

Тригонометрия

1. $\sin \alpha = \frac{a}{c}$; $\cos \alpha = \frac{b}{c}$; $\operatorname{tg} \alpha = \frac{a}{b}$; $\operatorname{ctg} \alpha = \frac{b}{a}$;
(a, b - катеты, c - гипотенуза прямоугольного треугольника, α - угол, лежащий против катеты a).
2. $\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$; $\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$.
3. $\operatorname{tg} \alpha \operatorname{ctg} \alpha = 1$.
4. $\sin\left(\frac{\pi}{2} \pm \alpha\right) = \cos \alpha$; $\sin(\pi \pm \alpha) = \mp \sin \alpha$.
5. $\cos\left(\frac{\pi}{2} \pm \alpha\right) = \mp \sin \alpha$; $\cos(\pi \pm \alpha) = -\cos \alpha$.
6. $\operatorname{tg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp \operatorname{ctg} \alpha$; $\operatorname{ctg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp \operatorname{tg} \alpha$.
7. $\sec\left(\frac{\pi}{2} \pm \alpha\right) = \mp \operatorname{cosec} \alpha$; $\operatorname{cosec}\left(\frac{\pi}{2} \pm \alpha\right) = \sec \alpha$.
8. $\sin^2 \alpha + \cos^2 \alpha = 1$.
9. $1 + \operatorname{tg}^2 \alpha = \sec^2 \alpha$.
10. $1 + \operatorname{ctg}^2 \alpha = \operatorname{cosec}^2 \alpha$.
11. $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$.
12. $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$.
13. $\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg} \alpha \pm \operatorname{tg} \beta}{1 \mp \operatorname{tg} \alpha \operatorname{tg} \beta}$.
14. $\operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg} \alpha \operatorname{ctg} \beta \mp 1}{\operatorname{ctg} \beta \pm \operatorname{ctg} \alpha}$.
15. $\sin 2\alpha = 2 \sin \alpha \cos \alpha$.
16. $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$.
17. $\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$.
18. $\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2 \operatorname{ctg} \alpha}$.
19. $\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$.
20. $\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$.

$$21. \left| \sin \frac{\alpha}{2} \right| = \sqrt{\frac{1 - \cos \alpha}{2}}.$$

$$22. \left| \cos \frac{\alpha}{2} \right| = \sqrt{\frac{1 + \cos \alpha}{2}}.$$

$$23. \left| \operatorname{tg} \frac{\alpha}{2} \right| = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}.$$

$$24. \operatorname{tg} \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}.$$

$$25. \left| \operatorname{ctg} \frac{\alpha}{2} \right| = \sqrt{\frac{1 + \cos \alpha}{1 - \cos \alpha}}.$$

$$26. \operatorname{ctg} \frac{\alpha}{2} = \frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha}.$$

$$27. 1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2}.$$

$$28. 1 - \cos \alpha = 2 \sin^2 \frac{\alpha}{2}.$$

$$29. \sin \alpha \pm \sin \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \mp \beta}{2}.$$

$$30. \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}.$$

$$31. \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}.$$

$$32. \operatorname{tg} \alpha \pm \operatorname{tg} \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cos \beta}.$$

$$33. \operatorname{ctg} \alpha \pm \operatorname{ctg} \beta = \frac{\sin(\beta \pm \alpha)}{\sin \alpha \sin \beta}.$$

$$34. \sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)].$$

$$35. \sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)].$$

$$36. \cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)].$$

37. Простейшие тригонометрические уравнения:

$$\left. \begin{aligned} \sin x = a, |a| \leq 1; x &= (-1)^n \arcsin a + \pi n; \\ \cos x = a, |a| \leq 1; x &= \pm \arccos a + 2\pi n; \\ \operatorname{tg} x = a, x &= \operatorname{arctg} a + \pi n; \\ \operatorname{ctg} x = a, x &= \operatorname{arcctg} a + \pi n \end{aligned} \right\} n \in \mathbf{Z}.$$

$$38. \arcsin x + \arccos x = \frac{\pi}{2}, \quad |x| \leq 1.$$

$$39. \operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2}.$$

$$40. \operatorname{arcsec} x + \operatorname{arccosec} x = \frac{\pi}{2}, \quad |x| \geq 1.$$

$$41. \sin(\arcsin x) = x, \quad x \in [-1; +1].$$

$$42. \arcsin(\sin x) = x, \quad x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right].$$

$$43. \cos(\arccos x) = x, \quad x \in [-1; +1].$$

$$44. \arccos(\cos x) = x, \quad x \in [0; \pi].$$

$$45. \operatorname{tg}(\operatorname{arctg} x) = x, \quad x \in \mathbf{R}.$$

$$46. \operatorname{arctg}(\operatorname{tg} x) = x, \quad x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right).$$

$$47. \operatorname{ctg}(\operatorname{arcctg} x) = x, \quad x \in \mathbf{R}.$$

$$48. \operatorname{arcctg}(\operatorname{ctg} x) = x, \quad x \in (0; \pi).$$

$$49. \arcsin x = \arccos \sqrt{1-x^2} = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}} = \operatorname{arcctg} \frac{\sqrt{1-x^2}}{x}, \quad 0 < x < 1.$$

$$50. \arccos x = \arcsin \sqrt{1-x^2} = \operatorname{arctg} \frac{\sqrt{1-x^2}}{x} = \operatorname{arcctg} \frac{x}{\sqrt{1-x^2}}, \quad 0 < x < 1.$$

$$51. \operatorname{arctg} x = \arcsin \frac{x}{\sqrt{1+x^2}} = \arccos \frac{1}{\sqrt{1+x^2}} = \operatorname{arcctg} \frac{1}{x}, \quad 0 < x < +\infty.$$

$$52. \operatorname{arcctg} x = \arcsin \frac{1}{\sqrt{1+x^2}} = \arccos \frac{x}{\sqrt{1+x^2}} = \operatorname{arctg} \frac{1}{x}, \quad 0 < x < +\infty.$$

$$53. \arcsin x + \arcsin y = \begin{cases} \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{если } xy \leq 0 \text{ или } x^2 + y^2 \leq 1; \\ \pi - \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{если } x > 0, y > 0 \text{ и } x^2 + y^2 > 1; \\ -\pi - \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{если } x < 0, y < 0 \text{ и } x^2 + y^2 > 1. \end{cases}$$

$$54. \arcsin x - \arcsin y = \begin{cases} \arcsin(x\sqrt{1-y^2} - y\sqrt{1-x^2}), & \text{если } xy \geq 0 \text{ или } x^2 + y^2 \leq 1; \\ \pi - \arcsin(x\sqrt{1-y^2} - y\sqrt{1-x^2}), & \text{если } x > 0, y < 0 \text{ и } x^2 + y^2 > 1; \\ -\pi - \arcsin(x\sqrt{1-y^2} - y\sqrt{1-x^2}), & \text{если } x < 0, y > 0 \text{ и } x^2 + y^2 > 1. \end{cases}$$

$$55. \arccos x + \arccos y = \begin{cases} \arccos(xy - \sqrt{(1-x^2)(1-y^2)}), & \text{если } x + y \geq 0; \\ 2\pi - \arccos(xy - \sqrt{(1-x^2)(1-y^2)}), & \text{если } x + y < 0. \end{cases}$$

$$56. \arccos x - \arccos y = \begin{cases} -\arccos(xy + \sqrt{(1-x^2)(1-y^2)}), & \text{если } x \geq y; \\ \arccos(xy + \sqrt{(1-x^2)(1-y^2)}), & \text{если } x < y. \end{cases}$$

$$57. \arctg x + \arctg y = \begin{cases} \arctg \frac{x+y}{1-xy}, & \text{если } xy < 1; \\ \pi + \arctg \frac{x+y}{1-xy}, & \text{если } x > 0 \text{ и } xy > 1; \\ -\pi + \arctg \frac{x+y}{1-xy}, & \text{если } x < 0 \text{ и } xy > 1. \end{cases}$$

$$58. \arctg x - \arctg y = \begin{cases} \arctg \frac{x-y}{1+xy}, & \text{если } xy > -1; \\ \pi + \arctg \frac{x-y}{1+xy}, & \text{если } x > 0 \text{ и } xy < -1; \\ -\pi + \arctg \frac{x-y}{1+xy}, & \text{если } x < 0 \text{ и } xy < -1. \end{cases}$$

$$59. 2 \arcsin x = \begin{cases} \arcsin(2x\sqrt{1-x^2}), & \text{если } |x| \leq \frac{\sqrt{2}}{2}; \\ \pi - \arcsin(2x\sqrt{1-x^2}), & \text{если } \frac{\sqrt{2}}{2} < x \leq 1; \\ -\pi - \arcsin(2x\sqrt{1-x^2}), & \text{если } -1 \leq x < -\frac{\sqrt{2}}{2}. \end{cases}$$

$$60. 2 \arccos x = \begin{cases} \arccos(2x^2 - 1) & \text{при } 0 \leq x \leq 1; \\ 2\pi - \arccos(2x^2 - 1) & \text{при } -1 \leq x < 0. \end{cases}$$

$$61. 2 \arctg x = \begin{cases} \arctg \frac{2x}{1-x^2}, & \text{если } |x| < 1; \\ \pi + \arctg \frac{2x}{1-x^2}, & \text{если } x > 1; \\ -\pi + \arctg \frac{2x}{1-x^2}, & \text{если } x < -1. \end{cases}$$

$$62. \frac{1}{2} \arcsin x = \begin{cases} \arcsin \sqrt{\frac{1-\sqrt{1-x^2}}{2}}, & \text{если } 0 \leq x \leq 1; \\ -\arcsin \sqrt{\frac{1-\sqrt{1-x^2}}{2}}, & \text{если } -1 \leq x < 0. \end{cases}$$

$$63. \frac{1}{2} \arccos x = \arccos \sqrt{\frac{1+x}{2}}, \text{ если } -1 \leq x \leq 1.$$

$$64. \frac{1}{2} \arctg x = \begin{cases} \arctg \frac{\sqrt{1+x^2}-1}{x}, & \text{если } x \neq 0; \\ 0, & \text{если } x = 0. \end{cases}$$