Computer Science Journal of Moldova, vol.5, no.1(13), 1997

Mathematical methods and computer facilities in economic reserches

M.Dyakonova E.Naval

Mathematical modeling of economic processes has become lately one of major directions of development of the economic theory. With the aid of mathematical methods and computer facilities it is possible to solve such economic problems, which by other means it is impossible to solve , it is possible also to improve the system of preparation and processing of the economic information, and to speed up economic calculations.

Researches, aimed at application of economico-mathematical modeling the methods both in theory and in practice may be divided into the two following parts:

1. Mathematical modeling of economic processes and development of methods of their implementation in conditions of a planned economy;

2. Macroeconomic modeling of a market type economy.

Thus the role of economico-mathematical models in a economy of a market type differs from their role in a planned economy. If for a planned economy result of implemention of models are data, enabling one to compare the scheduled tasks with a actual situation of things, for a market economy these data should serve for orientation and analysis in the field of a economy and society.

In the planned economy data concerning production volumes and gross output prevail, while monetary parameters are smaller interest. Aggregated data often are not the subject to the publication, while microdata are not anonymous.

©1997 by M.Dyakonova, E.Naval

For a market economy the higher significance have the data, concerning to finance and money, prices, and incomes. In it the microdata have confidential character, while macrodata may be published widely both at national and international levels.

The distinctions between the market economy and planned one may be explained by divergences in organization of economic processes. In a system of the market economy as opposed to the scheduled economy, in the field of making the economic decisions there coexists both the private initiative and the state one. Bodies of state management and the society are not a single whole, the lot of various organizations bear the main responsibility or for a financial policy, either for a policy in the field of incomes. At the same time the system of a market economy is not free of state interference.

Nevertheless both in the market economy, and in the planned economy for the researches scheduled for implementation in the field of modeling of economic processes the following rules are common:

1. Positing of a problem and application of theoretical researches in the form adequate for practical implementation, namely, in the form of modeling of economic processes;

2. Development of algorithms implementing the economico-mathematical models and performing the necessary calculations;

3. Carrying out the analysis of results of the model running. Construction of the economic scenarious and simulation calculations.

1 Balance models

The basis of macroeconomic accounting is the system of the national accounts (SNA), which reflect the income circulation. In such form SNA is based on:

- Calculation of generalizing parameters, measuring results of economic activity of a country;

- Analysis of influence of measures of an economic policy on economic processes and on the supposed future development.

SNA serves as an auxiliary tool of development of the economic theory, gives a general view of the level and course of development of important economic processes. However SNA is not free of faults, therefore at the international level the questions of addition of a system of the national accounts with other macroeconomic calculation methods are discussed. In particular the following is offered:

- To concord the "western" and "eastern" systems of macroeconomic calculations,

- To expand the system of national accounts by inclusion in it additional statistical methods, such as interbranch balance.

While macroeconomic calculations within the limits of SNA are based on choice of units using an institutional criterion, Input-Output Tables proceed from a functional criterion, which permits to examine the flow of each product from its manufacture to its use. The interbranch balance model makes the basis for the Input-Output analysis and is an important theoretical and practical tool to develop the economic forecasts.

From the mathematical point of view the interbranch balance model is represented by a system of linear equations, describing the processes of production of material values [17]. The correlation

$$x_i = \sum_{j=1}^{n} x_{ij} + b_i,$$

$$x_i = \sum_{j=1}^{n} a_{ij} x_j + b_i, \quad i = \overline{1, n} \tag{1}$$

establishes a dependence between use of production in various branches and their gross production. Here x_i are gross output of branches i, x_{ij}, a_{ij}, b_i are accordingly the interbranch flows of branches i to j, coefficients of direct material expenses and final consumption, n is the number of production branches.

The concept of production of material values excludes non-material services, taken into account in the system of national accounts, i.e. the flows $x_{i+k,j+k}$, $k = \overline{1,l}$, $i,j = \overline{1,n}$, , where l is the number of non-material service branches. Thus the net national income of the interbranch balance is less than the gross domestic product in the case, when flows from the non-productive sphere to the industrial one, are less than added value of a non-productive sphere.

In the matrix form the equation (1) will be copied as:

$$x = Ax + b. \tag{2}$$

The elements of matrix A satisfy to natural economic requirements

$$a_{ij} \geq 0, \quad \max_{1 \leq i \leq n} \sum_{j=1}^{n} a_{ij} < 1, \quad b_i \geq 0$$

when an interbranch value balance is considered. In this case to obtain the given vector of final consumption it is necessary to produce the goods in the following volume:

$$x = (I - A)^{-1} b, \tag{3}$$

where $(I - A)^{-1}$ is the matrix of coefficients of total inputs. However the necessary manufacture sizes determined thus for each kind of production can turn out to be unreasonable, because the possibilities of manufacture are limited by resources in cash.

In other words, the interproduct balance models give the answer to a question, what gross output is required to obtain a final product with a given structure. But in them the possibilites to manufacture this product are not taken into account, there are no reflections in them of such industrial factors as main production assets and number of people engaged in production of material values.

Some of these industrial factors are taken into account in an extended intebranch balance model, in which the manufacture volumes are co-ordinated with the parameters of comissioning of main production assets during a considered period [19].

In the extended interbranch balance model the vector of a final product is represented as $\overline{b} = b - D\Delta\Phi$, where $\Delta\Phi$ is the vector-column of putting into operation of the fixed industrial assets, D is the matrix of structural factors of capital investments, showing the capital investments of kind i necessary to put into operation the fixed industrial assets of kind j.

The balance of manufacture and distribution of product in the extended model can be written as:

$$x = Ax + D\Delta\Phi + \overline{b}. \tag{4}$$

Besides that the extended model is supplemented by a system of the fixed industrial asset balances, which permits to connect the requirements in fixed industrial assets with their volumes at the beginning of a year for branches of production of material values calculated as average annual values:

$$Fx + R = \Phi + \Lambda\Delta\Phi, \tag{5}$$

where F is the diagonal matrix of asset capacity factors, R is the vector-column of average annual exceed of fixed industrial assets put out of action, Φ is the vector-column of availability of fixed industrial assets at the beginning of a year, Λ is the diagonal matrix of coefficient to transform the actual fixed assets put into an average annual values, $\Delta\Phi$ is the vector-column of fixed industrial assets put into operation.

The solution of this model (volumes of product manufacture and fixed assets put into operation) can be obtained using various analitical or approximative methods. However the solution of systems of equations with bades stipulated matrices the approximative methods are more preferable.

Let us describe one of the methods to determine the mutual balanced solution of extended interbranch balance model [9,19].

On the first iteration of process as an initial approximation to the fixed industrial assets put into operation parameter we accept a zero value, the volumes of manufacture are determined from (4):

$$x^0 = (I - A)^{-1}\overline{b}.$$

Having substituted the obtained value x^0 in equations (5) we find the first approximation to the fixed industrial assets put into operation parameter:

$$\Delta\Phi^0 = \Lambda^{-1}(Fx^0 - \Phi + R).$$

At the following iteration:

$$x^1 = (I - A)^{-1}(D\Delta\Phi^0 + \overline{b}),$$

$$\Delta\Phi^1 = \Lambda^{-1}(Fx^1 - \Phi + R).$$

The process proceeds before the following conditions are satisfied:

$$\max_{1\le i\le n} |x_i^m - x_i^{m+1}| < \epsilon,$$

$$\max_{1\le i\le n} |\Delta\Phi^m - \Delta\Phi^{m+1}| < \epsilon.$$

Where ϵ is the given accurancy of an approximate solution of the model.

A model similar to conditions (4)–(5) could be considered, for which the reproduction of quantity of qualified workers is an endogenic variable. It is because on the one hand, the availability of the qualified staff is the factor, which determines manufacture volumes, while on the other hand, the process of personnel training requires expenses of branches of production of material values. Then the vector of a final product will be written as:

$$\tilde{b} = b - \tilde{D}\Delta L,$$

where $\tilde{D}$ is the a matrix of factors of product expenditure for branches of material values production, connected with preparation of experts for various professional groups, ΔL is the vector-column of preparation of the experts for various professional groups. In this case the balance of manufacture and the distribution of production could be written as:

$$x = Ax + \tilde{D}\Delta L + \tilde{b}, \tag{6}$$

and balance of labour resources in the section of professional groups:

$$L + \tilde{\Lambda}\Delta L = \tilde{T}x + \tilde{S}. \tag{7}$$

Here L is the vector-column of availability of experts of appropriate professional groups at the beginning of a year, $\tilde{\Lambda}$ is the diagonal matrix of factors to transform the actual receipt of the experts from a system of training of personnel to the year average, $\tilde{T}$ is the matrix of factors of labour input of production by professional groups, $\tilde{S}$ is the balance of year average interprofessional redistribution of labour resources due to change of specialisties by the workers both inside enterprises, and with moving to a new place of work, but contenting also the year average exceed of the experts of appropriate professional groups.

The extended models of the interbranch balance are a more perfect way to reflect the interrelations between manufacture and final use of production. They make a transitive stage from static to dynamic models [8].

However for implementation of the model, described above there exists a problem of parameter aggregation and disaggregation. The known methods of classical aggregation permit us to obtain integrated parameters, which enable analysis only for macromodels. Actually the large interest presents study of behaviour of initial detailed parameters.

A general theory, if under these term we understand the macroeconomic theory of the highest aggregation degree, remains only a general theory, hence, any progress can be achieved, when the researches will be directed to disaggregation.

Coordination of the solutions at various aggregation degree of parameters in cost model of the interbranch balance, named "the methods of iterative aggregation" were for the first time considered in [3].

The essence of the iterative aggregation process consists in the following:

Let a system is given, consisting from central body, the task of which is coordination of development of its subordinated branches. Let {1,2,..., n} be the set of indexes of products of the detailed nomenclature and m be the number of branches. We shall assume, that set {1,2,..., n} is broken into m groups:

$M_1, \ldots, M_m, \qquad m \le n$ such, that

$$\bigcup_{k=1}^{m} M_k = \{1, \ldots, n\}, \quad M_k \bigcap M_l = 0, \quad k \neq l$$

and the coordinated development of branches requires determination of production output parameters in the detailed nomenclature $(x_i)_1^n$, as it is a joint problem of a centre and branches.

Let on a step σ the approximation to the solution of a system of equations (2) was obtained: $x^\sigma = (x_i^\sigma)_1^n$. This approximation is used to determine specific weight of separate products in aggregates:

$$p_i^\sigma = x_i^\sigma / \sum_{j \in M_l} x_j^\sigma, \quad i \in M_l, \quad l = \overline{1, m} \tag{8}$$

and the factors of direct material inputs of production in the integrated nomenclature are calculated

$\hat{A}^\sigma = (\hat{a}_{kl}^\sigma)_1^m$:

$$\hat{a}_{kl}^\sigma = \sum_{i \in M_k} \sum_{j \in M_l} a_{ij} p_j^\sigma, \quad k, l = \overline{1, m}. \tag{9}$$

The obtained spefications are transmitted to the centre, where using the interbranch balance model of the calculation of z^σ. That is the volume of production output in the aggregated nomenclature is carried out:

$$z^\sigma = \hat{A}^\sigma z^\sigma + \hat{b}, \tag{10}$$

where

$$\hat{b} = (\hat{b}_k)_1^m, \quad \hat{b}_k = \sum_{i \in M_k} b_i, \quad k = \overline{1, m}$$

The results of the solution of an aggregated system (10) arrive to the branch level, then the disaggregation is carried out:

$$x^{\sigma+1} = A\hat{x}^{\sigma} + b, \tag{11}$$

where $\hat{x} = (\hat{x}_i^{\sigma})_1^n$, $\hat{x}_i^{\sigma} = p_i^{\sigma} z_k^{\sigma}$; $\quad i \in M_k, \quad k = \overline{1,m}$.

Further on the basis of the obtained solution $(x_i^{\sigma+1})_1^n$ the centre offers to branches to recalculate under the formula (8)–(9) the weight of each product in an appropriate aggregate and $(\hat{a}_{kl}^{\sigma+1})_1^m$.

The process is repeated until the following condition will be satisfied:

$$\max_{1 \le i \le n} |x_i^{\sigma} - x_i^{\sigma-1}| \le \epsilon, \tag{12}$$

where $\epsilon > 0$ is the given accuracy.

At present in the literature a wide class of problems is described reasonably, for implementation of which iterative aggregation algorithms are developed. To their number belongs and the problem of the optimal interbranch balance [2,10,13-16,22].

To investigation the convergence of the iterative aggregation process in models of the interbranch balance a number of works [18,20,21] is devoted. In them is theoretically proven its convergence for some special methods of variable aggregation and with the restriction $\|A\| < 1$, or with an arbitraty method of variable aggregation and $\|A\| < 1/3$.

With the purpose of finding — out of convergence of the iterative aggregation process in the general case in Moscow, Kiev, Kishinev numerical researches of this process [1,4,5,11,15] were conducted. Dependence of speed of convergence of process on the norm of the matrix of the initial system was investigated, as well as speeds of convergence of iterative aggregation process (with various methods of matrix aggregation of the initial system) with other approximate methods. Was investigated also the dependence of the speed of convergence of the iterative aggregation process on the method of aggregation and on the choice of initial approximation.

The results of numerical researches have shown, that the iterative aggregation method permits to obtain the approximate solution of a system of linear equations irrespective of the method of aggregation. In addition:

- If the norm of the matrix of the initial system of equations is less than 0.1, the number of iterations, necessary to obtain the solution of the system of linear equations using the method iterative aggregation does not depend on the number of aggregated variables;
- If the norm of the matrix of the initial system of equations is close to one, the number of iterations, necessary to obtain the solution the system, depends on the accuracy of calculations. With growth of number of aggregated variables total number of iterations, necessary to obtain of the solution with given accurancy decreases, in the case when not all variables at the initial problem participate in creation of aggregates.

For numerical comparison of speed of convergence of the iterative aggregation process with other approximate methods the same system of equations (2) was resolved by Zeidel method and successive approximation method.

The results of calculations have shown, that the method of iterative aggregation permits to obtain the solution of the model of the interbranch balance and simultaneously of the intersectorial balance. Thus speed of convergence of process of iterative aggregation is not lower than speed of convergence of a Zeidel method and is higher than speed of convergence of a successive approximation method.

We shall note, that in the iterative aggregation process described above the implementation of aggregation stages (8)–(10) and disaggregation one (11) can be carried out in various ways. Namely, the system of equations (10) can be resolved not only exactly, but also by one of approximate methods with various finishing criteria. In this case we obtain some updates of the iterative aggregation process [10,12].

It is possible to offer and other iterative processes for implementation of interbranch balance models. So, for example, at the disaggregation stage (11) of the iterative aggregation process as a basis a successive iteration method, i.e. iterative process is taken:

$$x^{\sigma} = Ax^{\sigma-1} + b. \quad (13)$$

If at disaggregation stage (11) of the iterative aggregation process when finding x^{σ} depending on $\hat{x}^{\sigma-1}$ one uses not a single iteration of a successive approximation method (13), but a single iteration of a method of simple iteration, or Zeidel, or Necrasov method, then a version of the iterative aggregation process [6,7] could be obtained.

Three versions of the iterative aggregation process in which at a disaggregation stage with reference as concerns to values $(\hat{x}_i^{\sigma})_1^n$ iteration of one of the methods, listed above is used instead of the iteration of a successive approximation method was considered.

The numerical researches were conducted in conformity with the following scheme. The system of equations of the cost interbranch balance was resolved at various initial approximations

1. Zeidel, Necrasov, successive approximations, simple iteration methods;

2. Methods of iterative aggregation and its versions.

As a result of the solution of an initial problem on the indicated scheme there was revealed, that by use of the second group of methods for solution of the system of equations with any initial approximation, the least number of iterations is necessary in the case, when the system is resolved with the help of versions of the iterative aggregation process constructed on the basis of a Necrasov and Zeidel methods. However, as for the iterative aggregation process, as for its versions, the number of iterations, necessary to obtain the solution of the system with given accuracy, decreases at increase of the number of aggregated variables. When comparing these results with the corresponding parameters, when the system (2) is resolved by known Zeidel, Necrasov, simple iteration and successive approximations methods, we notice, that even in the case, when is the aggregation of the initial problem to the one-dimensional one is carried out when solving the system (2) using the iterative aggregation method and its versions to obtain the solution with given accurancy it is necessary to execute less number of iterations, than by approximate methods, listed in 1 group.

2 Optimization models

Energy saving in agro-food complex

The permanently rising prices for the energetic resources on one hand and its consumption growth tendency on the other make the energy expenditures lowering problem be the top priority task in the agro-food complex branches.

There are diferent appoaches to the solution of this problem. We'll consider one of them in which the agro-food complex of the region is reprezented as the "input- output" model

$$x - Ax = y,$$

$$Bx = R \tag{1}$$

where A is the direct material expenditures coefficients matrix, B is the resources expenditures matrix, X is the vector of gross out-put, Y is the vector of the final product, R — vector of resources distribution.

The energy expenditures are considered the summary energy expenditures, which are composed of industrial expenditures and resources expenditures, utilized for the production of the whole set of products $x = (x_1, x_2, \ldots, x_n)$.

$$F = \sum_1^n \sum_1^n a_{ij} ek_i x_j + \sum_1^m \sum_1^n b_{ij} es_i x_j, \tag{2}$$

where $ek = (ek_j)_1^n$ is the vector of product energy equivalent, $es = (es_i)_1^m$ is the vector of resources energy equivalents.

This approach permits us to determine the branches, at which there are big energy expenditures, or in other words to find the products and resources, upon which the energy expenditures should be lowered. At the some time the approach doesn't show the way this can be done.

So it is expedient to consider the optimization models based on the model [23]. Let's formulate the problem of rational energy consumption in the regional AFC as a linear programming problem and its dual one. We'll consider the prices in the form of two vector-rows $ek = (ek_1, ek_2, \ldots, ek_n) \geq 0$, $es = (es_1, es_2, \ldots, es_m) \geq 0$, ek_j is the energy equivalent of j-th product; es_i is the energy equivalent of i-th resource, all ek_j, es_i are nonnegative and part of them are strictly positive.

The direct problem will be formulated as the maximization problem for the energy mainteinance of the final product, by choosing nonnegative gross outputs of all the products, which satisfy the ballance and resource distribution equations. Mathematically the problem is written in the follwing form:

$$\begin{aligned} &\max_x eky \\ &x = Ax + y \\ &Bx \leq R. \end{aligned} \tag{3}$$

Excluding y from the cost function using the first equation from (3), and introdusing the notation $EK = (I - A)ek$, I is an unitary matrix, we'll rewrite (3) in the following form:

$$\begin{aligned} &\max_x EKx \\ &Bx \leq R \\ &x \geq 0. \end{aligned} \tag{4}$$

Since cost function characterizes the energy value of the final product, the variables of the dual problem will be the resources energy equivalents

$$\begin{aligned} &\max_{es} esR \\ &ekA + esB \geq ek \\ &es \geq 0, \end{aligned} \tag{5}$$

that is means that it is necessary to minimize the energy cost of the resources by choosing nonnegative values of the energy equivalents of all the resources, which satisfy the condition that the energy expenditures for the production of any product are not less than the energy equivalent of the corresponding product.

But this means, that as applied to our system, in the equilibrium state

$$\sum_{1}^{n} ek_l a_{lj} + \sum_{1}^{m} es_i b_{ij} \quad \geq \quad ek_j, \tag{6}$$

$ek_l a_{lj}$ are energy expenditures for the product, necessary to produce one unit of the j-th product; $es_i b_{ij}$ are energy expenditures of the i-th resource, necessary to produce one unit of j-th product. The inequation (6) characterizes the entropy of the system, or that part of its internal energy that is being constrained.

The considered problems can be formulated in a same different way:

$$\begin{aligned} &\max_{ek} eky \\ (I-A)ek \quad &\leq \quad esB, \\ ek \quad &\geq \quad 0, \end{aligned} \tag{7}$$

$$\begin{aligned} &\min_{x} esBx \\ (I-A)x \quad &\geq \quad y \\ x \quad &\geq \quad 0 \end{aligned} \tag{8}$$

In this case we suppose that the restrictions of the resources are given in the form of equalities, but the production and distributioin of the products are given in the form of inequalities, so as $(I-A)x \quad \geq \quad y$, $Bx \quad = \quad R$. In the direct problem it is necessary to define such a nonnegative vector-row of the products energy equivalents (ek), that when the conditions (7) are fulfilled we obtain the maximal energy output of the final product. In the dual problem it is necessary to define such a vector-column of gross outputs, that when the conditions (8) are fulfilled the energy expenditures of the resources are minimized.

Such a formulation of the optimization problem permits us to find the energy equivalents of the products in the direct problem and the gross outputs which corresponds to this equivalents in the dual one. Solving the problems (4)–(5), (7)–(8) we can find the energy expenditures for the production of the given set of products $x = (x_1, x_2, \dots, x_n)$.

So the problem of the optimal utilization of energy resources exhaustes itself as far as the resources are not infinite and the energy containance is also limited and the technological methods of production and the expenditure matrix are fixed.

In connection to this it is expedient to consider the definition of the problem of optimal energy utilization, in which we can change the technological methods as a whole, or some of its components, to have the possibility to liquidate the losses of eneergy at the production of products or resources.

Further on we'll consider the energy saving problem in the form of a linear programming problem with variable coefficients, when we surppose that one or some of the columns of the resource expenditure matrix are variable. Such problems were considered in [3].

In connection with this we'll formulate the following problem of nonlinear programming:

1. The set T^n of vectors $A^j = (a_{j1}, a_{j2}, \dots, a_{jn})$, that describe technological methods of production is given, $T^n \subset E^n$ — n-dimensional Euclidian space of limited sequences. The totality of vectors reprezents the technological matrix A, $(\sum a_{ij} < 1)$.

2. The resources expenditures matrix is given $B^i = (b_{ij})_1^m$ - m-dimensional vector-column.

3. The vector-column of restricitions upon resources is given: - $R = (r_1, \dots, r_m)$.

4. The value of the final product vector-column is defined: - $Y = (y_1, \dots, y_n)$.

5. The energy equivalents vectors of the products and resources are given.

It is necessary to find the set of technological methods A_j, (j can take all the values from 1 to n or only a part of them), which moves up the matrix A, and the gross output vector $x = (x_1, x_2, \dots, x_n)$, which coresponds to it, such that the following conditions are satisfied

$$X(A) - AX(A) = Y \tag{14}$$

$$BX(A) \leq R \tag{15}$$

and the cost function $F = (ekA + esB)X(A)$ is minimized, that is

$$\min F$$

$$A^j \in E^n, j = 1, \dots, n \tag{16}$$

We'll rewrite the problem (14)–(16), using the equality (14) in the following form

$$\min_{A^j \in E^n, j=\overline{(1,n)}}, (ekA + esB)(I - A)^{-1}Y \tag{17}$$

$$\begin{aligned} B(I-A)^{-1}Y &\leq R, \\ (A^j)_1^n &\geq 0 \end{aligned} \tag{18}$$

in such a way we have obtained a nonlinear programming problem(17)–(18), for which we'll formulate the Cuhn-Tacker conditions. For this purpose we'll introduce the vector-row of Lagrange multipliers $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$, the number of wich is equal to the number of restrictions. We'll also define the Lagrange's function

$$L(\Lambda, A) = (ekA + esB)(I - A)^{-1}Y - \Lambda(B(I - A)^{-1}Y - R), \tag{19}$$

with the help of which the Cuhn-Tacker conditions will be written in the following way

$$\frac{\partial L}{\partial A} = (ek(I-A)^{-1} + Aek\frac{\partial (I-A)^{-1}}{\partial A})Y +$$

$$(esB - \Lambda B)\frac{\partial (I-A)^{-1}}{\partial A}Y \geq 0$$

$$\frac{\partial L}{\partial A} = A((ek(I-A)^{-1} + Aek\frac{\partial (I-A)^{-1}}{\partial A})Y +$$

$$(esB - \Lambda B)\frac{\partial (I-A)^{-1}}{\partial A}Y) = 0 \qquad (20)$$

$$\frac{\partial L}{\partial \Lambda} = -B(I-A)^{-1}Y + R \quad \leq \quad 0$$

$$\frac{\partial L}{\partial \Lambda}\Lambda = \Lambda(R - B(I-A)^{-1}Y) = 0 \qquad (21)$$

$\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m) \geq 0$, $A = (A^1, A^2, \ldots, A^n) \geq 0$. Thise conditions are necessary and suffisient for the existence of the local minimum, if the cost function is convex and the restriction functions are concave, and besides this the regularity of restrictions condition is fulfilled. This condition consists of the following: in some points of the admissible set all the restriction- inequalities are satisfied as strict inequalities.

Cuhn-Tacker Theorem:

Let the function $F(A)$ be a convex one, the functions $g_i = R_i - B^i X(A)$ be concave and the restrictions regularity condition be satisfied, than the matrix A is the solution of the problem (16)–(17) then and only then, when exists such a vector Λ, that A^*, Λ^* is the saddle points of the Lagrange function, this means that $L(A, \Lambda^*) \leq L(A^*, \Lambda^*) \leq L(A^*, \Lambda)$ for all $A \geq 0$, $\Lambda \geq 0$. The proof of the theorem is given in [24]. Although the Cuhn-Tacker condition gives the full characteristic of the solution, they do not contain the constructive method for finding the solution. To find the solution we can use gradient methods, that are based on the fact that the gradients of the cost function $\frac{\partial F}{\partial A} = ekY(I-A)^{-1} + ekAY\frac{\partial (I-A)^{-1}}{\partial A} + esBY\frac{\partial (I-A)^{-1}}{\partial A}$ give the direction of the fastest growth of the cost function. Particularly, the gradient projection method can be used. In this method the restrictions and the displacements of the points are in in the direction of the gradient. In the case when the motion in the gradient direction can move us out of the domain's boundary, the displacement is in the direction of the gradient's projection on the plane, which is tangent to the boundary. Such a displacement does't take the point out of the admissible domain and is permanently increasig the cost function value.

In practice we can take the following cost function: $F = (ekA + esB)E$, $E = (1, 1, \dots 1)$ is an unitary vector-column. With the help of this cost function the energy expenditures, necessary to produse a unitary vector of the product set is found. Since $F = (F_1, F_2, \dots, F_n) = (ekA^1 + esB^1, ekA^2 + esB^2, \dots, ekA^n + esB^n)E$, then $\frac{\partial F}{\partial A} = (\frac{\partial F}{\partial A^1}, \frac{\partial F}{\partial A^2}, \dots, \frac{\partial F}{\partial A^n}) = (ek_1, ek_2, \dots, ek_n)$, this means that the cost function gradient is the product energy equivalents vector.

Optimal control and its use at the modeling of economic processes

The theory of optimal control for distributed parameter systems is adjusted for solving the problems of determining the controled temperature fields of hidrotehnical construction with the aim of appointment of the tehnical and technological operations, which assure their integrity. The problem is simulated with the help of the conduction equation with initial and boundary condition. The control function is introduced as the heat flowing, boundary influence or as the thermal diffusivity coefficient.

An important place among the technical problems belongs to the continuos heat apportionment systems. The heat emission stipulates the appearance of complex temperature fields in the systems, that causes the appearance and concentration of temperature tensions, that further bring to the formation of cracks, stratifications and other defects, which lead to the distruction of the buildings and constructions. An example of sach systems can be the massive concrete constructions.

The cause of appearance of cracks in them is the temperature tensions, which appear at the internal heat apportionment, because of great differences between the temperatures in the internal and external parts of the concrete massive and deformation at its cooling to the normal exploitation temperature.

To present the formation of cracks in the concrete laying in the period of building of hydrotechnical constructions it's necessary to support certain temperature regimes, at which the tensions values do not exceld the permisible ones.

In this connection the problem of computing the controlled temperature fields of such constructions is of high importance. The computed temperature fields permit to take off temperature picks, and also to carry out corresponding technical and technological measures, which assume the achievement of given temperatures in a period of construction.

Mathematically this problem is given by the parabolic equation with initial and boundary conditions. The unstationary control action is found in the right part of the heat conduction equation as the densites of sources distribution, and in the boundary conditions as the heat flowing or is the thermal diffusivity coefficient [26]-[30].

The criterium of optimization is considered the square cost function. The one- two- and three-dimensional control problems are considered depending of the aims of investigation.

Control problem.

First we make the same assumptions concerning the erection of massive concrete constructions, as we'll state the optimal control problem of the temperature regime of the cooling concrete massive.

- Concrete is considered a quasihomogeneous izotropic material.
- The thermophysical characteristics of the concrete are constant values, that do not depend neither on time, nor on temperature, except the case when one of the thermophysical coefficients is a control influence.
- After the concrete laying a heat emision process begins, which is determined by the expression:

  $$q = q_0 e^{-mt},$$

  where q is the quantity of heat, emitted in a unit time, q_0, m are constants, t is the time.
- We'll consider that the control $p(t, x) \in L_2$. In the case when $p(t, x)$ enters in the heat conduction equation or in the boundary

condition as the heat flowing, this can be technically performed as pipe cooling system with pipes situated enouqh close to one another. In the case, when the control is a thermal diffusivity coefficient, this can be one of the compound components with corresponding properties.

An unbounded concrete wall of thickness R, one side of which is thermo-isolated, and on the other the heat exchange with the environment takes place is considered. The controlled process is given by the parabolic equation:

$$\frac{\partial U(t,x)}{\partial t} = \frac{\partial (ap(x)\frac{\partial U(t,x)}{\partial x})}{\partial x} + q_0 e^{-mt} + bp(t,x), \tag{A}$$

$$t \in (0, t_1], \ \ x \in (0, R)$$

with the initial condition

$$U(0,x) = \varphi(x), \ \ x \in (0, R) \tag{B}$$

and boundary conditions

$$\frac{\partial U(t,0)}{\partial x} = 0, \frac{\partial U(t,R)}{\partial x} = \alpha[cp(t) - U(t,R)], \ \ 0 < t \leq t_1, \tag{C}$$

here a is the thermal diffusivity coefficient, α is the heat exchange coefficient.

Now let's formulate the optimal control problem. Let t_1 be fixed constant. It's necessary to find such a control function $p(x)$, that at the moment of time $t = t_1$ the corresponding solution of the boundary problem (A) - (C) satisfies the condition:

$$U(t_1, x) = y(x), \ \ y(x) \in L_2(0, R), \tag{D}$$

and the cost function

$$J(p) = \int_0^R \int_0^{t_1} [ap^2(x) + bp^2(t,x) + cp^2(t)]dtdx \tag{E}$$

achieves the minimal value. a, b, c are constants, b and c get the value 1 or 0, depending of the control function considered.

Opportunity of application of the optimal control theory for the description of macroeconomic development by means of dynamic model [25] is considered. A problem of optimal control and necessary conditions of the optimality in the form of Pontreagin's principle of maximum is formulated.

It is supposed, that all economy is agregated in one unique branch, in which set of the manufacturers makes one unique product — national income. For manufacturing of unit of production is used a unique resource — labour. Industrial units differ one from other only at the expense of capacities and labour costs λ, necessary for manufacture of unit of production. The Technological structure of economy is determined with the help of a function of distribution of a share of capacities $M(t)$, appropriate to scales of necessary labour costs.

The small independent firms operate manufacture so that proceeding from a market conjuncture, maximize the curent income, forming thus demand for the industrial goods and labour resources. All products and resources are purchased in the market on one whole price and are paid by one legal tender.

The social structure of a system is formed by two groups: workers and proprietors. The Incomes of workers are determined by a wages, which they spend for consumption. The incomes of proprietors are formed of the account dividents of firms and banks and at the expense of interests payments on the lending capital, the part of these incomes is used for consumption while the other part is used for savings.

We shall introduce production function $f(x)$, which describes dependence between volume of production $Y(t)$, made in a time unit on the capacity output $M(t)$ and quantity of used labour resources $R^L(t)$:

$$M(t) = M(t) * f(x), \qquad f(x) = R^L(t)/M(t). \tag{22}$$

The function $f(x)$ is defined with the help of following dependences,

$$f(x) = \int_{\nu}^{\xi(x)} \Psi(\lambda) d\lambda,$$

$$x = \int_{\nu}^{\xi(x)} \lambda * \Psi(\lambda) d\lambda, \tag{23}$$

$$x \leq \overline{x} = \int_{\nu^-}^{\nu^+} \lambda * \Psi(\lambda) d\lambda,$$

where λ is labour costs, used for manufacturing of a production unit $\nu^- = \inf(\lambda|\Psi(\lambda) > 0)$, $\nu^- = \sup(\lambda|\Psi(\lambda) > 0)$, it has the following properties

$$f(0) = 0, \quad f(\overline{x}) = 1, \qquad f'(x) = 1/\xi, \qquad f''(x) < 0, \tag{24}$$

when $0 \leq x \leq \overline{x}$.

The demand for labour resources $R^L(t)$ is also defined by means of function $f(x)$ knowing the price for production $p(t)$ and wages $s(t)$,

$$R^L(t) = \overline{x} * M(t), \quad \overline{x} = 0 \quad if \quad p(t)/s(t) = \nu, \tag{25}$$

$$f'(x) = s(t)/p(t), \;\; if \;\; \nu < p(t)/s(t) < \nu^+, \quad \overline{x} = x^*, \;\; if \;\; p/s \geq \nu^+$$

We shall proceed to the description of process of expansion of manufacture. Thus the change of all capacities of a system will be described with the help of the following equation

$$dM(t)/dt = I(t) - \mu * M(t), \tag{26}$$

μ is the constant rate of capacities drop out, $I(t)$ newly created industrial capacities in time unit. It is necessary for firms to acquire $I(t)$ in the market the goods in quantity, for creation of the capacity output $X^I = b * I(t)$ where b is rate of increasing fund capacity For purchase of the given goods $I(t)$ it is necessary to acquire means

$$\Phi^I(t) = p * b * I(t). \tag{27}$$

It is supposed, that all these means firms receive as the long-term credits under interest from banks, and the received loans are completely used on the capital investments.

The change in time of the volume of long-term loans, granted by banks to firms L, occurs owing to issue by bank of the new loans on a sum $\Phi^I(t)$, debt servicing by firms $G(t)$ and charge of interests payments $r_1 * L(t)$ under the indeptedness of firms. Thus:

$$dL(t)/dt = \Phi^I(t) - G(t) + r_1 * L(t). \tag{28}$$

The balance cost of fixed capital $K(t)$ is considered as maintenance of the loans. The change of the balance cost is described by an equation

$$dK(t)/dt = \Phi^I(t) - \beta * K(t), \tag{29}$$

β — rate of amortization.

It is supposed, that the firms hold maximally possible sum of the loans

$$K(t) = L(t), \tag{30}$$

then from (29) and (30) we obtain, that

$$dL(t)/dt = \Phi^I(t) - \beta * L(t), \tag{31}$$

$$G(t) = (r_1 + \beta) * L(t). \tag{32}$$

We shall consider, that the demand of firms for the long-term credits is described by a following parity

$$\Phi^I = p(t) * \sigma(r_1) * M(t), \tag{33}$$

r_1 is the nominal interest rate for the credit. The function $\sigma(r_1$ describes in a aggregated kind a policy of firms in the field of expansion of manufacture.

The balance of current payments of firm will be formed of the proceeds at the expense of sale of the goods at the moment of time $(t-\theta)$, $\Phi(t-\theta) = \Phi(t) - \theta * \Phi'(t)$. (θ — time of a turn-overof legal tenders in a system). This sum is spent for the salary $s(t) * R^L(t)$ engaged in manufacture and debt servicing to bank $G(t)$. The remaining part is wholly paid to the proprietors in a kind of dividents on a sum:

$$d = \Phi(t) - \theta * d\Phi(t)/dt - G(t) - s(t) * R^L(t). \tag{34}$$

Substituting in (13) expression for from (11), we receive

$$\Phi(t) - d\Phi(t)/dt = d + s(t) * R^L(t) + r_1 * L(t) + \beta * L(t). \qquad (35)$$

Source of the long-term credits are the savings of the proprietors, located in a kind of the contributions in a bank system. In these contributions all incomes of the proprietors address, and from them means are withdrawed, which proprietors spend for consumption Φ^0. The consumer charges ofthe proprietors depend on the interest rate r_2, which bank pays under the contributions, and from some size $p(t)*Y(t)$, describing flow of incomes of proprietors

$$\Phi^0 = \eta(r_2) * \Pi, \qquad (36)$$

where $\Pi = p(t) * Y(t)$ or $\Pi = d$, and $\eta(r_2)$ is the function of propensity to consumption.

We shall consider now functioning of a bank system and of the capital market . The bank accepts from the proprietors the contributions, on which pay interest payments. The sum of these contributions will form liabilities of bank $D(t)$. A part of means assembled thus bank leaves in a kind of reserve $R(t)$ and other part grants to firms in a kind of the loans under percent rate r_1. It is supposed, that the long-term credits bank gives only to firms. Therefore the sum of indebtedness of firms coincides sum whith the loans given by bank. A reserve $R(t)$ and the sum of the loans given by bank make asset of bank. In each moment of a time the sum of assets should be equal for a sum of liabilities, i.e. balance of bank bank should

$$L(t) + R(t) = D(t). \qquad (37)$$

The reserve is necessary for bank, to ensure issue of the contributions under the first requirement. Usually the legislation requires, that the reserve made a certain share $R(t) \geq \chi * D(t)$ from the deposits, norm of reservation χ, controllable by state. All free means the bank system aims to grant on credit, to extract a profit. So actually the reserve keeps at a minimum and allowable level

$$R(t) = \chi * D(t). \tag{38}$$

We shall consider, that the reserve will be formed by a part of a made product, which we shall conditionally name as gold. To fill up a reserve, the bank should accept the contributions not only in kind of legal tenders issued to them, but also directly in a kind of gold. The price of gold is considered fixed. It is supposed, that the cost of gold made in time unit $E(t)$ is proportional to a manufactured product $E(t) = k * Y(t)$, where k is a constant. All this sum arrives in bank in a kind of the contributions and falls in a bank reserve. Therefore

$$dR(t)/dt = k * Y(t). \tag{39}$$

The profit, which can be formed from a difference of interest rates r_1 and r_2 and arrives to the proprietors of bank in a kind of dividends and level with the other incomes of the proprietors increases their contributions. Thus bank liabilities (contributions of the proprietors in bank) consits from the account of: a) dividents of firms d, b) dividents of bank d_B, c) of the gold contributions $k * Y(t)$ and d) at the expense of charge of interests payments under the early made contributions. Hence, the change in a time of bank liabilities D is subject to an equation

$$dD(t)/dt = d + d_B + r_2 * D(t) + k * Y(t) - \Phi^0. \tag{40}$$

For completion of the description of functioning of a bank system we shall define interest rates r_1 and r_2. From equations (38)–(37), (39) and (31) we give following corelation

$$\Phi^I(t) = (1/\chi - 1) * k * Y(t) + \beta * L(t). \tag{41}$$

which defines proposal of the loans from the bank.

It is supposed, that the market of the loans is constantly in balance, i.e. the demand for the loans is equal for their proposal. Then we shall receive by virtue of (33) equation for dtermination interest rate r_1

$$\Phi^I(t) = p * \sigma(r_1) * M(t) = (1/\chi - 1) * k * Y(t) + \beta * L(t). \qquad (42)$$

For its appreciation, it is necessary to know size of d_B included in equation (19). To define size is possible, having assumed, that the rate of bank profit is equal to rate of return on the capital for manufacture. Rate of return of firms is calculated as the ratio of a profit of firms to the balance cost of their capacity output. Rate of return of bank is calculated as the ratio of a bank profit d to the own capital of bank $L(t)$. The analysis of data shows, that the reserve of bank only slightly differs from the own capital of bank. Therefore the condition of equality of rates of return can be recorded in a kind

$$d_B/R(t) = d/R(t) \qquad or \qquad d_B = d * \chi/(1 - \chi) \qquad (43)$$

or $\chi << 1$ as far as we shall here in after neglect size in comparison with. Then (40) accepts a kind

$$dD(t)/dt = d + k * Y(t) - \Phi^0 + r_2 * D(t). \qquad (44)$$

From (17) follows, that interest rate should be established at such level, that to ensure to bank inflow of the deposits $dD(t)/dt = k * Y(t)/\chi$. Then from (44) we receive a equation for r_2

$$r_2 = (-d + \Phi^0 + k * Y(t) * (1/\chi - 1))/D(t). \qquad (45)$$

For completion of the description of model it is necessary to define the proposal of labour resources $\tilde{R}^L(t)$, price of a product $p(t)$ and rate of a wages $s(t)$.

The proposal of labour by the workers is determined by given quantity able-bodied and level of their material consumption

$$\tilde{R}^L(t) = P^A(t) * W(m), \quad \text{where } m = s(t) * R^L(t)/p(t) * P(t), \quad (46)$$

where $P(t)$ — number of the workers, quantity able-bodied, $P^A(t)$ - number of the workers, quantity able-bodied, m - level of material consumption, and the kind of function $0 < W(m) \leq 1$, $m \in [0, \infty]$ may be reasonably widely varied.

The change of the rate of a wages is described by a equation

$$ds(t)/dt = 1/\Delta * \max[0, p(t) * f(\overline{x}) - s], \qquad \overline{x} = \tilde{R}^L(t)/M(t). \quad (47)$$

The change of the price of a product is described by a equation

$$dp(t)/dt = -\alpha * Q(t)/M(t), \quad (48)$$

where $Q(t)$ is surplus of a stock of a product. Thus

$$Q(t)/dt = Y(t) - \Phi(t)/p(t). \quad (49)$$

And

$$\Phi(t) = \Phi^I(t) + s(t) * R(t) + \Phi^0 \quad (50)$$

is the cumulative proceeds of firms.

System of equations : (22), (23), (26), (45), (46), (25), (47-50), (26), (31-33), (35), (37), (39-40), (43) is closed and permits to define a state of economic system in any moment of a time, if is given a initial condition:

$$M(t_{0)} = M_0, Q(t_0) = Q_0, \Phi(t_0) = \Phi_0,$$

$$p(t_0) = p_0, s(t_0) = p_0, L(t_0) = L_0$$

and function of a time $P(t)$ and $P^A(t)$.

We shall formulate a problem of optimal control. In quality of control function we shall choose the investment $I(t) = u(t)$, which we shall rename in with the area of change $U = \{u(t)|u_1 \leq u \leq u_2\}$. Through $Z(t) = \{M(t), L(t), \Phi(t), s(t), p(t), Q(t), Y(t)\}$ we shall designate a vector. And then the problem of optimal control is defined as follows: It is necessary to determine such a control function $u(t) \in U, t \in [t_0, t_1]\}$ and the solution appropriate to it of a system : such, that the cost function reaches a minimum

$$I(\overline{u}) = \min_{u \in U} 1/2 \int_{t_0}^{t_1} [Y(t) - \overline{Y}(t)]^2 dt \quad (51)$$

Following theorem takes place.

Theorem (maximum‘s principle):

In order the process $(z(t), u(t)) \quad t_0 \leq t \leq t_1$ be optimum in a sense of criterion (30) it is necessary existence of a constant Ψ_0 and nonzero for any moment of time vector $\Psi(t) = \{\Psi_1(t), \Psi_2(t), \ldots, \Psi_m(t), t_0 \leq t \leq t_1\}$ that a following condition to fulfile

$$H(\Psi(t), Z(t), \overline{u}(t)) = \max_{u \in U} H(\Psi(t), Z(t), u(t)), \quad (52)$$

where $H = -F_0(Z, u) + \sum_1^m \Psi^k * F^k(Z, u)$

$$F^0 = 1/2[Y(t) - \overline{Y}(t)]^2,$$

$$F^1 = u(t)/p(t) * b - \mu * M(t),$$

$$F^2 = u(t) - \beta * L(t),$$

$$F^3 = (u(t) + \Phi_0 - d - (r_1 + \beta) * L(t)/\theta, \quad (53)$$

$$F^4 = 1/ \; delta \max 0, p(t)/\nu - s(t),$$

$$F^5 = -\alpha * Q(t)/M(t),$$

$$F^6 = Y(t) - u(t)/p(t),$$

and thus vector-function $\Psi(t)$, which satisfies to a system of equations

$$dPsi_i(t)/dt = -\frac{\partial H(\Psi, Z, u)}{\partial Z_i}, \quad (54)$$

$$i = 1, 2, \ldots, m, \quad \Psi_i(t_1) = 0.$$

References

[1] Arhangelskii Iu., Vahutinskii I.IA., Dudkin L.M., Dyakonova M.E., Novoselskii I.A., Homiacov V.A. Numerical researches of the iterative aggregation methods for the decision of a problem of the interbranch balance. - Automatics and telemechanics, Moscow, 1985, N 7 (Russian).

[2] Dudkin L.M. A system of accounts optimum economic plan. - Moscow, 1972 (Russian).

[3] Dudkin L.M., Ershov E.B. The interbranch balance and material balances of separate products. - A scheduled economy, Moscow, 1965, N 5 (Russian).

[4] Dyakonova M.E. Algorithm of iterative aggregation.- The Newsletter Algorithms and programs, Moscow, 1973, N 2 (Russian).

[5] Dyakonova M.E. Application of the iterative agregation method for the decision of systems of linear equations of large dimension.- The Newsletter: Algorithms and programs, Moscow, 1975, N 2 (Russian).

[6] Dyakonova M.E., Kojuhar P.V. Numerical researches of some versions and updatings of iterative agregation process for the decision of a problem of the interbranch balance.- Mathematical researches, Kishinev, 1989, N 110 (Russian).

[7] Dyakonova M.E., Kojuhar P.V., Novoselskii I.A. Models and methods of planning of interbranch proportions, Kishinev, 1990 (Russian).

[8] Dyakonova M.E., Kojuhar P.V., Naval E.S., Smolina L.I. Use of balance dynamic model for planning of manufacture and distribution of production of the republic. - Mathematical researches, Kishinev, 1990, N 114 (Russian).

[9] Dyakonov M.E., Kojuhar P.V.,Naval E.S., Smolina L.I. Experimental accounts on extended model of the interbranch balance (

on a example Moldavian Republic).- Systems of optimization and data processing. Kishinev, 1988 (Russian).

[10] Dyakonova M.E., Novoselskii I.A. Algorithm of the decision of one generalized model of the interbranch balance. - Applied mathematician and programming. Kishinev, 1975 (Russian).

[11] Dyakonova M.E., Novoselskii I.A. Research of the iterative aggregation algorithm for the decision of a problem of the interbranch balance. A economy and mathematical methods, Moscow, 1976, v. XII, N 3 (Russian).

[12] Dyakonova M.E., Novoselskii I.A. About one property of iterative aggregation process.- Of model of planning and management of branches and enterprises, Kishinev, 1976 (Russian).

[13] Dyakonova M.E., Novoselskii I.A. Numerical researches of the iterative agregation process for the decision a optimum interbranch balance problem.- Mathematical researches, Kishinev, 1979, N 52 (Russian).

[14] Dyakonova M.E., Novoselskii I.A. Experimental accounts on optimum interbranch balance model .- Mathematical researches, Kishinev, 1980, .60 (Russian).

[15] Dyakonova M.E. The iterative aggregation in the interbranch and optimum interbranch models. - Of news of Moldavian Academy of Science. A series physical and mathematical sciences, 1980, N 3 (Russian).

[16] Dyakonova M.E., Novocelskii I.A. About experimental accounts on optimum interbranch balance model.- A economy and mathematical methods, Moscow, 1982, v. XYIII, N 1 (Russian).

[17] Kossov V.V. Interbranch models. - Moscow, 1973 (Russian).

[18] Leashenko I.N. The iterative aggregation in interbranch balance.- Research of operations and MANAGEMENT INFORMATION SYSTEM. Kiev, 1980, N 15 (Russian).

[19] Modeling of economic processes / Under edition Dadaean, Moscow, 1973 (Russian).

[20] Hizder L.A. The proof of convergence of the iterative aggregation method in general case.- Of research on mathematical economies and adjacent questions, Moscow, 1971 (Russian).

[21] Scennico B.A. Application of the iterative aggregation method for the decision linear systems.- A economy and mathematical methods, 1966, v. II, N 5 (Russian).

[22] The theory of iterative aggregation. L.M.Dudkin ed., New York, 1980.

[23] L. Batirmurzaev, M. Diaconov, P. Cojuhari, E. Naval, L. Smolin. Constraction the energy balance report of the regional agro-food complex. Mathematical research, ed.121, Kichinev "Knowledge",1991.(Russian)

[24] M. Intriligator. Mathematical Optimization and Economic Theory. - M., Progress, 1975.(Russian)

[25] A.P. Crutov,I.G. Pospelov. To the system analisys of the developing economy: Influense of the banking system. Technics cybernetics, N 6,Moscow,1981 (Russian)

[26] A.Egorov, E.Naval. About optimal control of heat conductivity process. Applyed mathematic and programming. Kishinev, Knowledge. 1975, Vol.13, p.44-63 (Russian)

[27] E.Naval Abaut optimal control of heat conductivity process in the hollow cylinder. Mathematic research. Kishinev, Knowledge. 1980, Vol.57, p.77-82 (Russian)

[28] E.Naval Optimal control of two-dimensional temperature fields. Mathematic research. Kishinev, Knowledge. 1988, Vol.101, p.89-97 (Russian)

[29] E.Naval About rational location of cooling pipes in the dig (one dimensional problem). News of Academy of sciences of Moldova, Ph.-Tec.and Math.series. 1988, No 2,p.14-19. (Russian)

[30] E.Naval Thermal diffuzivity control of the parabolic system. Mathematic research. Kishinev, Knowledge. 1991, Vol.121, p.107-113 (Russian)

M.Dyakonova, E.Naval,
Institute of Mathematics,
Academy of Sciences, Moldova
5, Academiei str., Kishinev,
277028, Moldova
e-mail: 32zamdk@math. moldova.su

Received August 23, 1996