

Errata to “Shortest single axioms with neutral element for groups of exponent 2 and 3”

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The purpose of this note is to correct an error in [1] pointed out in [2] and to explain the source of this error. Upon initial review of [1], one reviewer requested that multiplicative dots be used to reduce the large number of parentheses in the identities. For instance, $(ey)((((yy)(xz))z)z) = x$ would be changed to $ey \cdot ((yy \cdot xz)z \cdot z) = x$. A program was written to facilitate this conversion for the the large number of identities contained in Theorems 6.1 and 6.2 of [1]. Given that the right hand side of every identity in Theorem 6.1 should be x , the right hand side of every identity in Theorem 6.2 was inadvertently changed to x as well. For example, $e((x(xy)z)y) = z$ became $e(x(xy \cdot z) \cdot y) = x$ instead of the correct $e(x(xy \cdot z) \cdot y) = z$. Below is the corrected version of Theorem 6.2 of [1].

Theorem. *The following 173 identities with neutral element (and their mirrors) are single axioms with neutral element for Boolean groups.*

$e(x(xy \cdot z) \cdot y) = z$	$ex \cdot (xy \cdot zy) = z$	$e(x \cdot y(x \cdot zy)) = z$
$e(xy \cdot (x \cdot zy)) = z$	$ex \cdot y(x \cdot zy) = z$	$(e \cdot xy)(y \cdot zx) = z$
$e((x \cdot yz) \cdot xy) = z$	$ex \cdot y(z \cdot xy) = z$	$e(x \cdot y(z \cdot yx)) = z$
$e((x \cdot yz)y \cdot x) = z$	$ex \cdot y(z \cdot yx) = z$	$e(x \cdot yz) \cdot yx = z$
$(e \cdot (x \cdot yz)z)x = y$	$x(ey \cdot (x \cdot zy)) = z$	$xe \cdot y(x \cdot zy) = z$
$xe \cdot (yx \cdot z)y = z$	$(xe \cdot y)(x \cdot zy) = z$	$x \cdot (ey \cdot x)zy = z$
$x \cdot (ey \cdot x)zy = y$	$(xe \cdot yx)z = y$	$xe \cdot y(yz \cdot x) = z$
$x(ey \cdot (z \cdot xy)) = z$	$x((e \cdot yz) \cdot xy) = z$	$x(ey \cdot z) \cdot xy = z$
$x((e \cdot yz)x \cdot z) = y$	$xe \cdot (y \cdot zx)z = y$	$xe \cdot (yz \cdot x)z = y$
$((xe \cdot y) \cdot zx)z = y$	$((x \cdot ey)z \cdot x)z = y$	$((xe \cdot y)z \cdot x)z = y$
$x(e \cdot y(zy \cdot x)) = z$	$x((ey \cdot z) \cdot yx) = z$	$(xe \cdot y)(zy \cdot x) = z$
$x(ey \cdot z) \cdot yx = z$	$(x(e \cdot yz) \cdot y)x = z$	$((xe \cdot y)z \cdot y)x = z$
$x(e \cdot yz) \cdot z \cdot x = y$	$(xe \cdot yz)z \cdot x = y$	$x(x(ey \cdot z) \cdot y) = z$
$x \cdot (xe \cdot yz)y = z$	$x(x(y \cdot ez) \cdot y) = z$	$x(y \cdot e(x \cdot zy)) = z$
$x(ye \cdot (x \cdot zy)) = z$	$xy \cdot (ex \cdot z)y = z$	$x(y \cdot (e \cdot yz)x) = z$
$x(ye \cdot (yz \cdot x)) = z$	$xy \cdot (ey \cdot z)x = z$	$(x \cdot ye)(y \cdot zx) = z$
$(xy \cdot (e \cdot yz))x = z$	$((x \cdot ye) \cdot yz)x = z$	$x(y \cdot e(z \cdot xy)) = z$
$(x \cdot ye)(zx \cdot y) = z$	$x(y \cdot ez) \cdot xy = z$	$((x \cdot ye)z \cdot x)y = z$
$(xy \cdot ez) \cdot xz = y$	$(x \cdot ye)z \cdot xz = y$	$(xy \cdot ez)x \cdot z = y$
$x(y \cdot ez) \cdot yx = z$	$((x \cdot ye) \cdot zy)x = z$	$x((y \cdot ez) \cdot zx) = y$
$(xy \cdot ez) \cdot zx = y$	$(x(y \cdot ez) \cdot z)x = y$	$e(x \cdot (x \cdot yz)y) = z$
$(ex \cdot (xy \cdot z))y = z$	$e(x \cdot (yx \cdot z)y) = z$	$e(xy \cdot (xz \cdot y)) = z$
$ex \cdot (yx \cdot zy) = z$	$(e \cdot xy)(x \cdot zy) = z$	$(e \cdot x(y \cdot xz))y = z$
$e(xy \cdot xz) \cdot y = z$	$(ex \cdot (yx \cdot z))y = z$	$((e \cdot xy) \cdot xz)y = z$
$(ex \cdot (y \cdot xz))z = y$	$(ex \cdot (yx \cdot z))z = y$	$e(xy \cdot (yz \cdot x)) = z$

$$\begin{array}{lll}
ex \cdot y(yz \cdot x) = z & (ex \cdot y)(yz \cdot x) = z & ((e \cdot xy) \cdot yz)z = x \\
e(x \cdot (yz \cdot x)y) = z & ex \cdot (yz \cdot xy) = z & (e \cdot x(y \cdot zx))y = z \\
e((xy \cdot z)x \cdot z) = y & ex \cdot (yz \cdot xz) = y & ex \cdot (yz \cdot x)z = y \\
(e(xy \cdot z) \cdot x)z = y & e(x \cdot y(zx \cdot x)) = z & e((x \cdot yz) \cdot yx) = z \\
ex \cdot (yz \cdot yx) = z & (ex \cdot yz) \cdot yx = z & e((x \cdot yz)y \cdot z) = x \\
e(xy \cdot z) \cdot yz = x & e(xy \cdot zy) \cdot z = x & ((e \cdot xy) \cdot zy)z = x \\
(e(x \cdot yz) \cdot y)z = x & (ex \cdot yz)z \cdot x = y & e((x \cdot yz)z \cdot y) = x \\
e(xy \cdot z) \cdot zy = x & x \cdot e(yx \cdot zy) = z & x((ey \cdot x) \cdot zy) = z \\
x((e \cdot yx)z \cdot y) = z & (x \cdot ey)(xz \cdot y) = z & (x \cdot e(yx \cdot z))y = z \\
x(ey \cdot xz) \cdot y = z & (xe \cdot (y \cdot xz))y = z & ((xe \cdot y) \cdot xz)y = z \\
xe \cdot (y \cdot xz)z = y & (x \cdot e(yx \cdot z))z = y & ((xe \cdot y) \cdot xz)z = y \\
x(e \cdot y(yz \cdot x)) = z & x \cdot e(yz \cdot xy) = z & x(e \cdot (yz \cdot x)y) = z \\
x((ey \cdot z) \cdot xy) = z & x((e \cdot yz)x \cdot y) = z & (x \cdot e(y \cdot zx))y = z \\
((xe \cdot y) \cdot zx)y = z & x(e \cdot (yz \cdot x)z) = y & (xe \cdot yz) \cdot xz = y \\
x \cdot e(yz \cdot yx) = z & x(ey \cdot (z \cdot yx)) = z & (xe \cdot yz) \cdot zx = y \\
x((x \cdot ey) \cdot zy) = z & x((xe \cdot y)z \cdot y) = z & (x \cdot (x \cdot ey)z)y = z \\
x(xy \cdot (e \cdot zy)) = z & x(x(ye \cdot z) \cdot y) = z & x \cdot (xy \cdot ez)y = z \\
x((xy \cdot e)z \cdot y) = z & x(xy \cdot ez) \cdot y = z & xy \cdot e(x \cdot zy) = z \\
xy \cdot e(xz \cdot y) = z & (xy \cdot e)(x \cdot zy) = z & (x \cdot y(e \cdot xz))y = z \\
x(ye \cdot xz) \cdot y = z & (xy \cdot (e \cdot xz))y = z & ((xy \cdot e) \cdot xz)y = z \\
x(y \cdot (ey \cdot z)x) = z & xy \cdot e(y \cdot zx) = z & xy \cdot e(yz \cdot x) = z \\
(xy \cdot e)(y \cdot zx) = z & ((xy \cdot e) \cdot yz)x = z & x((y \cdot ez)x \cdot y) = z \\
(x \cdot y(e \cdot zx))y = z & x((y \cdot ez) \cdot xz) = y & x \cdot (ye \cdot zx)z = y \\
x((y \cdot ez)x \cdot z) = y & (x \cdot y(e \cdot zx))z = y & (x \cdot y(ez \cdot x))z = y \\
x(ye \cdot zx) \cdot z = y & ((xy \cdot e) \cdot zx)z = y & x(y \cdot e(zy \cdot x)) = z \\
x(y \cdot (ez \cdot y)x) = z & (x(y \cdot ez) \cdot y)x = z & x(ye \cdot z) \cdot zx = y \\
e(x \cdot (xy \cdot z)y) = z & e(x(yx \cdot z) \cdot y) = z & (ex \cdot (y \cdot xz))y = z \\
e(x(yx \cdot z) \cdot z) = y & ((e \cdot xy) \cdot yz)x = z & (ex \cdot yz) \cdot xy = z \\
(e(x \cdot yz) \cdot y)x = z & x(ye \cdot z) \cdot xz = y & e((xy \cdot z) \cdot yz) = x \\
x(e(x \cdot yz) \cdot y) = z & x(e(xy \cdot z) \cdot y) = z & xe \cdot (x \cdot yz)y = z \\
x \cdot e(xy \cdot zy) = z & x \cdot (ex \cdot yz)y = z & x((e \cdot xy)z \cdot y) = z \\
xe \cdot (xy \cdot z)y = z & (x \cdot e(xy \cdot z))y = z &
\end{array}$$

A finite model of any of the following 5 identities with neutral element (or their mirrors) is a Boolean group with neutral element e .

$$\begin{array}{lll}
(ex \cdot y)z \cdot xz = y & ((ex \cdot y)z \cdot x)z = y & (ex \cdot y)z \cdot yz = x \\
(ex \cdot yz)y \cdot z = x & (ex \cdot yz)z \cdot y = x &
\end{array}$$

Any additional single axioms with neutral element for Boolean groups are among these 5 identities with neutral element (up to renaming, mirroring, and symmetry).

References

- [1] N. C. Fiala and K. M. Agre, *Shortest single axioms with neutral element for groups of exponent 2 and 3*, Quasigroups and Related Systems **21** (2013), 69 – 82.
- [2] A. Krapež, *Fiala–Agre list of single axioms for Boolean groups is wrong*, Quasigroups and Related Systems **22** (2014), 255 – 256.

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