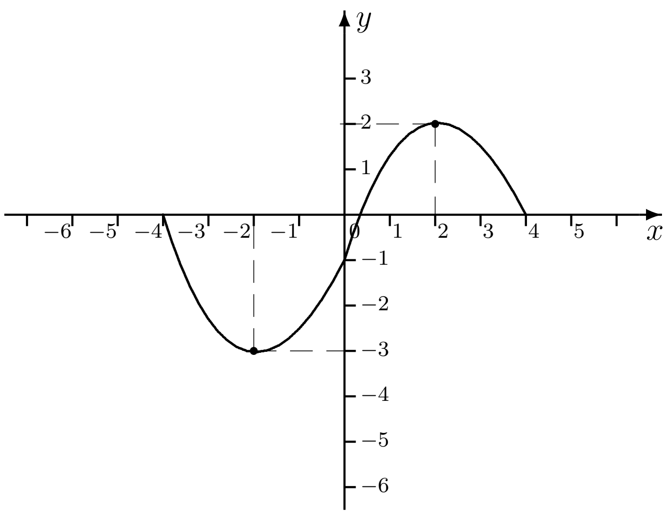
Ministerul Educatiei al Republicii Moldova
Directia Invatamant Preuniversitar
Examenul de bacalaureat la matematica, iunie 2004
Profilul real

Timp alocat: 180 minute.

In itemii 1-3 incercuiti litera corespunzatoare variantei corecte de raspuns.

1. Functia $f : [-4; 4] \to \mathbb{R}$ este reprezentata grafic. Care propozitie este adevarata?



a) $f'(0) = 0$ b) $f'(0) > 0$ c) $f'(0) < 0$ d) $f'(0)$ nu exista.

2. Dreapta definita de ecuatia $ax + by = 1$ este paralela cu axa absciselor daca
a) $a = 0$ si $b = 0$ b) $a = 0$ si $b \neq 0$ c) $a \neq 0$ si $b = 0$ d) $a \neq 0$ si $b \neq 0$.

3. Care ecuatie admite o singura solutie pe intervalul $(0; 2\pi]$?
a) $\text{tg}x = 1$ b) $\cos x = 0$ c) $\text{ctg}x = -3$ d) $\sin x = 1$.

4. Completati caseta libera cu unul dintre semnele $<, =, >$, astfel incat propozitia sa fie adevarata.

$$\lg \text{tg} 40^o + \lg \text{ctg} 40^o \boxed{}\ 0.$$

Argumentati raspunsul.

5. Calculati limita $\lim\limits_{x \to 6} \dfrac{x^2 - 36}{\sqrt{x+3} - 3}$.

6. Determinati termenul din mujloc al dezvoltarii binomului $\left(\sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}\right)^6$.

7. Rezolvati in $\mathbb{R}$ inecuatia $D(x) \leq 0$, unde $D(x) = \begin{vmatrix} 1 & x & x^2 & x^3 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \end{vmatrix}$.

8. Baza unei piramide triunghiulare este triunghiul dreptunghic ABC, unde $m(\angle A) = 90^o$, $m(\angle B) = 60^o$, $|AC| = 12\sqrt{3}$ cm. Determinati volumul piramidei, daca muchiile laterale sunt congruente si au lungimea egala cu 13 cm.

9. Determinati extremele locale ale functiei $f : \mathbb{R} \to \mathbb{R}, f(x) = x \cdot e^{1-2x^2}$.

10. Rezolvati in $\mathbb{R}$ ecuatia $\log_3 x^2 - \log_3^2(-x) + 3 = 0$.

11. Unul dintre zerourile unei primitive a functiei $f : \mathbb{R} \to \mathbb{R}, f(x) = x^2 - 4x + 1$ este egal cu 2. Determinati celelalte zerouri ale acestei primitive.

12. Intr-un trapez isoscel bazele au lungimi egale cu 21 cm si 9 cm, iar lungimea inaltimii este egala cu 8 cm. Determinati raza cercului circumscris acestui trapez.

13. Fie polinomul $P(X) = X^4 - X^3 - aX^2 + (b-2)X + a$, unde $a, b \in \mathbb{R}$. Se stie ca $\alpha = 1 + i$ este radacina a polinomului $P(X)$. Determinati valorile parametrilor reali a si b.

14. Rezolvati in $\mathbb{R}$ inecuatia $\dfrac{\sqrt{3^{2x+1} - 4 \cdot 3^x + 1}}{x^2 - x - 6} \leq 0$.

Solutii

1. Raspuns corect b) $f'(0) > 0$.

2. Raspuns corect b) $a = 0$ si $b \neq 0$.

3. Raspuns corect d) $\sin x = 1$.

4. $\lg \operatorname{tg}40^o + \lg \operatorname{ctg}40^o = \lg(\operatorname{tg}40^o \cdot \operatorname{ctg}40^o) = \lg 1 = 0$.

5. $$\lim_{x\to 6} \frac{x^2 - 36}{\sqrt{x+3} - 3} = \lim_{x\to 6} \frac{(x-6)(x+6)(\sqrt{x+3}+3)}{(\sqrt{x+3}-3)(\sqrt{x+3}+3)} = \lim_{x\to 6} \frac{(x-6)(x+6)(\sqrt{x+3}+3)}{(x+3-9)} =$$
$$= \lim_{x\to 6}(x+6)(\sqrt{x+3}+3) = 12 \cdot 6 = 72.$$

Raspuns: 72.

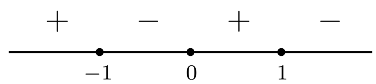
6. Cum $n = 6$, dezvoltarea binomului contine 7 termeni si termenul din mijloc este T_4.

$$T_4 = T_{3+1} = C_6^3 (x^{\frac{1}{3}})^{6-3} (x^{-\frac{1}{3}})^3 = \frac{6!}{3!3!} x^{\frac{1}{3}\cdot 3} x^{-\frac{1}{3}\cdot 3} = \frac{3! \cdot 4 \cdot 5 \cdot 6}{3! \cdot 1 \cdot 2 \cdot 3} x^0 = 20.$$

Raspuns: $T_4 = 20$.

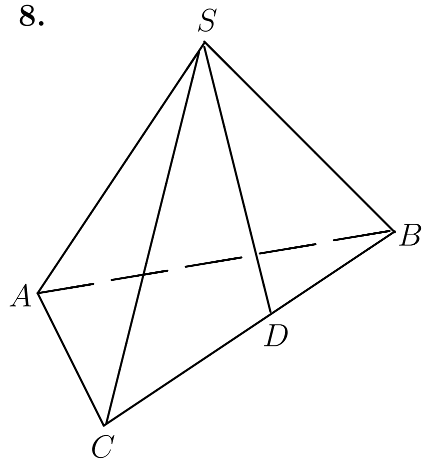
7. Descompunem determinantul dupa linia a 4 si obtinem:
$$D(x) = 1\cdot(-1)^{4+1}\cdot \begin{vmatrix} x & x^2 & x^3 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix} = -x \begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -2x\cdot(-1)^{2+3}\cdot \begin{vmatrix} 1 & x^2 \\ 1 & 1 \end{vmatrix} = 2x(1-x^2) =$$
$$= 2x(1-x)(1+x).$$

Utilizand metoda intervalelor, pentru inecuatia $D(x) \le 0 \Leftrightarrow 2x(1-x)(1+x) \le 0$, se obtine $x \in [-1; 0] \cup [1; +\infty)$.



Raspuns: $x \in [-1; 0] \cup [1; +\infty)$.

8.



Cum muchiile laterale sunt congruente, piciorul D al inaltimii $SD = h$ se afla in mijlocul ipotenuzei BC. Determinam ipotenuza BC:

$$BC = \frac{AC}{\sin\angle B} = \frac{12\sqrt{3}}{\frac{\sqrt{3}}{2}} = 24(\text{cm}).$$

Rezulta $BD = \dfrac{24}{2} = 12$. Din $\triangle SDB$ (dreptunghic in D) aflam inaltimea piramidei h:

$$h = SD = \sqrt{SB^2 - BD^2} = \sqrt{13^2 - 12^2} = \sqrt{1 \cdot 25} = 5(\text{cm}).$$

Aflam aria bazei:

$$S_{\triangle ABC} = \frac{1}{2}AC \cdot BC \cdot \sin C = \frac{1}{2}12\sqrt{3} \cdot 24 \cdot \sin(90^o - 60^o) = \frac{1}{2}12\sqrt{3} \cdot 24 \cdot \frac{1}{2} = 72\sqrt{3}\ (\text{cm}^2).$$

Aflam volumul piramidei:

$$V = \frac{1}{3}S_{\triangle ABC} \cdot SD = \frac{1}{3}72\sqrt{3} \cdot 5 = 120\sqrt{3}\ (\text{cm}^3).$$

Raspuns: $V = 120\sqrt{3}\ (\text{cm}^3)$.

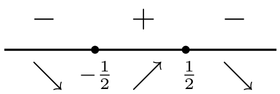
9. Aflam derivata functiei f:

$$f'(x) = x'e^{1-2x^2} + xe^{1-2x^2}(1-2x^2)' = e^{1-2x^2} + xe^{1-2x^2}(-4x) = = e^{1-2x^2}(1-4x^2).$$

Aflam punctele critice, rezolvand ecuatia $f'(x) = 0$:

$$f'(x) = 0 \quad \Leftrightarrow \quad e^{1-2x^2}(1-4x^2) = 0 \quad \Leftrightarrow \quad x_1 = -\frac{1}{2} \text{ si } x_2 = \frac{1}{2}.$$

Determinam semnul functiei f' pe intervale $(-\infty; -\frac{1}{2}), (-\frac{1}{2}, \frac{1}{2})$ si $(\frac{1}{2}; +\infty)$



si obtinem extremele locale: $x = -\frac{1}{2}$ punct de minim, $f_{min} = f(-\frac{1}{2}) = -\frac{\sqrt{e}}{2}$ si $x = \frac{1}{2}$ punct de maxim, $f_{max} = f(\frac{1}{2}) = \frac{\sqrt{e}}{2}$.

Raspuns: $f_{min} = -\frac{\sqrt{e}}{2}$, $f_{max} = \frac{\sqrt{e}}{2}$.

10. DVA: $x < 0$.

$$\log_3 x^2 - \log_3^2(-x) + 3 = 0 \;\Leftrightarrow\; 2\log_3|x| - \log_3^2(-x) + 3 = 0 \;\Leftrightarrow$$

$$\Leftrightarrow \log_3^2(-x) - 2\log_3|x| - 3 = 0 \;\Leftrightarrow\; \left[\begin{array}{l} \log_3(-x) = -1, \\ \log_3(-x) = 3, \end{array}\right. \;\Leftrightarrow\; \left[\begin{array}{l} -x = 3^{-1}, \\ -x = 3^3, \end{array}\right. \;\Leftrightarrow\; \left[\begin{array}{l} x = -\frac{1}{3} \in \text{DVA}, \\ x = -27 \in \text{DVA}. \end{array}\right.$$

Raspuns: $x \in \{-\frac{1}{3}; -27\}$.

11. $F(x) = \int (x^2 - 4x + 1)\, dx = \dfrac{x^3}{3} - 2x^2 + x + C$.

Cum $F(2) = 0$, rezulta: $\dfrac{8}{3} - 8 + 2 + C = 0$, de unde $C = \dfrac{10}{3}$. Prin urmare, $F(x) = \dfrac{x^3}{3} - 2x^2 + x + \dfrac{10}{3}$. Determinam, celelalte radacini:

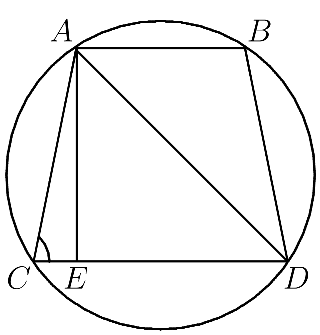
$$F(x) = 0 \;\Leftrightarrow\; x^3 - 6x^2 + 3x + 10 = 0 \;\Leftrightarrow\; x^3 - 2x^2 - 4x^2 + 8x - 5x + 10 = 0 \;\Leftrightarrow$$

$$\Leftrightarrow\; x^2(x-2) - 4x(x-2) - 5(x-2) = 0 \;\Leftrightarrow\; (x-2)(x^2-4x-5) = 0 \;\Leftrightarrow\; (x-2)(x+1)(x-5) = 0,$$

de unde $x_2 = -1, x_3 = 5$.

Raspuns: $x_2 = -1, x_3 = 5$.

12.



Fie $AB = 9$ cm, $CD = 21$ cm, $AE \perp DC$, $AE = 8$ cm.

Cum trapezul $ABCD$ este isoscel $CE = \dfrac{CD - AB}{2} = \dfrac{21 - 9}{2} = 6$ (cm), atunci $DE = 21 - 6 - 15$ (cm), $AD = \sqrt{AE^2 + DE^2} = \sqrt{64 + 225} = 17$ (cm) (din $\triangle AED$ dreptuinghic in E). Din $\triangle AEC$ (dreptunghic in E) aflam AC: $AC = \sqrt{CE^2 + AE^2} = \sqrt{64 + 36} = 10$ (cm).

Atunci

$$\sin\angle ACE = \frac{AE}{AC} = \frac{8}{10} = \frac{4}{5}.$$

Conform teoremei sinusurilor $\dfrac{AD}{\sin\angle ACE} = 2R$, de unde

$$R = \frac{17\cdot 5}{2\cdot 4} = \frac{85}{8} = 10\frac{5}{8}.$$

Ramane de observat ca raza cercului circumscris $\triangle ACD$ coincide cu raza cercului circumscris trapezului.

Raspuns: $R = 10\frac{5}{8}$ cm.

13. Cum $\alpha = 1+i$ radacina a polinomului $P(X)$, rezulta $P(\alpha) = 0$ si

$$(1+i)^4 - (1+i)^3 - a(1+i)^2 + (b-2)(1+i) + a = 0 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad 1+4i^3+6i^2+4i^3+i^4-1-3i-3i^2-i^3-a(1+2i+i^2)+(b-2)+i(b-2)+a=0 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad 1+4i-6-4i+1-1-3i+3+i-2ai+(b-2)+i(b-2)+a=0 \quad \Leftrightarrow$$

$$\Leftrightarrow \quad a-2+b-2+i(b-2-2-2a)=0 \quad \Leftrightarrow \quad a+b-4+i(b-2a-4)=0.$$

Utilizand definitia egalitatii a doua numere complexe, obtinem $\begin{cases} a+b-4=0, \\ b-2a-4=0, \end{cases}$ de unde $\begin{cases} a=0, \\ b=4. \end{cases}$

Raspuns: $a = 0,\ b = 4$.

14.

$$\frac{\sqrt{3^{2x+1}-4\cdot 3^x+1}}{x^2-x-6} \le 0 \Leftrightarrow \left[\begin{array}{l} \begin{cases} 3^{2x+1}-4\cdot 3^x+1=0, \\ x\in\mathbb{R}\setminus\{-2;3\}, \end{cases} \\ \begin{cases} 3^{2x+1}-4\cdot 3^x+1>0, \\ x^2-x-6<0, \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} \left[\begin{array}{l} 3^x=1, \\ 3^x=\dfrac{1}{3}, \end{array}\right. \\ x\in\mathbb{R}\setminus\{-2;3\}, \end{cases} \\ \begin{cases} (3^x-1)\left(3^x-\dfrac{1}{3}\right)>0, \\ -2<x<3, \end{cases} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \quad \left[\begin{array}{l} \{\ x=0,\ x=-1, \\ \begin{cases} \left[\begin{array}{l} x>0, \\ x<-1, \end{array}\right. \\ -2<x<3, \end{cases} \end{array}\right. \quad \Leftrightarrow \quad x\in(-2;-1]\cup[0;3).$$

Raspuns: $x\in(-2;-1]\cup[0;3)$.