Computer Science Journal of Moldova, vol.2, no.3(6), 1994

Complex analysis of interbranch connections in national economy

M.Dyakonova P.Kojuhar

Abstract

Diferrent aspects of investigation of the interbranch connections of the national economy of the republic are prezented in this article which use not only money estimations but energy estimations also as a method of evaluation of production. Some approaches to the reflection in of import products and calculation of balanced volumes of the production in IB (intersectoral balance) are considered here.

At present during the transition to new methods of economic management it is necessary to study thoroughly the state of republic's economy, developing in the complicated network of the interbranch interconnections. The method of the interbranch analysis (input — output analysis), based on the usage of the intersectoral balance tables is usually used for its studying, forecasting and regulation. During the process of investigation of the interbranch connections we shall consider cost and energy aspects besides material aspects.

It is necessary that the price mechanism and incomes will be formed during the process of creation, distribution and redistribution of the national income under conditions of equality of aggregate costs of the final product and conventionally net products at cost aspect.

Energy aspect of the investigation of the interbranch connections means the usage of energy units of measurement instead of monetary units for the evaluation of the production with the aim of conducting of energy analysis and finding energy and economical ways of developing of republic's economy.

At first, we shall write in a general aspect the model input — output [1]:

$$x = Ax + y. \tag{1}$$

Here

$x = (x_i)_1^n$ — are the volumes of the production's output;

$A = (a_{ij})_{1,1}^{n,n}$ — is the coefficient matrix of direct material expenses;

$y = (y_i)_1^n$ — is the vector of the final product.

For the republics with the evident open character of reproduction as Moldova is, it is necessary to pay special attention to the problem of import of products and resources. This fact needs to do the corresponding modifications in the model of intersectoral balance allowing to determine the volumes of import and to take into account separately its influence on the level of production of intermediate and final products.

In this connection we shall choose from the vector of the final product y import vector $V = (v_i)_1^n$, then correlation (1) will be rewritten as:

$$x = Ax + \overline{y} - V. \tag{2}$$

Let R_{ij} — is consumption of the product i, made in republic, at the output of x_j unit of the production of the branch j; S_{ij} — is consumption of the product i, imported to the republic at the production of x_j units of the product j; $y^r = (y_i^r)_1^n$ — is the vector of the final product, formed only with the help of the production, made in the republic; $y^v = (y_i^v)_1^n$ — is the vector of the final product, formed only owing to the production, imported to the republic.

Let's consider the model of the intersectorial balance (IB) for the case when the expenses of the imported production and made in the republic are not separated from each other for productive and final consumption. Then the coefficients of the direct material expenses and import vector have the view:

$$a_{ij} = (R_{ij} + S_{ij})/x_j = r_{ij} + s_{ij}; \;\; i,j = \overline{1,n}. \tag{3}$$

$$v_i = S_{ij} + y_i^v, \;\; i = \overline{1,n}, \tag{4}$$

and the balanced volumes of the output is calculated according to the formula:

$$x = (I - A)^{-1}(\overline{y} - V). \tag{5}$$

There is another approach to the reflection in IB of consumption of the imported production: productive and final consumption of the republic's branches are separated from the expenses of the product, imported to the republic for the same productive and final consumption. In this case, taking into account the correlations (4),

$$x = (I - R)^{-1} y^r, \tag{6}$$

$$V = S(I - R)^{-1} y^r + y^v. \tag{7}$$

Here $R = (r_{ij})_1^n$ and $S = (s_{ij})_1^n$ — are coefficient matrices of direct material expenses of republican's and imported production. It follows from (3), that $A = R + S$ and considering the common case, when the imported production goes for final and productive consumption ($y^v - V \leq 0$), it is not difficult to show that in order that the results of realization of the given models (formula (5) and (6)) will be identical it is necessary, that

$$[(I - R) - S]^{-1} \geq (I - R)^{-1}.$$

However, in reality the model of the first type is usually used, as the model of IB with isolated expenses of the imported and republican's production can't be built in practice because of the absence of the necessary information. Besides, the coefficient of the matrix A are more stable than coefficients $(r_{ij})_1^n$ and $(s_{ij})_1^n$ — as they depend on economic and technological circumstances of the production and their meanings are not sensitive to the fluctuations of the expenses of the import and the republican production [2]. In this connection we often use the model of competitive and import type, in which the import vector is assumed to be proportional to the total interrepublican consummption. Then the coefficient of import for every branch is such a way:

$$k_i = v_i/(\sum_{j=1}^{n} a_{ij}x_j + y_i^s), \;\; i = \overline{1,n}, \tag{8}$$

where y_i^s — is a part of the final product of the branch i, without including the corresponding value — balance of export–import.

We denote the diagonal matrix of coefficient of import through $\overline{V}$

$$\overline{V} = \begin{pmatrix} k_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & k_n \end{pmatrix}$$

and througt y_i^z — export of the product i outside the republic $y_i^s + y_i^z = \overline{y_i}$, then model IB we can write down as:

$$x = Ax + y^s + y^z - \overline{V}(Ax + y^s).$$

and then the volumes of the production which provide the given final product are calculated according the formula:

$$x = [I - (I - \overline{V})A]^{-1}[y^z + (I - \overline{V})y^s]. \tag{9}$$

So the correlations (5),(6),(9) for every concrete case allow to coordinate the levels of production with the levels of the final consumption. However it may happen, that the vector of the forecasting final consumption assumes such volumes of production which it can't achieve at existing power.

In this case we can expect that in those branches where the productive powers are limited, the periods of delivery should be prolonged and prices should arise. As a result of this the consumers of the production of these branches should find the means of decreasing of its consumption. And the branches with the lack of productive powers under the stimulating influence of more high prices should try to increase powers, though in practice we shall need much time for that. The violations in the date of delivery and deficits will take place until the prices fulfil the function of balance between demand and supply.

Models (1) and (2) don't take into account such a situation, but nevertheless they are useful in other aspects. In particular, we can calculate the influence of the limited productive powers of certain branches on the final consumption.

Let's denote through I a set of indexes of all branches of matereal production, and through I_1 — a set of indexes of branches with the insufficient productive powers. We shall conduct the analysis, picking out 2 groups of branches I_1 and $I \setminus I_1$. Then the correlation (1) in block form will be rewritten in the following way:

$$\begin{pmatrix} x_r \\ x_s \end{pmatrix} = \begin{pmatrix} A_{rr} & A_{rs} \\ A_{sr} & A_{ss} \end{pmatrix} \begin{pmatrix} x_r \\ x_s \end{pmatrix} + \begin{pmatrix} y_r \\ y_s \end{pmatrix}, \ \ r \in I \setminus I_1, s \in I_1.$$

Let's assume that the level of production for the branches belonging to the set of I_1 is sufficiently high, so that we can limit in them the production by the given maximum volumes of production.

Then it is necessary to make the following volumes of the production in order to satisfy the final consumption $y_r, r \in I \setminus I_1$:

$$x_r = A_{rr}x_r + A_{rs}x_s + y_r$$

or

$$x_r = (I - A_{rr})^{-1}(A_{rs}x_s + y_r), \ \ r \in I \setminus I_1, s \in I_1.$$

The vector of the final consumption is defined from the following correlations:

$$y_s = [I - A_{ss} - A_{sr}(I - A_{rr})^{-1}A_{rs}]x_s - A_{sr}(I - A_{rr})^{-1}y_r$$

for the branches with the insufficient working powers ($s \in I_1$), which achieve the maximum levels of the production.

We must note, that the above mentioned models are built on the basis of intersectorial balance, the elements of this balance must satisfy the correlation's expressing the material and cost aspects of gross output.

It is possible to determine the balanced prices or energy estimation of considered products on the level of the cost, using the scheme of IB (intersectoral balance) in the natural expression and information of the third quadrant of the table.

Let $p = (p_i)_1^n$ — is the column vector of prices and q_i — the total quantity of the added value per unit of the production of the branch i. For every branch this quantity includes the following l items: depreciation deductions, wages, profit, all kinds of taxes per unit of production. That is, vector $q = (q_i)_1^n$ is obtained as a result of aggregation of l line of the matrix $(z1_{ij})_{1,1}^{l,n}$, characterizing the creation and the first distribution of the national income (the third quadrant of the scheme IB), where n — is the measure vector and is devided into corresponding volumes of production.

In its turn, as a result of intersection of l line of the matrix $(z1_{ij})_{1,1}^{l,n}$ and m columns of the matrix $(y'_{ik})_{1,1}^{n,m}$ forming the final product $(y_i)_1^n$:

$$\sum_k^m y'_{ik} = y_i, \quad i = \overline{1, n},$$

a new matrix $(u_{ij})_{1,1}^{l,m}$ (IY quadrant of IB scheme) is obtained the elements of the matrix show how the depreciation deductions should be used for a simple and an extended reproduction, that go for accumulation and consumption and so on.

Characterizing the balance of the production and the use of conditionally net products, of the matrices

$$(z1_{ij})_{1,1}^{l,n}; \qquad \text{and} \qquad (u_{ik})_{1,1}^{l,m}$$

should satisfy the correlation:

$$\sum_i^n z1_{ti} = \sum_j^m u_{tj}, \qquad t = \overline{1, l}. \tag{10}$$

As the elements of the matrix $(u_{tj})_{1,1}^{l,m}$ reflect the partial redistribution of the national income, as a result of which final incomes of the branches of material production of non–productive sphere and inhabitants are formed it is necessary the structure of the final product to correspond to the structure of the final incomes:

$$\sum_{i}^{n} y'_{ik} = \sum_{t}^{l} u_{tk}, \qquad k = \overline{1, m}.$$

As a result of summing up of the last correlation on k and correlation (10) on t we shall obtain:

$$\sum_{t=1}^{l} \sum_{i=1}^{n} z1_{ti} = \sum_{k=1}^{m} \sum_{i=1}^{n} y'_{ik}. \tag{11}$$

Using the correlation (1) for cost IB the equality is justifiable:

$$\sum_{i=1}^{n} a_{ij} x_j + q_j x_j = x_j, \qquad j = \overline{1, n}.$$

If we assume that only the unit of the production is produced in every branch then the last correlation should be rewritten in the model of balanced prices:

$$p = A'p + q \tag{12}$$

and for determination of a set of prices $(p_i)_1^n$, providing the given vector q, it is necessary to solve the system (12) relative to p:

$$p = [(I - A)^{-1}]'q. \tag{13}$$

Here A' — is the transposed matrix and indexes of the II and III quadrants of the scheme IB are conected by the correlation (11) or

$$\sum_{i=1}^{n} (y_i - q_i x_i) = 0. \tag{14}$$

As in the case whith the model IB in the natural expression and for the model in the cost expression (12) we can find the reasons for making of some changes in some prices. In order to difine how these changes influence the other prices we shall introduce the following denotations: I_2 — a set of indexes of branches, for which prices of the corresponding products are fixed, $I \setminus I_2$ — a set of indexes of branches, for which meaning of the added value is given. Then the correlation (12) may be rewritten in the block view:

$$\begin{pmatrix} p_r \\ p_s \end{pmatrix} = \begin{pmatrix} A'_{rr} & A'_{rs} \\ A'_{sr} & A'_{ss} \end{pmatrix} \begin{pmatrix} p_r \\ p_s \end{pmatrix} + \begin{pmatrix} q_r \\ q_s \end{pmatrix}, \; r \in I \setminus I_2, s \in I_2.$$

According to the given meaning of $q_r, r \in I \setminus I_2$ the balanced prices are determined:

$$p_r = (I - A'_{rr})^{-1}(A'_{rs}p_s + q_r), \;\; r \in I \setminus I_2, s \in I_2,$$

but the meanings of the elements of the aggregated added value per unit of products, corresponding to the given prices $p_s, s \in I_2$ are calculated as:

$$q_s = [I - A'_{ss} - A'_{sr}(I - A'_{rr})^{-1}]p_s - A'_{sr}(I - A'_{rr})^{-1}q_r, \;\; s \in I_2, r \in I \setminus I_2.$$

Concerning the intersectoral balance in energy expression, under this term we mean the studying of the problem of production and distribution of the products from the point of view of energy inputs.

If $e = (e_i)_1^n$ — is a set of energy estimations for the products IB (energy equivalents) and wavy line under the variable means that indexes are qiven in energy expression, $(z_{ij})_1^n$ — is the matrix of interbranch flow then we can give the following correlations:

$$\tilde{x}_j = e_j x_j; \;\; \tilde{z}_{ij} = e_i z_{ij}; \;\; \tilde{y}_j = e_j y_j;$$

$$\tilde{q}_j = e_j q_j; \;\; \tilde{a}_{ij} = e_i a_{ij}/e_j, \;\; i, j = \overline{1, n}$$

Let's denote through $\tilde{A} = (\tilde{a}_{ij})_1^n$ — the matrix of coefficients of the direct material expenses in energy expression, through $\tilde{E}$ — the diagonal matrix of energy estimations and through $\tilde{E}^{-1}$ — the matrix inverse to $\tilde{E}$ (it is evident that $e_i \neq 0, \;\; i = \overline{1, n}$).

$$\tilde{E} = \begin{pmatrix} e_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e_n \end{pmatrix}, \qquad \tilde{E}^{-1} = \begin{pmatrix} 1/e_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/e_n \end{pmatrix}.$$

Then the correlation between coefficients of direct material expenses in natural and energy expressions and in reverse order are prezented in the following view:

$$\tilde{A} = \tilde{E}A\tilde{E}^{-1}; \;\; A = \tilde{E}^{-1}\tilde{A}\tilde{E}.$$

The transition from IB in natural expression to energy expression and connection between indexes of the second and the third quadrants of the last we shall write as:

$$\tilde{E}x = \tilde{E}Ax + \tilde{E}y, \tag{15}$$

$$\sum_{i}^{n}(\tilde{y}_i - \tilde{q}_i x_i) = 0.$$

If the left part of the correlation (15) presents the value of energy contents of the manufactured production, for every branch of the balance, than the right part presents the energy inputs of the given branch for the production of the whole set of the production's output $(x_i)_1^n$ (energy inputs on intermediate consumption) and supply of the corresponding energy contents in the final product.

We shall note that for the transition to the balance (15) it is necessary to solve another very important problem - to define in the aspect of the underlined products the corresponding energy equivalents. It may be realized, to our mind, with the help of solving the simultaneaus equations:

$$e_i = \sum_{j=1}^{n} a_{ji}e_j + \tilde{q}_i, \qquad i = \overline{1, n}$$

or in matrix view:

$$e = A'e + \tilde{q}, \tag{16}$$

where $e = (e_i)_1^n$ is the column vector of energy equivalents, $\tilde{q} = (\tilde{q}_i)_1^n$ — are the total inputs of energy of added value elements on production of per unit of products of every branch, A' — is transposed matrix of coefficients of direct material expenses in natural expression. Hence it follows

$$e = [(I - A)^{-1}]'\tilde{q}. \tag{17}$$

There is another approach to the determination of energy equivalents. That is: let's assume, that energy equivalents are determined for some products of plant — growing and for nutrition. Let's denote through I_3 — a set of indexes of branches for which energy equivalents are given. Then correlation (16) will be written as:

$$\begin{pmatrix} e_r \\ e_s \end{pmatrix} = \begin{pmatrix} A'_{rr} & A'_{rs} \\ A'_{sr} & A'_{ss} \end{pmatrix} \begin{pmatrix} e_r \\ e_s \end{pmatrix} + \begin{pmatrix} \tilde{q}_r \\ \tilde{q}_s \end{pmatrix}, \quad r \in I \setminus I_3, s \in I_3$$

and

$$e_r = (I - A'_{rr})^{-1}(A'_{rs}e_s + \tilde{q}_r), \qquad r \in I \setminus I_3, s \in I_3.$$

In order to define the meaning of the added value in the energy expression, which corresponds to the given meanings of energy equivalents we shall use the following formula:

$$\tilde{q}_s = [I - A'_{ss} - A'_{sr}(I - A'_{rr})^{-1}A'_{rs}]e_s - A'_{sr}(I - A'_{rr})^{-1}e_r,$$

$$s \in I_3, \qquad r \in I \setminus I_3.$$

We must note that the inversed matrix of the type $(I - A)^{-1}$ is the basis of the calculations of the balanced volumes of production, balanced prices and of the first approach to the determination of energy estimations. Coefficient matrix of total inputs $(I - A)^{-1} = B = (b_{ij})_1^n$ used for the determination of the output may be used for the conducting the analysis of the development of the national economy. As the coefficients $b_{ij} \quad (i, j = \overline{1, n})$ mean that it is necessary to increase the gross output of product i on the value b_{ij}, in order to increase the meaning of the final product of the branch j on unit, it follows that the change of the final product on the value Δy may be achieved due the changes of the output $x = (x_i)_1^n$ on the value $\Delta x = (I - A)^{-1}\Delta y$.

Besides, we can calculate the induced volumes of the production, obtained by different elements of the final product $(y'_{ik})_{1,1}^{n,m}$ [3]. For example, the induced volume of the production x^h, obtained by the element h of the final product $y^h \quad (y^h = (y'_{ik})_{1,k=h}^n)$ is defined as:

$$x^h = (I - A)^{-1}y^h.$$

The contribusion of every element $k \quad (k = \overline{1, m})$ of the final product y^k into the gross production is calculated as ratio of induced volumes of the production on elements of the final product to the total volume of the induced production. As a result of multiplying of x^h by the coefficients of the added value we obtain the volumes of the induced added value. Further, depending on the variation of the added value we may define how the structure of prices or the corresponding energy estimations will change.

$$\Delta p = [(I - A)^{-1}]' \Delta q,$$

$$\Delta e = [(I - A)^{-1}]' \Delta \tilde{q},$$

where Δq, $\Delta \tilde{q}$ — are the variations of the added value (correspondingly in cost and energy expressions), Δp, Δe — are the meanings of price variations and energy equivalents correspondingly.

References

[1] V.Kossov. Interbranch models. M.,Economics. 1973.

[2] The problems of optimal planning materials of the International scientific seminar on the questions of optimization of planning and intersectoral balance. M., Economics. 1966.

[3] Mathematical economics on a personal computer. M., Finances and statistics. 1991.

M.Dyakonova, Ph.D.,
P.Cojuhari, Ph.D.,
Institute of Mathematics,
Academy of Sciences, Moldova
5, Academiei str., Kishinev,
277028, Moldova
e-mail: 32*zamdk@math.moldova.su*

Received 23 April, 1994