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Fuzzy logic foundations of optimal inference

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Abstract

In this paper we propose to solve the problem of the optimal fuzzy model designing for the dynamic systems controlling, to develop new mathematical models of fuzzy inference, logical schemes of hardware support based on these models, software support, intellectual system based on these models. The proposed schemes will be able to perform an entire inference process required for real–time fuzzy control. Each scheme works independently of the number of control rules in the knowledge base. The necessary accuracy of the output results can be provided. Among the advantages of suggested architectures are: gain in memory size, simplicity in architectural decisions, fast implementation. The proposed intellectual system gives the new approaches to fuzzy logics acquisitionin the ES and FLC, based on t–norms approach. The system is supplied by cognitive graphics interface. The main functions of the system are: visualization of fuzzy logics by multi–color tables, fuzzy logics acquisition, simulation the fuzzy reasoning processes of the system, testing of fuzzy logics.

1 Introduction

The traditional control methods applied to dynamic systems use the precise formalization of the problem, carefully selecting the parameters and the values that they can have. Such a method is very effective when the relationship between objects are simple, well formalized and, first of all, well known. However, this approach becomes unworkable in applications, such as intelligent control ones, where relationships between variables may be obscure or even unknown. Therefore the possibility to perform a modeling by means of a quantitative/qualitative description is more effective. Fuzzy logic, offering such a capability through the use of linguistic variables and the associated fuzzy sets, replace the traditional methods in an even greater number of new applications. It is important to underline that it is not that fuzzy logic based techniques and traditional ones are antagonist, but rather are complementary.

Let us start with a general formulation of the problem. The dynamic systems which are of the type ARB are considered, where A denotes fuzzy set of input, an B stands for a fuzzy set of output and R denotes a fuzzy relation expressing the relationship between input and output. (For example the relation R can simulate the experience of human operators which is naturally expressed as linguistic fuzzy control rules.) We are seeking for the optimal relation R from the particular performance index point of view (e.g. sampling time period, the computational precise etc.).

To achieve optimum the dependencies are considered between the value of the performance index and the fuzzy partition of the inverse, or the choice of the underlying fuzzy algebra.

The problem under the consideration was investigated earlier but the optimization procedure did not concern the optimization of the fuzzy structure of the inverse. The known optimization procedures were directed to construct the cell–to–cell mapping using the precise model (A.Pagni et all), the appropriate composition operator for connection between A and B (W.Pedrycz), the parameters of fuzzy model (T.Procyk etc.).

2 Optimal inference from the collection of rules

For solving the problem of the development of the extended math models of the fuzzy logic inference from the collection of linguistic rules, the idea of the replacement the collection of control rules by the special fuzzy relation is used. More thoroughly, suppose that the scheme of inference, based on the collection of fuzzy implication rules $A_i \longrightarrow B_i, \quad i = 1, 2, \ldots, N$, have to be implemented. Each collection

$\{A_i\}$ or $\{B_i\}$ describes the fuzzy partition of the corresponding universes X and Y. The existing fuzzy logic implementation schemes of control devices based on using the composition rule of inference:

$$B'(y) = \sup_{i=1}^{N} [\min(A'(x), (A_i \longrightarrow B_i)(x,y))], \quad y \in Y \tag{1}$$

where A' is fuzzy input and B' is fuzzy output. So to obtain the desired set B' it is necessary to process through all the implementation rules. It's not optimal procedure in the case where the number of rules N is large or is compatible with the number of elements in X.

The other approavh, suggested in this paper, based on the replacement the whole collection of implivation rules $(A_i \longrightarrow B_i)$, $\quad i - 1, 2, \ldots, N$, by the unique relation R so that the formula (1) can be rewritten as:

$$B'(y) = \sup_{x \in X} [\min(A'x), R(x,y)], \quad y \in Y \tag{2}$$

The desired relation R must satisfy border conditions:

$$B_i(y) = \sup_{x \in X} [\min(A_i(x), R(x,y)], \quad y \in Y, \quad i = 1, 2, \ldots, N. \tag{3}$$

So the desired relation R is the decision of the system (3) of equations. Let us consider the formula

$$R = \bigcap_{i=1}^{N} \hat{R}_i, \quad \text{where } \hat{R}_i = A_i \alpha B_i, \quad i = 1, 2, \ldots, N. \tag{4}$$

where α stands for the α–composition of A and B. The membership function of $\hat{R}_i$ is calculated from

$$\hat{R}_i(x,y) = \begin{cases} 1, & \text{if } A_i(x) \leq B_i(y), \\ B_i(y), & \text{if } A_i(x) > B_i(y), \end{cases} \tag{5}$$

Unfortunately the relation R does not satisfy the system (3) in general. The necessary and sufficient condition for the solvability of this system is established in [2]. This condition described the partitions of the universes X, Y.

So the optimal choice of fuzzy sets A_i and B_i, $i = 1, 2, \ldots, N$, which are used in the rules of inference can be constructed. This very important fact has to be in mind when the fuzzy rule base is projected. Also the mutual properrties of A' and B' with respect to their relative position in the collections $\{A_i\}$ and $\{B_i\}$ are considered in [1].

The comparison of the proposed model with the existing ones reveals the following advantages in its implementation scheme:

1. the usage of the considerably less memory units,
2. the usage of the considerably less memory min circuits when N is large or is compatible with $|X|$.
3. the significznt speed advantage.

Note that the implementation scheme based on formula (2) is not principally in contrast to the existing ones based on collection of rules.

3 Optimal inference based on the choice of basic operations

The great number of logics can be generated by t–norms axiom system. t–norm $T(p,q)$ performs a conjunction operator on the degrees of certainty of two or more conditions in the same premise, satisfying the following properties:

$$
\begin{aligned}
&T(0,0) = 0;\\
&T(p,1) = T(1,p) = p;\\
&T(p,q) = T(q,p);\\
&T(p,q) < T(r,s), \quad \text{if } p < r \text{ and } q < s;\\
&T(p,(T(q,r)) = T(T(p,q),r).
\end{aligned}
$$

A t–conorm $S(p,q)$ computes the degree of certainty of a conclusion derived from two or more rules. It is a disjunction operator satisfying the de Morgan Laws properties. Some additional restrictions can be added to these properties. A t–norm and A t–conorm are dual for suitable negation operator: $T(p.q) = 1 - S(1-p, 1-q)$.

When it is impossible to get exact mathematical dependencies between the characteristics of the system (e.g. length of inference) and choice of logic the result may be got by simulation methods. It is connected with the fact, that very many different logics based on A t–norms axiomatic is used.

The proposed intellectual system (FLAMES) gives the new approaches to fuzzy logics acquisition in the ES and FLC, based on A t–norms approach. The system is supplied by cognitive graphics interface. The main functions of the system are: visualization of fuzzy logics by multi–color tables, fuzzy logics acquisition, simulation the fuzzy reasoning processes of the system. testing of fuzzy logics.

References

[1] I.Perfilyeva. The basic relations between fuzzy sets. Proc. 1st Europ. Congr. on Fuzzy & Intel.Tecn. 3(1993) 1410–1415 (Aachen, 7–11 Sept., 1993)

[2] I.Perfilyeva. Necessary and sufficient conditions for the identification problem. Aachen (1994), to appear.

[3] A.Averkin. Fuzzy sets in artificial intelligence and control systems. Nauka Pbl., 1986, Moscow (Russian)

[4] A.Averkin. Fuzzy modelling relation in artificial intelligence and psychology. Fuzzy sets and Systems, v. 22, No.1–2, 1987

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