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A Nonlinear Compromise Scheme in Multicriteria Problems of Decision-Making

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Abstract: The concept of nonlinear trade-off scheme in multicriteria problems of evaluation and optimization is presented. It is shown that the problem is to approximate correctly the decision-maker's utility function and construct a substantial mathematical model (scalar convolution) adequate to the given situation to solve various multicriteria problems.

Keywords: multicriteria problems, situation of decision-making, adaptation, model of utility function, nonlinear compromise scheme.

1 Problem Description

Finding a multicriteria solution is inherently a compromise and is based on using subjective information. Given this information and a compromise scheme selected, it is possible to pass from the general vector expression to the scalar convolution of partial criteria, which provides a basis for a constructive apparatus to solve multicriteria problems. Solving the problem is based on the hypothesis that there exists a utility function appearing in the DM's brain during the solution of a specific multicriteria problem. We may state that virtually all the approaches to determining the scalar convolution of criteria are reduced to constructing one mathematical model or another of the DM's utility function.

The problem is to approximate correctly the utility function and to construct a substantial mathematical model as a scalar convolution, adequate to the given situation, to solve different multicriteria problems.

2 Formalization of the Problem

A DM's utility function can generally be represented as $\Phi[y(x), r]$, where $x=\{x_i\}_{i=1}^{n} \in X$ is the vector of solutions defined in the feasible domain *X;* $y=\{y_{0k}\}_{k=1}^{s} \in M$ is the vector of normalized partial criteria defined in the domain $M=\{y \mid 0 \le y_{0k} \le 1, k \in [1,s]\}$; $A=\{A_k\}_{k=1}^{s}$ is the constraint vector; and $r \in R$ is the vector of external conditions defined on the set of feasible factors R.

The situation of making a multicriteria decision is defined by the factors of external conditions r. In solving multicriteria problems, it is usually assumed that the vector r is fixed and specified: $r = r^{\circ}$. Then the DM's utility function can be represented by the scalar convolution of criteria

$$\Phi[y_0(x), r]\big|_{r=r^{\circ}} = Y[y_0(x)]^{\circ},$$

where $Y[y_0(x)]^{\circ}$ is the scalar convolution constructed from the compromise scheme adequate to the given situation.

In most cases, solving multicriteria problems is restricted to a linearized model.

Though such an approach has a doubtless advantage (simplicity), it is characterized by shortcomings inherent in the linearization method. In practical multicriteria problems, it is expedient to construct a *nonlinear* model of the DM's utility function (the concept of a nonlinear compromise scheme).

3 Conceptual Analysis of the DM's Utility Function

In what follows, we will consider an optimization problem and assume for definiteness that all the criteria $y_0(x)$ are to be *minimized*. Then mathematically, the vector optimization problem can be represented as

$$x^* = \arg\min_{x \in X} Y\left[y_0(x)\right].$$

Let us introduce the concept of the intensity of a situation as a measure of how normalized relative partial criteria are close to the limit value (unity):

$$\rho_k = 1 - y_{0k}, \rho_k \in [0;1], k \in [1, s].$$

If a multicriteria decision is made in an intense situation, then in the conditions specified, one or several partial criteria may appear dangerously close to the limit values ($\rho_k \approx 0$). And if one of the criteria achieves the limit (or is outside it), this event will not be compensated by a possible small level of other criteria (violating any of the constraints is usually prohibited).

In such a situation, it is necessary to interfere (in every possible way) the dangerous increase of the most adverse (i.e., the closest to the limit) partial criterion irrespective of the behavior of other criteria. And in the first polar case (ρ_k=0), the DM leaves only this unique, most unfavorable partial criterion for consideration and neglects the others. Hence, a minimax Chebyshev model (egalitarian principle)

$$x^* = \arg\min_{x \in X} Y[y_0(x)]^{(1)} = \arg\min_{x \in X} \max_{k \in [1,s]} y_{0k}(x)$$

adequately expresses the compromise scheme in an intense situation.

In the second polar case ($\rho_k \approx 1$), the situation is quiet, partial criteria are small, and there is no threat to violate the constraints. The DM considers that a unit deterioration of any partial criterion is compensated quite well by an equivalent unit improvement of any other criterion. Such a scheme can be expressed by the model of integral optimality (utilitarian principle)

$$x^* = \arg\min_{x \in X} Y[y_0(x)]^{(2)} = \arg\min_{x \in X} \sum_{k=1}^{s} y_{0k}(x).$$

If we take the conclusions from this analysis as a logic basis for formalizing the choice of a compromise scheme, we can present various constructive concepts such as the concept of a nonlinear compromise scheme.

4 Nonlinear Compromise Scheme

Below there is example for Definition, Theorem and Corollary layout. Also pattern for Example is given. These layouts are recommended, but not obligatory.
From the formalization standpoint, it is expedient to replace the problem of choosing a compromise scheme with the equivalent problem of synthesis of a unified scalar convolution of partial criteria which would express different principles of optimality in different situations

Thus, a universal convolution should express a compromise scheme adaptable to a situation. We may say that adaptation and adaptability are the main substantial essence of studying multicriteria systems. The scalar convolution should include the explicit characteristics of the situation intensity ρ.

Among the possible functions meeting the above requirements, let us consider an elementary one

$$Y(\alpha, y_0) = \sum_{k=1}^{s} \alpha_k [1 - y_{0k}(x)]^{-1}; \alpha \in \Gamma_\alpha ,$$

where α_k=const are formal parameters defined on a simplex and having double physical meaning. On the one hand, these are weight coefficients that express the DM's preferences in partial criteria, and on the other hand, these are coefficients of a substantial regression model of the DM's utility function on the concept of a nonlinear compromise scheme.

Thus, a nonlinear compromise scheme is associated with a vector optimization model, which explicitly depends on the characteristics of the situation intensity ρ:

$$x^* = \arg \min_{x \in X} \sum_{k=1}^{s} \alpha_k [1 - y_{0k}(x)]^{-1}.$$

In contrast to the linear model, defined in a small neighborhood of a working point, the nonlinear model is defined on the whole feasible region X and does not require coefficients α_k to be recalculated if the situation varies.

As is seen from the formula, if any relative partial criterion, for example, $y_{0i}(x)$, approaches the limit (unity), i.e., the situation becomes intense, the corresponding term $Y_i = 1/\alpha_i [1 - y_{0i}(x)]$ in the sum being minimized increases so that the minimization of the whole sum reduces to the minimization of only this worst term, i.e., of the criterion $y_{0i}(x)$. And this is a minimax model manifestation.

If relative partial criteria are far from unity, i.e., the situation is quiet, the proposed model operates equivalent to the integral optimality model. In intermediate situations, different degrees of partial alignment of criteria are obtained. Therefore, the nonlinear compromise scheme has the property of continuous adaptation to the situation of making a multicriteria decision.

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