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On left strongly simple ordered hypersemigroups

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Abstract. We present a structure theorem referring to the decomposition of ordered hypersemigroups into left strongly simple components, that is, into subhypersemigroups which are both simple and left quasi-regular. We prove that an ordered hypersemigroup is a semilattice of left strongly simple hypersemigroups if and only if it is a complete semilattice of left strongly simple hypersemigroups and we characterize this type of hypersemigroups in terms of intra-regular and semisimple hypersemigroups. We also characterize the chains of left strongly simple ordered hypersemigroups.

1. Introduction and prerequisites

The concept of the hypergroup introduced by the French Mathematician F. Marty at the 8th Congress of Scandinavian Mathematicians in 1933 is as follows: An hypergroup is a nonempty set H endowed with a multiplication xy such that (i) $xy \subseteq H$; (ii) $x(yz) = (xy)z$; (iii) $xH = Hx = H$ for every x, y, z in H (cf. [9]). Hundreds of papers appeared on hyperstructures since Marty introduced this concept, and in the recent years, many groups in the world investigate the hypersemigroups in research programs using the definition given by Marty. Being impossible to give a complete information regarding the bibliography, we will refer only some recent books and articles such as the [1–7, 9–11].

The present paper deals with the decomposition of ordered hypersemigroups into their hypersemigroups which are left strongly simple, that is, both simple and left quasi-regular. In this respect, we characterize the ordered hypersemigroups which are semilattices of left strongly simple hypersemigroups. We prove that for ordered hypersemigroups, the concepts of semilattices of left strongly simple hypersemigroups and complete semilattices of left strongly simple hypersemigroups are the same. Moreover, we prove that an ordered hypersemigroup S is a semilattice of left strongly simple hypersemigroups if and only if it is a union of left strongly simple hypersubsemigroups. We show that an ordered hypersemigroup S is a semilattice of left strongly simple hypersemigroups if and only if every left hyperideal of S is an intra-regular hypersubsemigroup or a semisimple hypersubsemigroup of S. This type of ordered hypersemigroups are the ordered hypersemigroups in which $a \in (S \circ a^2 \circ S \circ a]$ for every $a \in S$. Finally, we prove that the chains and the complete chains of left strongly simple ordered hypersemigroups coincide and they

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are characterized as the ordered hypersemigroups in which, for every $a, b \in S$, we have $a \in (S \circ a \circ b \circ S \circ a]$ or $b \in (S \circ a \circ b \circ S \circ b]$. The corresponding results for hypersemigroups (without order) can be also obtained as application of the results of this paper, and this is because every hypersemigroup endowed with the equality relation is an ordered hypersemigroup. Left strongly simple semigroups (without order) have been considered in [8].

Let $(S, \circ, \leq)$ be an ordered hypersemigroup. For a hypersubsemigroup T of S and a subset H of T, we denote by $(H]_T$ the subset of T defined by

$$(H]_T := \{t \in T \mid t \leq h \text{ for some } h \in H\}.$$

In particular, for $T = S$, we write $(H]$ instead of $(H]_S$. So, for $H \subseteq S$, we have

$$(H] := \{t \in S \mid t \leq h \text{ for some } h \in H\}.$$

A nonempty subset A of S is called a *left* (resp. *right*) hyperideal of S if (1) $S \circ A \subseteq A$ (resp. $A \circ S \subseteq A$) and (2) if $a \in A$ and $b \in S$, $b \leq a$, then $b \in A$. A is called a *hyperideal* of S if it is both a left and a right hyperideal of S. We denote by $L(a)$ (resp. $R(a)$) the left (resp. right) hyperideal of S generated by a, and by $I(a)$ the hyperideal of S generated by a ($a \in S$). We have $L(a) = (a \cup S \circ a]$, $R(a) = (a \cup a \circ S]$ and $I(a) = (a \cup S \circ a \cup a \circ S \cup S \circ a \circ S]$ for every $a \in S$. A left (resp. right) hyperideal A of S is clearly a hypersubsemigroup of S i.e. $A \circ A \subseteq A$. S is called *simple* if for every hyperideal T of S, we have $T = S$. A hypersubsemigroup L of S is called *intra-regular* if for each $a \in L$ there exist $x, y \in L$ such that $a \leq x \circ a^2 \circ y$, equivalently if $a \in (L \circ a^2 \circ L]_L$ for every $a \in L$ or $A \subseteq (L \circ A^2 \circ L]_L$ for every nonempty subset A of L. An equivalence relation σ on S is called *congruence* if $(a, b) \in \sigma$ implies $(a \circ c, b \circ c) \in \sigma$ and $(c \circ a, c \circ b) \in \sigma$ for every $c \in S$, in the sense that for every $x \in a \circ c$ and every $y \in b \circ c$ we have $(x, y) \in \sigma$ and for every $x \in c \circ a$ and every $y \in c \circ b$, we have $(x, y) \in \sigma$. A congruence σ on S is called *semilattice congruence* if, for every $a, b \in S$, we have $(a^2, a) \in \sigma$ meaning that $x \in a \circ a$ implies $(x, a) \in \sigma$ and $(a \circ b, b \circ a) \in \sigma$ in the sense that if $x \in a \circ b$ and $y \in b \circ a$, then $(x, y) \in \sigma$. If σ is a semilattice congruence on S, then the σ-class $(x)_\sigma$ of S containing x is a hypersubsemigroup of S for every $x \in S$. A semilattice congruence σ on S is called *complete* if $a \leq b$ implies $(a, a \circ b) \in \sigma$, that is, if $x \in a \circ b$, then $(a, x) \in \sigma$. Recall that if σ is a complete semilattice congruence on S then, the relation $a \leq a$ implies $(a^2, a) \in \sigma$, so the complete semilattice congruences on S can be also defined as the congruences on S such that $(a \circ b, b \circ a) \in \sigma$ and $a \leq b$ implies $(a, a \circ b) \in \sigma$ for every $a, b \in S$. A hypersubsemigroup F of S is called a *hyperfilter* of S if (1) for any $a, b \in S$, $(a \circ b) \cap A \neq \emptyset$ implies $a, b \in F$ and (2) $a \in F$ and $S \ni b \geq a$ implies $b \in A$. We denote by $\mathcal{N}$ the relation on S defined by $\mathcal{N} := \{(x, y) \mid N(x) = N(y)\}$ where $N(a)$ denotes the hyperfilter of S generated by a ($a \in S$). The relation $\mathcal{N}$ is the least complete semilattice congruence on S. We say that S is a *semilattice of left strongly simple hypersemigroups* (resp. *complete semilattice of left strongly simple hypersemigroups*) if there exists a semilattice congruence (resp. complete semilattice congruence) σ on S such that the σ-class $(x)_\sigma$ of S containing x is a left strongly simple hypersubsemigroup of S for every $x \in S$. An equivalent definition is the following: The ordered hypersemigroup S is a semilattice of left strongly simple hypersemigroups if there exists a semilattice Y and a nonempty family $\{S_\alpha \mid \alpha \in Y\}$ of left strongly simple hypersubsemigroups of S such that

(1) $S_\alpha \cap S_\beta = \emptyset$ for every $\alpha, \beta \in Y$, $\alpha \neq \beta$

(2) $S = \bigcup\limits_{\alpha \in Y} S_\alpha$

(3) $S_\alpha \circ S_\beta \subseteq S_{\alpha\beta}$ for every $\alpha, \beta \in Y$.

In ordered hypersemigroups, the semilattice congruences are defined exactly as in hypersemigroups (without order) so the two definitions are equivalent. An ordered hypersemigroup S is a complete semilattice of left simple hypersemigroups if and only if in addition to (1), (2) and (3) above, we have the following:

(4) $S_\beta \cap (S_\alpha] \neq \emptyset$ implies $\beta = \alpha\beta$.

We say that S is a *chain* (resp. *complete chain*) *of left strongly simple hypersemigroups* if there exists a semilattice congruence (resp. complete semilattice congruence) σ on S such that the σ-class $(x)_\sigma$ of S containing x is a left strongly simple hypersubsemigroup of S for every $x \in S$, and the set S/σ of (all) $(x)_\sigma$-classes of S endowed with the relation $(x)_\sigma \preceq (y)_\sigma \iff (x)_\sigma = (x \circ y)_\sigma$ is a chain.

2. Main results

Definition 2.1. A hypersubsemigroup L of an ordered hypersemigroup S is called *left* (resp. *right*) *quasi-regular* if $a \in (L \circ a \circ L \circ a]_L$ (resp. $a \in (a \circ L \circ a \circ L]_L$) for every $a \in L$.

Definition 2.2. An ordered hypersemigroup S is called *left* (resp. *right*) *strongly simple* if it is simple and left (resp. right) quasi-regular.

Definition 2.3. A hypersubsemigroup L of an ordered hypersemigroup S is called *semisimple* if $a \in (L \circ a \circ L \circ a \circ L]_L$ for every $a \in L$.

It might be noted that an ordered hypersemigroup S is semisimple if and only if the hyperideals of S are idempotent, that is, for every hyperideal A of S, we have $(A \circ A] = A$.

Lemma 2.4. *An ordered hypersemigroup S is simple if and only if $(S \circ a \circ S] = S$ for every $a \in S$.*

Proof. $(\Rightarrow)$. For an element a of S, the set $(S \circ a \circ S]$ is an hyperideal of S. Indeed, the set $(S \circ a \circ S]$ is a nonempty subset of S, $S \circ (S \circ a \circ S] = (S] \circ (S \circ a \circ S] \subseteq (S^2 \circ a \circ S] \subseteq (S \circ a \circ S]$, $(S \circ a \circ S]S \subseteq (S \circ a \circ S]$, and $((S \circ a \circ S]] = (S \circ a \circ S]$. Since S is simple, we have $(S \circ a \circ S] = S$.

($\Leftarrow$). Let T be an hyperideal of S. We get an arbitrary element b of T (such an element exists since T is nonempty). Then $S \circ b \circ S \subseteq S \circ T \circ S \subseteq T$, so $(S \circ b \circ S] \subseteq (T] = T$. On the other hand, by hypothesis, we have $(S \circ b \circ S] = S$. Thus we have $S \subseteq T$, and $T = S$. $\square$

Lemma 2.5. *An ordered hypersemigroup S is left strongly simple if and only if $a \in (S \circ b \circ S \circ a]$ for every $a, b \in S$.*

Proof. ($\Rightarrow$). Let $a, b \in S$. Since S is simple, by Lemma 2.4, we have $(S \circ b \circ S] = S$, then $a \in (S \circ b \circ S]$. On the other hand, since S is left quasi-regular, we have $a \in (S \circ a \circ S \circ a]$. Thus we get

$$\begin{aligned} a \in (S \circ a \circ S \circ a] &\subseteq (S \circ (S \circ b \circ S] \circ S \circ a] \\ &= (S \circ (S \circ b \circ S) \circ S \circ a] \\ &\subseteq (S \circ b \circ S \circ a]. \end{aligned}$$

($\Leftarrow$). If $a \in S$, by hypothesis, we have $a \in (S \circ a \circ S \circ a]$, so S is left quasi-regular. If $a, b \in S$, by hypothesis, we have

$$\begin{aligned} a \in (S \circ b \circ S \circ a] &\subseteq (S \circ b \circ S \circ (S \circ b \circ S \circ a]] \\ &= (S \circ b \circ S \circ (S \circ b \circ S \circ a)] \\ &\subseteq (S \circ b \circ S], \end{aligned}$$

thus we have $S \subseteq (S \circ b \circ S]$ and $(S \circ b \circ S] = S$, and so S is simple. $\square$

Lemma 2.6. *If S is an intra-regular ordered hypersemigroup, then for the complete semilattice congruence $\mathcal{N}$ on S, the class $(x)_{\mathcal{N}}$ is a simple hypersubsemigroup of S for every $x \in S$.*

Which means that the intra-regular ordered hypersemigroups are complete semilattices of simple hypersemigroups.

Theorem 2.7. *Let $(S, \circ, \leq)$ be an ordered hypersemigroup and σ a complete semilattice congruence on S. Then S is left quasi-regular if and only if $(a)_\sigma$ is a left quasi-regular hypersubsemigroup of S for every $a \in S$.*

Proof. ($\Rightarrow$). Let $b \in (a)_\sigma$. Then there exist elements $u, v \in (a)_\sigma$ such that $b \leq u \circ b \circ v \circ b$. In fact: Since $b \in S$ and S is left quasi-regular, $b \leq s \circ b \circ t \circ b$ for some $s, t \in S$. Then we have

$$\begin{aligned} b &\leq s \circ b \circ t \circ (s \circ b \circ t \circ b) \leq s \circ b \circ t \circ s \circ b \circ t \circ (s \circ b \circ t \circ b) \\ &= (s \circ b \circ t \circ s) \circ b \circ (t \circ s \circ b \circ t) \circ b. \end{aligned}$$

Moreover we have $s \circ b \circ t \circ s, t \circ s \circ b \circ t \in (a)_\sigma$. In fact, since $b \leq s \circ b \circ t \circ b$ and σ is a complete semilattice congruence on S, we have $(b, b \circ s \circ b \circ t \circ b) \in \sigma$, then $(b, s \circ b \circ t \circ b) \in \sigma$. Since $(a, b) \in \sigma$, we have $(a, s \circ b \circ t \circ b) \in \sigma$. Since $(t \circ b, b \circ t) \in \sigma$, we have $(s \circ b \circ t \circ b, s \circ b^2 \circ t) \in \sigma$, then $(a, s \circ b \circ t) \in \sigma$, $(a, s \circ b \circ t \circ s) \in \sigma$, and $s \circ b \circ t \circ s \in (a)_\sigma$. Moreover, since $(a, s \circ b \circ t) \in \sigma$, we have $(a, s \circ b \circ t^2) \in \sigma$, $(a, t \circ s \circ b \circ t) \in \sigma$, and $t \circ s \circ b \circ t \in (a)_\sigma$.

($\Leftarrow$). Let $a \in S$. Since $(a)_\sigma$ is left quasi-regular, we have

$$a \in ((a)_\sigma \circ a \circ (a)_\sigma \circ a]_{(a)_\sigma} \subseteq (S \circ a \circ S \circ a],$$

so S is left quasi-regular. □

Theorem 2.8. *Let $(S, \circ, \leq)$ be an ordered hypersemigroup. The following are equivalent:*

(1) *S is a complete semilattice of left strongly simple hypersemigroups.*

(2) *S is a semilattice of left strongly simple hypersemigroups.*

(3) *S is a union of left strongly simple hypersubsemigroups of S.*

(4) *$a \in (S \circ a^2 \circ S \circ a]$ for every $a \in S$.*

(5) *Every left hyperideal of S is an intra-regular hypersubsemigroup of S.*

(6) *Every left hyperideal of S is a semisimple hypersubsemigroup of S.*

Proof. The implications (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are obvious.

(3) $\Rightarrow$ (4). Let S be the union of the left strongly simple hypersubsemigroups S_α, $\alpha \in Y$, and let $a \in S$. Suppose $a \in S_\alpha$ for some $\alpha \in Y$. Since S_α is a left strongly simple hypersemigroup and $a, a^2 \in S_\alpha$, by Lemma 2.5, we have

$$a \in (S_\alpha \circ a^2 \circ S_\alpha \circ a]_{S_\alpha} \subseteq (S \circ a^2 \circ S \circ a].$$

(4) $\Rightarrow$ (5). Let L be a left hyperideal of S and $a \in L$. Since $a, a^2 \in S$, by (4), we have

$$\begin{aligned} a \in (S \circ a^2 \circ S \circ a] &\subseteq (S \circ (S \circ a^4 \circ S \circ a^2] \circ S \circ a] \\ &= (S \circ (S \circ a^4 \circ S \circ a^2) \circ S \circ a] \\ &\subseteq ((S \circ a^2) \circ a^2 \circ (S \circ a^2 \circ S \circ a)]. \end{aligned}$$

Since $a^2 \in L$, we have $S \circ a^2 \subseteq SL \subseteq L$ and $S \circ a^2 \circ S \circ a \subseteq S \circ L \subseteq L$. Thus we have $a \in (L \circ a^2 \circ L] = (L \circ a^2 \circ L]_L$, and L is intra-regular.

(5) $\Rightarrow$ (6). Let L be a left hyperideal of S and $a \in L$. Since L is intra-regular, we have

$$\begin{aligned} a \in (L \circ a^2 \circ L]_L &\subseteq (L \circ a \circ (L \circ a^2 \circ L]_L \circ L]_L \\ &= (L \circ a \circ (L \circ a^2 \circ L) \circ L]_L \\ &\subseteq (L \circ a \circ L \circ a \circ (S \circ L)]_L \\ &\subseteq (L \circ a \circ L \circ a \circ L]_L, \end{aligned}$$

and L is semisimple.

(6) $\Rightarrow$ (1). Let $a \in S$. By (6), $L(a)$ is a semisimple hypersubsemigroup of S i.e. $x \in (L(a) \circ x \circ L(a) \circ x \circ L(a)]_{L(a)} = (L(a) \circ x \circ L(a) \circ x \circ L(a)]$ for every $x \in L(a)$. Thus we have

$$\begin{aligned} a &\in (L(a) \circ a \circ L(a) \circ a \circ L(a)] \\ &= ((a \cup S \circ a] \circ a \circ (a \cup S \circ a] \circ a \circ (a \cup S \circ a]] \\ &= ((a \cup S \circ a) \circ a \circ (a \cup S \circ a) \circ a \circ (a \cup S \circ a)] \\ &= (a^2 \circ S \circ a^3 \cup S \circ a^2 \circ S \circ a]. \end{aligned}$$

Then

$$a^2 \in (a^2 \circ S \circ a^3 \cup S \circ a^2 \circ S \circ a] \circ (a] \subseteq (a^2 \circ S \circ a^4 \cup S \circ a^2 \circ S \circ a^2],$$

and

$$\begin{aligned} a^2 \circ S \circ a^3 &\subseteq (a^2 \circ S \circ a^4 \cup S \circ a^2 \circ S \circ a^2] \circ (S \circ a^3] \\ &\subseteq ((a^2 \circ S \circ a^4 \cup S \circ a^2 \circ S \circ a^2) \circ (S \circ a^3)] \\ &= (a^2 \circ S \circ a^4 \circ S \circ a^3 \cup S \circ a^2 \circ S \circ a^2 \circ S \circ a^3] \\ &\subseteq (S \circ a^2 \circ S \circ a]. \end{aligned}$$

Thus we have $a \in ((S \circ a^2 \circ S \circ a] \cup S \circ a^2 \circ S \circ a] = ((S \circ a^2 \circ S \circ a]] = (S \circ a^2 \circ S \circ a]$. Since $a \in (S \circ a^2 \circ S \circ a] \subseteq (S \circ a^2 \circ S], (S \circ a \circ S \circ a]$ for every $a \in S$, S is both intra-regular and left quasi-regular. Since S is intra-regular, by Lemma 2.6, $(x)_{\mathcal{N}}$ is a simple hypersubsemigroup of S for every $x \in S$. Since S is left quasi-regular and $\mathcal{N}$ a complete semilattice congruence of S, by Theorem 2.7, $(x)_{\mathcal{N}}$ is a left quasi-regular hypersubsemigroup of S for every $x \in S$. Since $\mathcal{N}$ is a complete semilattice congruence on S and $(x)_{\mathcal{N}}$ a left strongly simple hypersubsemigroup of S for every $x \in S$, S is a complete semilattice of left strongly simple hypersemigroups. □

Theorem 2.9. *An ordered hypersemigroup S is a chain of left strongly simple hypersemigroups if and only if, for every $a, b \in S$, we have*

$$a \in (S \circ a \circ b \circ S \circ a] \textit{ or } b \in (S \circ a \circ b \circ S \circ b].$$

Proof. ($\Rightarrow$). Suppose σ is a semilattice congruence on S such that $(x)_\sigma$ is a left strongly simple hypersubsemigroup of S for every $x \in S$ and the set S/σ endowed with the relation

$$(x)_\sigma \preceq (y)_\sigma \Longleftrightarrow (x)_\sigma = (x \circ y)_\sigma$$

is a chain. Let now $a, b \in S$. Since $(S/\sigma, \preceq)$ is a chain, we have $(a)_\sigma \preceq (b)_\sigma$ or $(b)_\sigma \preceq (a)_\sigma$. Let $(a)_\sigma \preceq (b)_\sigma$. Then $(a)_\sigma = (a \circ b)_\sigma$ and $\{a\}, a \circ b \subseteq (a)_\sigma$. Since $(a)_\sigma$ is a left strongly simple hypersemigroup, by Lemma 2.5, we have $a \in ((a)_\sigma \circ a \circ b \circ (a)_\sigma \circ a]_{(a)_\sigma} \subseteq (S \circ a \circ b \circ S \circ a]$. If $(b)_\sigma \preceq (a)_\sigma$, similarly we obtain $b \in (S \circ a \circ b \circ S \circ b]$.

($\Leftarrow$). Let $a \in S$. By hypothesis, we have $a \in (S \circ a^2 \circ S \circ a]$. For the semilattice congruence $\mathcal{N}$, the $\mathcal{N}$-class $(x)_\mathcal{N}$ is a left strongly simple hypersubsemigroup of S for every $x \in S$ (cf. the proof of (6) $\Rightarrow$ (1) in Theorem 2.8). Let now $(x)_\mathcal{N}, (y)_\mathcal{N} \in S/\mathcal{N}$. By hypothesis, we have $x \in (S \circ x \circ y \circ S \circ x]$ or $y \in (S \circ x \circ y \circ S \circ y]$. Let $x \in (S \circ x \circ y \circ S \circ x]$. Since $x \in N(x)$ and $x \leq t \circ x \circ y \circ h \circ x$ for some $t, h \in S$, we have $x \circ y \subseteq N(x)$, then $N(x \circ y) \subseteq N(x)$. Let $y \in (S \circ x \circ y \circ S \circ y]$. Since $y \in N(y)$ and $y \leq z \circ x \circ y \circ k \circ y$ for some $z, k \in S$, we have $x \circ y \subseteq N(y)$, so $N(x \circ y) \subseteq N(y)$. On the other hand, $x \circ y \subseteq N(x \circ y)$ implies $x, y \in N(x \circ y)$, then $N(x) \subseteq N(x \circ y)$ and $N(y) \subseteq N(x \circ y)$. Hence we have $N(x \circ y) = N(x)$ or $N(x \circ y) = N(y)$. Thus $(x)_\mathcal{N} = (x \circ y)_\mathcal{N}$ or $(y)_\mathcal{N} = (x \circ y)_\mathcal{N} = (y \circ x)_\mathcal{N}$, that is, $(x)_\mathcal{N} \preceq (y)_\mathcal{N}$ or $(y)_\mathcal{N} \preceq (x)_\mathcal{N}$. $\square$

Remark. An ordered hypersemigroup is a chain of left strongly simple hypersemigroups if and only if it is a complete chain of left strongly simple hypersemigroups.

Let us finish with the following examples which correspond to the definitions 2.1–2.3.

Example 2.10. We consider the ordered hypersemigroup $S = \{a, b, c, d, f\}$ defined by the hyperoperation given in the table and the order below.

$\circ$	a	b	c	d	f
a	$\{a, f\}$	$\{b, f\}$	$\{c, d, f\}$	$\{d\}$	$\{f\}$
b	$\{b, f\}$	$\{a, f\}$	$\{c, d, f\}$	$\{d\}$	$\{f\}$
c	$\{c, d, f\}$	$\{c, d, f\}$	$\{c, d, f\}$	$\{c, d, f\}$	$\{c, d, f\}$
d	$\{c, d, f\}$	$\{c, d, f\}$	$\{c, d, f\}$	$\{c, d, f\}$	$\{c, d, f\}$
f	$\{f\}$	$\{f\}$	$\{c, d, f\}$	$\{d\}$	$\{f\}$

$$\leq := \{(a, a), (b, b), (c, c), (d, c), (d, d), (f, a), (f, b), (f, c), (f, f)\}.$$

The covering relation of S is the following:

$$\prec = \{(d, c), (f, a), (f, b), (f, c)\}.$$

This is a left quasi-regular ordered hypersemigroup. As the left quasi-regular ordered hypersemigroups are also semisimple, this is an example of an ordered semisimple hypersemigroups as well. It is not simple as $(S \circ c \circ S] \neq S$.

Example 2.11. The ordered hypersemigroup defined by the hyperoperation and the covering relation below is left quasi-regular (also right quasi-regular) and simple.

$\circ$	a	b	c	d	e
a	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a\}$	$\{a, b, c\}$
b	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a\}$	$\{a, b, c\}$
c	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a\}$	$\{a, b, c\}$
d	$\{a, b, d\}$	$\{a, b, d\}$	S	$\{a, b, d\}$	S
e	$\{a, b, d\}$	$\{a, b, d\}$	S	$\{a, b, d\}$	S

$$\prec = \{(a,b),(b,c),(b,d),(c,e),(d,e)\}.$$

We wrote this paper in the usual way, and we will come back to this paper in a forthcoming paper.

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