2.15. $f(x) = e^{\sin x}$.

2.16. $f(x) = e^{-x}$.

2.17. $f(x) = 2^{\operatorname{tg} x}$.

2.18. $f(x) = 3^{\sqrt{x}}$.

2.19. $f(x) = \ln(\sin x)$.

2.20. $f(x) = \ln(\operatorname{arctg} x)$.

2.21. $f(x) = \operatorname{arctg} \sqrt{x}$. 2.22. $f(x) = \operatorname{arctg}\, e^x$.

2.23. $f(x) = \arcsin \sqrt{x}$. 2.24. $f(x) = \arcsin e^{-x}$.

3. Sa se calculeze derivata functiei:

3.1. $f(x) = \ln \operatorname{tg} \dfrac{x}{2}$. 3.2. $f(x) = \ln \left(x + \sqrt{x^2+1}\right)$.

3.3. $f(x) = \ln \sqrt[4]{\dfrac{1-\sin x}{1+\sin x}}$. 3.4. $f(x) = \ln \dfrac{x^2-1}{x^2+1}$.

3.5. $f(x) = \ln \sin \dfrac{2x+4}{x+1}$. 3.6. $f(x) = \ln \operatorname{tg} \left(\dfrac{x}{2} + \dfrac{\pi}{4}\right)$.

3.7. $f(x) = \operatorname{arctg} \sqrt{4x-1}$. 3.8. $f(x) = \sqrt{x} - \operatorname{arctg} \sqrt{x}$.

3.9. $f(x) = \operatorname{arctg} \dfrac{1+x}{1-x}$. 3.10. $f(x) = \operatorname{arctg} \dfrac{x}{1+\sqrt{1+x^2}}$.

3.11. $f(x) = \arcsin \dfrac{1-x}{\sqrt{2}}$. 3.12. $f(x) = \arcsin \sqrt{1-x^2}$.

3.13. $f(x) = \arccos \dfrac{1-x^2}{1+x^2}$. 3.14. $f(x) = \cos(2 \arccos x)$.

3.15. $f(x) = e^{\sqrt{\frac{1+x}{1-x}}}$. 3.16. $f(x) = e^{\operatorname{tg} \frac{1}{x}}$.

3.17. $f(x) = \operatorname{tg}^2 x + \ln \cos^2 x$. 3.18. $f(x) = \operatorname{arcctg}\left(\operatorname{ctg}^2 x\right)$.

3.19. $f(x) = \sqrt{2x^2 + \sqrt{x^2+1}}$. 3.20. $f(x) = \sqrt{2+x^2}\sqrt[3]{3+x^3}$.

4. Sa se calculeze derivata functiei:

4.1. $f(x) = \ln\left(2x - 3 + \sqrt{4x^2-12x+10}\right) - \operatorname{arctg}(2x-3)\sqrt{4x^2-12x+10}$.

4.2. $f(x) = x^2\sqrt{x^4+1} + \ln\left(x^2 + \sqrt{x^4+1}\right)$.

4.3. $f(x) = x + e^{-x} \operatorname{arctg} e^x - \ln \sqrt{1+e^{2x}}$.

4.4. $f(x) = \sqrt{49x^2+1} \operatorname{arctg} 7x - \ln\left(7x + \sqrt{49x^2+1}\right)$.

4.5. $f(x) = \arcsin e^{-2x} + \ln\left(e^{2x} + \sqrt{e^{4x}-1}\right)$.

4.6. $f(x) = \dfrac{3-\sin x}{2}\sqrt{\cos^2 x - 2\sin x} + 2\arcsin \dfrac{1+\sin x}{\sqrt{2}}$.

4.7. $f(x) = \operatorname{arctg} \sqrt{e^x} + e^x \arcsin \sqrt{\dfrac{e^x}{e^x+1}} - \sqrt{e^x}$.

4.8. $f(x) = 2\sqrt{3} \operatorname{arctg} \dfrac{\sqrt{3}}{1-2x^2} + \ln \dfrac{x^4-x^2+1}{x^4+2x^2+1}$.

4.9. $f(x) = \ln \dfrac{2\,(x^2+2x+2)}{2x^2+2x+1} + 4\operatorname{arctg}(x+1) - \operatorname{arctg}(2x+1).$

4.10. $f(x) = \dfrac{5x+2}{x^2+x+1} + \ln \sqrt[3]{\dfrac{(x-1)^2}{x^2+x+1}} + \dfrac{8}{\sqrt{3}}\ \operatorname{arctg}\dfrac{2x+1}{\sqrt{3}}.$

4.11. $f(x) = x \ln\left(\sqrt{1-x} + \sqrt{1+x}\right) + \dfrac{1}{2}(\arcsin x - x).$

4.12. $f(x) = (3x-2)^4 \arcsin\dfrac{1}{3x-2} + \left(3x^2-4x+2\right)\sqrt{9x^2-12x+3}.$

4.13. $f(x) = e^{2\arcsin x}\left[\cos(2\arcsin x) + \sin(2\arcsin x)\right].$

4.14. $f(x) = \sqrt{1+\sqrt[3]{1+\sqrt[4]{1+x^4}}}.$

4.15. $f(x) = \dfrac{2}{3x-2}\sqrt{12x-9x^2-3} + \ln\dfrac{1+\sqrt{12x-9x^2-3}}{3x-2}.$

4.16. $f(x) = x\,(2x^2+5)\sqrt{x^2+1} + 3\ln\left(x+\sqrt{x^2+1}\right).$

4.17. $f(x) = \sqrt{x^2+5x+4} + 3\ln\left(\sqrt{x+4}+\sqrt{x+1}\right).$

4.18. $f(x) = \dfrac{x\arcsin 2x}{\sqrt{1-4x^2}} + \ln\sqrt{1-4x^2}.$

4.19. $f(x) = \dfrac{1}{4\sqrt{3}}\ln\dfrac{\sqrt{x^2+2}-x\sqrt{3}}{\sqrt{x^2+2}+x\sqrt{3}} + \dfrac{1}{2}\operatorname{arctg}\dfrac{\sqrt{x^2+2}}{x}.$

4.20. $f(x) = \dfrac{\cos x}{3(2+\sin x)} + \dfrac{4}{3\sqrt{3}}\operatorname{arctg}\dfrac{2\operatorname{tg}\frac{x}{2}+1}{\sqrt{3}}.$

4.21. $f(x) = \dfrac{1}{\cos x} + \dfrac{1}{3\cos^3 x} - \dfrac{1}{2}\ln\dfrac{1+\cos x}{1-\cos x}.$

4.22. $f(x) = 2\sqrt{1-x^2}\arcsin x - 2x + x(\arcsin x)^2.$

4.23. $f(x) = \dfrac{\ln(1+\sin x)}{\operatorname{tg} x} + x - \ln\operatorname{tg}\dfrac{x}{2}.$

4.24. $f(x) = \log_{\frac{1}{2}}\left(x+\dfrac{1}{2}\right)^2 + \log_2\sqrt{4x^2+4x+1}.$

4.25. $f(x) = x^x.$

4.26. $f(x) = \sin x^{\cos x}.$

4.27. $f(x) = x + x^x + x^{x^x}.$

4.28. $f(x) = x^{e^x}.$

4.29. $f(x) = x^{e^{\sin x}}.$

4.30. $f(x) = x^{3^x}2^x.$

5. Sa se studieze derivabilitatea urmatoarelor functii:

5.1. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\left|x^3-4x\right|$.

5.2. $f:\left(-\frac{1}{3},\infty\right)\longrightarrow\mathbb{R}$, $f(x)=\begin{cases}\ln(1+3x), & \text{dacă} \quad -\dfrac{1}{3}<x\leq 0\\ 3x, & \text{dacă} \quad x>0.\end{cases}$

5.3. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\begin{cases}\sin^3 x\operatorname{sgn}x, & \text{dacă} \quad |x|\leq\frac{\pi}{4}\\ \dfrac{3\sqrt{2}}{4}x\operatorname{sgn}x-\dfrac{\sqrt{2}(3\pi-4)}{4}, & \text{dacă} \quad |x|>\frac{\pi}{4}.\end{cases}$

5.4. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\begin{cases}\operatorname{tg}\left(x^3+x^2\sin\dfrac{2}{x}\right), & x\neq 0\\ 0, & x=0.\end{cases}$

5.5. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\begin{cases}\sqrt[3]{1-2x^3\sin\dfrac{5}{x}}-1+x, & x\neq 0\\ 0, & x=0.\end{cases}$

5.6. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\dfrac{|x+1|-|4-x|}{|x|+|x-5|}$.

5.7. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=|\cos x|$.

5.8. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\begin{cases}x, & \text{dacă} \quad x\in\mathbb{Q}\\ 0, & \text{dacă} \quad x\in\mathbb{R}\backslash\mathbb{Q}.\end{cases}$

5.9. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\begin{cases}\operatorname{arctg}ax, & \text{dacă} \quad |x|\leq 1, a\in\mathbb{R}\\ b\operatorname{sgn}x+\dfrac{x-1}{2}, & \text{dacă} \quad |x|>1, b\in\mathbb{R}.\end{cases}$

5.10. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\begin{cases}2^{\frac{1}{x-1}}, & \text{dacă} \quad x<1\\ 0, & \text{dacă} \quad x=1\\ \ln(x^2-2x+2), & \text{dacă} \quad x>1.\end{cases}$

6. Sa se calculeze derivatele de ordinul n ($n\in\mathbb{Z}$, $n\geq 1$) ale functiilor urmatoare:

6.1. $f(x)=xe^{2x}$.

6.2. $f(x)=\dfrac{12x-1}{6x-1}$.

6.3. $f(x)=\sin 3x+\cos(x+2)$.

6.4. $f(x)=\ln(x+3)$.

6.5. $f(x)=(x-1)^n(x-2)^n$.

6.6. $f(x)=x^ne^{-x}$.

6.7. $f(x)=\sin x$.

6.8. $f(x)=\cos x$.

6.9. $f(x) = \sin^2 x$.

6.10. $f(x) = \sin^4 x + \cos^4 x$.

6.11. $f(x) = \dfrac{x}{x^2 - 4x - 12}$.

6.12. $f(x) = \dfrac{3}{x^2 - x - 2}$.

6.13. $f(x) = x \sin x$.

6.14. $f(x) = \operatorname{arctg} x$.

6.15. $f(x) = \dfrac{1}{\sqrt{x-1}}$.

6.16. $f(x) = e^x \sin x$.

6.17. $f(x) = e^x \cos 2x$.

6.18. $f(x) = \dfrac{\ln x}{x}$.

6.19. $f(x) = \dfrac{2x+1}{3x+2}$.

6.20. $f(x) = \sqrt[3]{e^{2x-1}}$.

7. Utilizand diferentiale, sa se calculeze cu aproximatie:

7.1. $f(x) = x^5$, $x = 3,01$.

7.2. $f(x) = x^6$, $x = 1,997$.

7.3. $f(x) = \sqrt[3]{x^2}$, $x = 1,029$.

7.4. $f(x) = \sqrt{3 + x + \cos x}$, $x = 0,01$.

7.5. $f(x) = \sqrt{\dfrac{3-x}{1+x}}$, $x = -0,85$.

7.6. $f(x) = \dfrac{1}{\sqrt{3x+2}}$, $x = 0,668$.

7.7. $f(x) = \arcsin x$, $x = 0,08$.

7.8. $f(x) = \operatorname{arctg} x$, $x = 1,03$.

7.9. $f(x) = \sin x$, $x = 31^\circ$.

7.10. $f(x) = \ln \operatorname{tg} x$, $x = 48^\circ$.

7.11. $f(x) = \dfrac{x + \sqrt{10 - x^2}}{2}$, $x = 0,99$.

7.12. $f(x) = \sqrt{x^2 + 12}$, $x = 1,98$.

8. Sa se calculeze derivata y'_x:

8.1. $\begin{cases} x = \sin^2 t, \\ y = \cos^2 t. \end{cases}$

8.2. $\begin{cases} x = e^{-t}, \\ y = t^2. \end{cases}$

8.3. $\begin{cases} x = \sqrt{t}, \\ y = \sqrt[3]{t}. \end{cases}$

8.4. $\begin{cases} x = e^t, \\ y = \arcsin t. \end{cases}$

8.5. $\begin{cases} x = \dfrac{3at}{1+t^3}, \\ y = \dfrac{3at^2}{1+t^3}. \end{cases}$

8.6. $\begin{cases} x = \dfrac{1}{t+1}, \\ y = \dfrac{t}{t+1}. \end{cases}$

8.7. $\begin{cases} x = \operatorname{arctg} e^t, \\ y = \sqrt{e^{2t} + 1}. \end{cases}$

8.8. $\begin{cases} x = \operatorname{arctg} t, \\ y = \ln \dfrac{1+t^2}{t+1}. \end{cases}$

8.9. $\begin{cases} x = \dfrac{t}{1-t^2}\arcsin t + \ln\sqrt{1-t^2}, \\ y = \dfrac{t}{\sqrt{1-t^2}}. \end{cases}$

8.10. $\begin{cases} x = \ln \operatorname{tg} t, \\ y = \operatorname{cosec}^2 t. \end{cases}$

8.11. $\begin{cases} x = \dfrac{5t^2+2}{5t^3}, \\ y = \sin\left(\dfrac{1}{3}t^3 + t\right). \end{cases}$

8.12. $\begin{cases} x = \sqrt{4-t^2}, \\ y = \operatorname{tg}\sqrt{2+t}. \end{cases}$

8.13. $\begin{cases} x = e^t \cos t, \\ y = e^t \sin t. \end{cases}$

8.14. $\begin{cases} x = a(\sin t - t\cos t), \\ y = a(\cos t + t\sin t). \end{cases}$

8.15. $xy + \ln y = 1.$

8.16. $\sqrt{x} + \sqrt{y} = 1.$

8.17. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1.$

8.18. $e^y + xy = 2e.$

8.19. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}.$

8.20. $y^5 + y^3 + y - x = 0.$

8.21. $\operatorname{arctg}\dfrac{y}{x} = \ln\sqrt{x^2+y^2}.$

8.22. $y^2 = 2px.$

8.23. $x^2 + y^2 - 6x + 10y - 2 = 0.$

8.24. $x^2 y + \operatorname{arctg}\dfrac{y}{x} = 0.$

9. Sa se scrie ecuatiile tangentelor la graficele functiilor in punctele specificate:

9.1. $f(x) = x^2 - x - 12\ ,\ x = 3.$

9.2. $f(x) = \dfrac{1}{3}(3x - x^3)\ ,\ x = 2.$

9.3. $f(x) = \dfrac{x^3+1}{x^3-1}\ ,\ x = 0.$

9.4. $f(x) = \dfrac{x}{x^2+1}\ ,\ x = -1.$

9.5. $f(x) = \dfrac{x^2-x-2}{x^2-3x}\ ,\ x = 2.$

9.6. $f(x) = \ln\dfrac{x^2-2x+1}{x^2+x+e}\ ,\ x = 0.$

9.7. $f(x) = \cos 2x - 2\sin x\ ,\ x = \dfrac{\pi}{2}.$

9.8. $f(x) = \operatorname{arctg}\dfrac{1}{x}\ ,\ x = 1.$

9.9. $f(x) = \dfrac{x}{\sqrt[3]{x+1}}\ ,\ x = -2.$

9.10. $f(x) = 4\operatorname{tg} x - \dfrac{\sin x}{\cos^2 x}\ ,\ x = \dfrac{\pi}{4}.$

10. Sa se determine in ce puncte si sub ce unghi se intersecteaza graficele functiilor:

10.1. $f_1(x) = \sin x,\ f_2(x) = \sqrt{3}\cos x.$

10.2. $f_1(x) = x^2,\ f_2(x) = x.$

10.3. $f_1(x) = x^3,\ f_2(x) = x^2.$

10.4. $f_1(x) = (x-2)^2,\ f_2(x) = 4 - x^2.$

10.5. $f_1(x) = \dfrac{1}{\sqrt[3]{x}},\ f_2(x) = x.$

10.6. $f_1(x) = \dfrac{1}{x^3},\ f_2(x) = x^2.$

10.7. $f_1(x) = 4x^2 + 2x - 8,\ f_2(x) = x^3 - x + 10.$

10.8. $f_1(x) = \ln x,\ f_2(x) = 2 - \dfrac{x}{e}.$

10.9. $f_1(x) = 3x - x^2,\ f_2(x) = x^2 - x.$

10.10. $f_1(x) = \sin x,\ f_2(x) = \cos x.$

11. Sa se studieze monotonia si sa se determine punctele de extrem pentru fiecare din functiile f pe domeniul lor maxim de definitie:

11.1. $f(x) = x^2 - x - 12$.

11.2. $f(x) = 6x - x^2$.

11.3. $f(x) = 3x^3 - 4x^2 + 1$.

11.4. $f(x) = x^3 - 6x^2 + 2$.

11.5. $f(x) = (x+1)^2 (x-4)^3$.

11.6. $f(x) = x^2 - 8\ln x$.

11.7. $f(x) = \sqrt[3]{(2-x)(1-x)^2}$.

11.8. $f(x) = (x-1)\sqrt{x^2-1}$.

11.9. $f(x) = \ln(1+x) - x + \dfrac{x^2}{2}$.

11.10. $f(x) = \dfrac{x^2}{x-1}$.

11.11. $f(x) = \ln\sqrt{1+x^2} + \operatorname{arctg} x$.

11.12. $f(x) = x^2 e^{\frac{1}{x}}$.

11.13. $f(x) = \ln(4x^2+1) - 8\operatorname{arctg} 2x$.

11.14. $f(x) = \dfrac{x^3}{3} e^{-x}$.

11.15. $f(x) = \ln x + \operatorname{arctg} x$.

10.16 $f(x) = x^2 \ln x$.

11.17. $f(x) = x - 2\operatorname{arctg}(x-1) - 1$.

11.18. $f(x) = \sin^3 x + \cos^3 x$.

11.19. $f(x) = \cos x + \dfrac{1}{2}\sin 2x$.

11.20. $f(x) = \dfrac{1}{x-1} + \dfrac{2(x-1)}{x^2-2x}$.

12. Sa se determine intervalele de concavitate, convexitate si eventualele puncte de inflexiune pentru functiile urmatoare:

12.1. $f(x) = 2x^4 - 3x^2 + 3x - 2$.

12.2. $f(x) = x^4 + 4x^3$.

12.3. $f(x) = 3x^2 - x^3 + 1$.

12.4. $f(x) = x + \cos x$.

12.5. $f(x) = e^{-x^2} + 2x$.

12.6. $f(x) = \ln(1+x^2)$.

12.7. $f(x) = \dfrac{(x+1)^2}{x^3}$.

12.8. $f(x) = \dfrac{\ln(x+1)}{\sqrt{x+1}}$.

12.9. $f(x) = \left(\dfrac{x}{2-x}\right)^4$.

12.10. $f(x) = \sin x + \dfrac{1}{3}\sin 3x$.

12.11. $f(x) = \sqrt[3]{x-1} - \sqrt[3]{x}$.

12.12. $f(x) = \sin x - \sin^3 x$.

12.13. $f(x) = \sin^4 x - \cos^4 x$.

12.14. $f(x) = x^5 - 10x^2 + 7x$.

12.15. $f(x) = \operatorname{tg} x + \cos x$.

12.16. $f(x) = x + \ln x^2$.

12.17. $f(x) = \ln\dfrac{x}{x-3}$.

12.18. $f(x) = \dfrac{\sqrt{x+1}}{x}$.

12.19. $f(x) = e^x - \dfrac{1}{2}x^2 + 1$.

12.20. $f(x) = 3x + 2\sin\dfrac{x}{2}$.

13. Sa se reprezinte grafic urmatoarele functii, $f : D \longrightarrow \mathbb{R}$, D – fiind domeniul maxim de definitie:

13.1. $f(x) = 3x - x^3$.

13.2. $f(x) = 2 - 3x - x^3$.

13.3. $f(x) = \frac{1}{16}x^2(x-4)^2$.

13.4. $f(x) = x^2 - x^4$.

13.5. $f(x) = x(2x^2 + 9x + 12)$.

13.6. $f(x) = (x-1)^2(3-x)^2$.

13.7. $f(x) = \frac{3x-2}{x^3}$.

13.8. $f(x) = \frac{x^3+4}{x^2}$.

13.9. $f(x) = \left(\frac{x}{x-1}\right)^2$.

13.10. $f(x) = \frac{3x^4+1}{x^3}$.

13.11. $f(x) = \frac{x^3}{x-1}$.

13.12. $f(x) = 3x + \frac{6}{x} - \frac{1}{x^3}$.

13.13. $f(x) = \frac{3}{x+2} - \frac{3}{x-2} - 1$.

13.14. $f(x) = \frac{\ln(x+1)}{\sqrt{x+1}}$.

13.15. $f(x) = \sin x - \sin^2 x$.

13.16. $f(x) = \cos 3x + 3\cos x$.

13.17. $f(x) = \sin x + \frac{1}{2}\sin 2x$.

13.18. $f(x) = \cos x \cos 3x$.

13.19. $f(x) = \arccos \frac{2x}{1+x^2}$.

13.20. $f(x) = \arcsin \frac{1-x^2}{1+x^2}$.

13.21. $f(x) = \ln x - x + 1$.

13.22. $f(x) = x^2 \ln x$.

13.23. $f(x) = \frac{\ln x}{x}$.

13.24. $f(x) = \ln\left(\frac{x-5}{x}\right) + 2$.

13.25. $f(x) = x \operatorname{arctg} x$.

13.26. $f(x) = \operatorname{arctg} \sin x$.

13.27. $f(x) = \ln(\sin x - \cos x)$.

13.28. $f(x) = x^{\frac{2}{3}} e^{-\frac{x^2}{3}}$.

13.29. $f(x) = \frac{e^{x+2}}{x+2}$.

13.30. $f(x) = e^{\sin x + \cos x}$.

13.31. $f(x) = \sqrt[3]{x(x^2-1)}$.

13.32. $f(x) = \sqrt[3]{(x-2)(x+1)^2}$.

14. Sa se calculeze limitele urmatoare folosind regula lui l'Hospital:

14.1. $\lim\limits_{x\to 1} \frac{x^3 - 5x^2 + 4}{2x^3 - x^2 - 1}$.

14.2. $\lim\limits_{x\to 1} \frac{x^5 - 1}{\ln x}$.

14.3. $\lim\limits_{x\to 0} \frac{\sin 5x}{2x}$.

14.4. $\lim\limits_{x\to \frac{\pi}{2}} \frac{\cos 5\pi x}{\cos 3\pi x}$.

14.5. $\lim\limits_{x\to 0}\dfrac{\operatorname{tg} x - x}{\sin x - x}$.

14.6. $\lim\limits_{x\to 0}\dfrac{e^x - e^{-x}}{\ln(1+x)}$.

14.7. $\lim\limits_{x\to\infty}\dfrac{\ln(1+\frac{1}{x^2})}{\pi - 2\operatorname{arctg} x}$.

14.8. $\lim\limits_{x\to 0}\dfrac{\ln\cos 2x}{\ln\cos 3x}$.

14.9. $\lim\limits_{x\to 0}\dfrac{\sin 2x - 2xe^x + 3x^2}{\operatorname{arctg} x - \sin x - \frac{x^3}{6}}$.

14.10. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\ln\operatorname{tg} x}{\operatorname{ctg} 2x}$.

14.11. $\lim\limits_{x\to\infty}\dfrac{\pi - 2\operatorname{arctg} x}{e^{\frac{2}{x}} - 1}$.

14.12. $\lim\limits_{x\to 1}\dfrac{\ln x - x + 1}{\operatorname{tg}^2(x-1)}$.

14.13. $\lim\limits_{x\to\infty}\dfrac{x^2}{e^x}$.

14.14. $\lim\limits_{x\to\infty}\dfrac{x^4}{e^x}$.

14.15. $\lim\limits_{x\to 1+}\dfrac{\ln(x-1)}{\operatorname{ctg}\pi x}$.

14.16. $\lim\limits_{x\to 0+}\dfrac{\ln x}{\ln\sin x}$.

14.17. $\lim\limits_{x\to 0} x\operatorname{ctg}\pi x$.

14.18. $\lim\limits_{x\to\frac{\pi}{2}}\left(x - \dfrac{\pi}{2}\right)\operatorname{tg} x$.

14.19. $\lim\limits_{x\to 0}(\operatorname{ctg} x \arcsin x)$.

14.20. $\lim\limits_{x\to 0}\sin x \ln(\operatorname{ctg} x)$.

14.21. $\lim\limits_{x\to 2}(x-2)\operatorname{tg}\dfrac{\pi x}{4}$.

14.22. $\lim\limits_{x\to 3}(x-3)\operatorname{ctg}\dfrac{\pi x}{3}$.

14.23. $\lim\limits_{x\to 1}\left(\dfrac{1}{x-1} - \dfrac{1}{\ln x}\right)$.

14.24. $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2} - \operatorname{ctg}^2 x\right)$.

14.25. $\lim\limits_{x\to 3}\left(\dfrac{2x-3}{x^2-7x+12} - \dfrac{1}{(x-2)\ln(x-2)}\right)$.

14.26. $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{1}{\arcsin x}\right)$.

14.27. $\lim\limits_{x\to 1}\left(\dfrac{2}{1-x^2} - \dfrac{3}{1-x^3}\right)$.

14.28. $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{1}{e^{2x}-1}\right)$.

14.29. $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x^2}}$.

14.30. $\lim\limits_{x\to\infty}\left(\dfrac{2}{\pi}\operatorname{arctg} x\right)^x$.

14.31. $\lim\limits_{x\to 0}(x+3^x)^{\frac{2}{x}}$.

14.32. $\lim\limits_{x\to 0}(x+e^x)^{\frac{1}{x}}$.

14.33. $\lim\limits_{x\to\pi+}(x-\pi)^{\sin x}$.

14.34. $\lim\limits_{x\to 0+}|\ln x|^{\frac{x}{2}}$.

14.35. $\lim\limits_{x\to 0+}\left(\dfrac{1}{x}\right)^{\sin x}$.

14.36. $\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^{\frac{1}{x^2}}$.