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# A note on a union of hyper K-algebras

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**Abstract**

In this paper, at first by some examples we show that the Theorem 3.7 of [1] is not true in general. Then we give a correct version of it. Moreover we give the notion of a union of a family of hyper K-algebras and investigate some of its properties. Finally by considering the concept of (closed set)-decomposable, we show that a hyper BCK-algebra is (closed set)-decomposable if and only if it is the union of a family of hyper BCK-algebras.

## 1. Introduction and Preliminaries

The study of BCK-algebra was initiated by Imai and Iséki [2] in 1966 as a generalization of the concept of set-theoretic difference and propositional calculi. Borzooei, et.al. [1, 3] applied the hyper structure to BCK-algebras and introduced the concept of hyper BCK-algebra and K-algebra in which each of them is a generalization of BCK-algebra. They have defined the notion of a union of two hyper K-algebra as an extension of a union of BCK-algebra [1]. Now we follow them and obtain some results such as mentioned in the abstract.

**Definition 1.** [1, 4] Let $H$ be a non-empty set containing a constant 0 and $\mathcal{P}^*(H)$ be the set of all non-empty subset of H. Then $H$ with hyperoperation $\circ : H \times H \to \mathcal{P}^*(H)$, where $(a,b) \mapsto a \circ b$, is called a *hyper K-algebra* (*hyper BCK-algebra*) if it satisfies $K1 - K5$ (respectively: $HK1 - HK4$)

| | |
|---|---|
| $K1$: $(x \circ z) \circ (y \circ z) < x \circ y$, | $HK1$: $(x \circ z) \circ (y \circ z) \ll x \circ y$, |
| $K2$: $(x \circ y) \circ z = (x \circ z) \circ y$, | $HK2$: $(x \circ y) \circ z = (x \circ z) \circ y$, |
| $K3$: $x < x$, | $HK3$: $x \circ H \ll x$, |
| $K4$: $x < y,\ y < x$ then $x = y$, | $HK4$: $x \ll y,\ y \ll x$ then $x = y$, |
| $K5$: $0 < x$, | |

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for all $x, y, z \in H$, where $x < y (x \ll y)$ means $0 \in x \circ y$. Moreover for any $A, B \subseteq H$, $A < B$ if $\exists\, a \in A$, $\exists\, b \in B$ such that $a < b$ and $A \ll B$ if $\forall\, a \in A$, $\exists\, b \in B$ such that $a \ll b$.

The readers could see some definitions and results about hyper K-algebra and hyper BCK-algebra in [1, 4, 5].

**Definition 2.** [6] Let $I$ and $S$ be non-empty subsets of $H$. Then we say that $I$ is *$S$-absorbing* if $x \in I$ and $y \in S$ then $x \circ y \subseteq I$.

**Theorem 1.** [6] *Let $H$ be a hyper BCK-algebra and $I$ be a hyper BCK-ideal or closed set. Then $I$ is $H$-absorbing.*

**Definition 3.** [6] A hyper K-algebra $H$ is called *$(P)$-decomposable* if there exists a family $\{A_i\}_{i\in\Omega}$ of subsets of $H$ with *$P$-property* such that:
$(i)$ $H \neq \{A_i\}$ for all $i \in \Omega$, $(ii)$ $H = \cup_{i\in\Omega} A_i$, $(iii)$ $A_i \cap A_j = \{0\}, i \neq j$.

In this case, we write $H = \oplus_{i\in\Omega} A_i(\mathrm{P})$ and say that $\{A_i\}_{i\in\Omega}$ is a (P)-decomposition for $H$. If each $A_i$, $i \in \Omega$, is S-absorbing we write $H \overset{\mathrm{S}}{=} \oplus_{i\in\Omega} A_i(\mathrm{P})$. Moreover we say that this decomposition is a *closed union*, in short (P)-CUD, if $\cup_{i\in\Delta} A_i$ has P-property for any non-empty subset $\Delta$ of $\Omega$. If there exists a (P)-CUD for $H$, then we say that $H$ is (P)-CUD.

**Theorem 2.** [6] *Let $H \overset{\mathrm{H}}{=} \oplus_{i\in\Omega} A_i$ (a hyper K-ideal). Then $H$ is (hyper K-ideal)-CUD.*

## 2. Union hyper K-algebras

In this section at first we show that Theorem 3.7 [1] as follows is not true in general, then we give a correct version of it.

Theorem 3.7 of [1]: *Let$(H_1, \circ_2, 0)$ and $(H_2, \circ_2, 0)$ be hyper K-algebras (resp. hyper BCK-algebras) such that $H_1 \cap H_2 = \{0\}$ and $H = H_1 \cup H_2$. Then $(H, \circ, 0)$ is hyper K-algebra (resp. hyper BCK-algebras), where the hyperoperation $\circ$ on $H$ is defined as follows:*

$$x \circ y := \begin{cases} x \circ_1 y & \text{if } x, y \in H_1, \\ x \circ_2 y & \text{if } x, y \in H_2, \\ \{x\} & \text{otherwise,} \end{cases}$$

*for all $x, y \in H$.*

**Remark 1.** This theorem is not true in general. Because we have $0 \circ 0$ is a subset of $H_1$ and $H_2$. Hence $0 \circ 0$ must be $\{0\}$ in any $H_i$, $i = 1, 2$, but the authors have not considered this fact, this yields that $\circ$ is not well-defined. To see this, consider the following example.

**Example 1.** Let $H_1 = \{0, 1, 2\}$ and $H_2 = \{0, 3, 4\}$. Then $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ with hyperoperations $\circ_1$ and $\circ_2$ as follows are hyper K-algebras.

| $\circ_1$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | {0,1} | {0} | {0} |
| 1 | {1} | {0,1} | {1} |
| 2 | {2} | {2} | {0,1,2} |

| $\circ_2$ | 0 | 3 | 4 |
|---|---|---|---|
| 0 | {0,3,4} | {0} | {0} |
| 3 | {1} | {0,3,4} | {3} |
| 4 | {4} | {4} | {0,3,4}. |

By the definition of hyper operation $\circ$ for the union of two hyper K-algebras in theorem 3.7 of [1] we have: $(i)$: $0 \circ 0 = \{0, 1\}$, because $0 \in H_1$. $(ii)$: $0 \circ 0 = \{0, 3, 4\}$, because $0 \in H_2$. $(iii)$: $0 \circ 0 = \{0\}$, because $0 \in H_1 \cap H_2$. Thus $\circ$ is not well-defined.

Theorem 3.7 [1] is not correct even if we assume $0 \circ 0 = \{0\}$. For this, consider the following example.

**Example 2.** Let $H_1 = \{0, 1, 2\}$, $H_2 = \{0, 3, 4\}$ with hyperoperation $\circ_1$ and $\circ_2$ respectively as follows:

| $\circ_1$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | {0} | {0,1} | {0,1,2} |
| 1 | {1} | {0,1,2} | {0,1} |
| 2 | {2} | {2} | {0,1,2} |

| $\circ_2$ | 0 | 3 | 4 |
|---|---|---|---|
| 0 | {0} | {0,3} | {0,4} |
| 3 | {3} | {0,3,4} | {0,3,4} |
| 4 | {4} | {3,4} | {0,3,4} |

Then $H_1 = (H, \circ_1, 0)$ and $H_2 = (H, \circ_2, 0)$ are hyper K-algebras and $H_1 \cap H_2 = \{0\}$. Thus by considering the hyperoperation $\circ$ of the theorem 3.7 of [1] we have $H = H_1 \cup H_2 = \{0, 1, 2, 3, 4\}$ with hyperoperation $\circ$ as follows:

| $\circ$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | {0} | {0,1} | {0,1,2} | {0,3} | {0,4} |
| 1 | {1} | {0,1,2} | {0,1} | {1} | {1} |
| 2 | {2} | {2} | {0,1,2} | {2} | {2} |
| 3 | {3} | {3} | {3} | {0,3,4} | {0,3,4} |
| 4 | {4} | {4} | {4} | {3,4} | {0,3,4} |

But $(H, \circ, 0)$ is not hyper K-algebra. Since $(1 \circ 1) \circ 3 = \{0, 1, 2, 3\} \neq (1 \circ 3) \circ 1 = \{0, 1, 2\}$, i.e., K2 does not hold.

Now we give not only a correct version of it, but also we extend it.

**Theorem 3.** *Let $(H_i, \circ_i, 0), i \in \Omega$ be a family of hyper K-algebras such that $H_i \cap H_j = \{0\}, i \neq j \in \Omega$ and 0 be a left scalar in each $H_i$, $i \in \Omega$. Then $(H, \circ, 0)$ is hyper K-algebra where, $H = \cup_{i\in\Omega} H$ and $\circ$ on $H$ is defined as follows:*

$$x \circ y := \begin{cases} x \circ_i y & \text{if } x, y \in H_i, \\ \{x\} & \text{if } x \in H_i, y \notin H_i. \end{cases}$$

*Proof.* Since 0 is a left scalar in $H_i$ for all $i \in \Omega$, then $0 \circ 0 = \{0\}$ and $\circ$ is well-defined. Now we prove $H$ satisfies K1-K5. If $x, y, z \in H_i$ for some $i \in \Omega$ then, by hypothesis, $H$ satisfies $K1 - K5$, otherwise we consider the following cases:

(I) $x \in H_i$ and $y, z \notin H_i$,

(II) $x, y \in H_i$ and $z \notin H_i$,

(III) $x, z \in H_i$ and $y \notin H_i$.

**K1**: $(x \circ z) \circ (y \circ z) < x \circ y, \forall x, y, z \in H$.

If (I) holds and $0 \notin y \circ z$, then $(x \circ z) \circ (y \circ z) = x \circ (y \circ z) = \{x\} < x \circ y = \{x\}$. Otherwise $(x \circ z) \circ (y \circ z) = x \circ (y \circ z) = x \circ 0 < x \circ y = \{x\}$, since $x \in x \circ 0$. If (II) holds, then $(x \circ z) \circ (y \circ z) = x \circ_i y < x \circ_i y$. If (III) holds, by considering $x \circ z \subseteq H_i$ and 0 is a left scalar, then $(x \circ z) \circ (y \circ z) = (x \circ_i z) \circ y = x \circ_i z < x \circ y = \{x\}$.

**K2**: $(x \circ y) \circ z = x \circ y = (x \circ z) \circ y, \forall x, y, z \in H$.

If (I) holds, then $(x \circ y) \circ z = x \circ z = \{x\} = (x \circ z) \circ y$. If (II) holds, then $(x \circ y) \circ z = x \circ_i y = (x \circ z) \circ y$, even if $0 \in x \circ y$. Since 0 is a left scalar and $x \circ y \subseteq H_i$, so we have $(x \circ y) \circ z = x \circ_i y$ and K2 holds. If (III) holds, then as proof (II) we have $(x \circ y) \circ z = x \circ_i z = (x \circ_i z) \circ y$, and K2 holds.

The proof of **K3** and **K5** are straightforward.

**K4**: If $x < y$ and $y < x$, then $x = y$. Let $x, y \in H$ be such that $x < y$ and $y < x$. We consider two cases (i): $x \in H_i$, $y \notin H_i$ and (ii): $x, y \in H_i$. If $(i)$ holds, then $0 \in x \circ y = \{x\}$ and $0 \in y \circ x = \{y\}$, hence $x = y = 0$. If $(ii)$ holds, then $H$ satisfies K4. Therefore $(H, \circ, 0)$ is a hyper K-algebra. □

**Definition 4.** Let $(H_i, \circ_i, 0), i \in \Omega$ be hyper K-algebras such that $H_i \cap H_j = \{0\}$, $i \neq j \in \Omega$ and 0 be a left scalar in each $H_i$, $i \in \Omega$. Then the hyper K-algebra $(H, \circ, 0)$ which has been defined in Theorem 3 is called the *union* of a family $\{H_i : i \in \Omega\}$ of hyper K-algebras and it is denoted by $H = \oplus_{i\in\Omega} H_i$(hyper K-algebra).

Now we consider some properties of $H_i$'s, $i \in \Omega$ in which they can be extended to $H = \oplus_{i\in\Omega} H_i$ (hyper K-algebra).

**Theorem 4.** *Let $H = \oplus_{i\in\Omega} H_i$ (hyper K-algebra). Then*

($i$) *whenever 0 is a right scalar, the hyper K-algebra $H$ is positive implicative if and only if each $H_i$, $i \in \Omega$ is positive implicative;*

($ii$) *whenever 0 is a right scalar, the hyper K-algebra $H$ is strong implicative if and only if each $H_i$, $i \in \Omega$ is strong implicative;*

($iii$) *the hyper K-algebra $H$ is weak implicative (implicative) if and only if each $H_i$, $i \in \Omega$ is weak implicative (implicative).*

*Proof.* Since the proof of ($\Rightarrow$) is clear, we prove only ($\Leftarrow$).

($i$) Let each $H_i$, $i \in \Omega$ satisfies the identity:

$$(x \circ y) \circ z = (x \circ z) \circ (y \circ z). \tag{1}$$

We have to show that $H$ satisfies (1) for all $x, y, z \in H$. For briefly, we denote $(x \circ y) \circ z$ by A and $(x \circ z) \circ (y \circ z)$ by B and proceed the proof by following cases.

**Case 1**: If $x, y, z \in H_i$ for some $i \in \Omega$, then the proof is clear.

**Case 2**: If $x, y \in H_i$ and $z \in H_j$ where $i \neq j \in \Omega$, then from the definition of $\circ$ and the fact that 0 is a left scalar in each $H_i$, we have $A = x \circ_i y = B$.

**Case 3**: If $x, z \in H_i$ and $y \in H_j$ where $i \neq j$, then by K2 and Case 2 the proof is obvious.

**Case 4**: If $x \in H_i$ and $y, z \in H_j$ where $i \neq j$, then $A = \{x\}$. Since $y \circ z \subseteq H_j$ and 0 is a right scalar, so we have B$=$ $x \circ (y \circ z) = \{x\}$, hence A$=$B.

**Case 5**: If $x \in H_i$, $y \in H_j$ and $z \in H_k$ where $i \neq j$, $i \neq k$, $j \neq k$ and $i, j, k \in \Omega$, then $A = \{x\} = B$, which completes the proof of ($i$).

($ii$) We know $H$ is strong implicative if $x \circ 0 \subseteq x \circ (y \circ x)$. Let $x, y \in H$. For $x, y \in H_i$, $i \in \Omega$, the proof is obvious. For $x \in H_i$, $y \in H_j$, where $i \neq j \in \Omega$, the fact that 0 is a right scalar implies $x \circ 0 = \{x\} \subseteq x \circ (y \circ x) = \{x\}$, which completes the proof.

($iii$) If $H$ is weak implicative (implicative) then $x < x \circ (y \circ x)$ (resp. $x \in x \circ (y \circ x)$). Thus the case $x, y \in H_i$, $i \in \Omega$, is obvious. In the case $x \in H_i$, $y \in H_j$, $i \neq j \in \Omega$, we have $x \circ (y \circ x) = \{x\}$. Hence $x < x \circ (y \circ x)$ (resp. $x \in x \circ (y \circ x)$). □

**Theorem 5.** *Let $H = \oplus_{i\in\Omega} H_i$ (hyper K-algebra). Then $H \overset{\mathrm{H}}{=} \oplus_{i\in\Omega} H_i$ (hyper K-ideal), moreover $H$ is (hyper K-ideal)-CUD.*

*Proof.* It is sufficient to prove that each $H_i$, $i \in \Omega$, is an H-absorbing hyper K-ideal in $H$. By Theorem 2, $H$ is (hyper K-ideal)-CUD. Let $x \neq 0$, $x \circ y <$

$H_i$ and $y \in H_i$. If $x \notin H_i$, then we have $0 \in (x \circ y) \circ t = \{x\}$, for some $t \in H_i$, hence $x = 0$ which is a contradiction. So, $x \in H_i$ and $H_i$ is a hyper K-ideal of $H$. Since $H_i \cap H_j = \{0\}$, $i \neq j$, and $H = \cup H_{i\in\Omega}$, by Definition 3 we conclude that $H = \oplus_{i\in\Omega} H_i$ (hyper K-ideal). Also $H_i$, $i \in \Omega$ is H-absorbing. Indeed, for $x, y \in H_i$ we have $x \circ y = x \circ_i y \subseteq H_i$. For $x \in H_i$, $y \notin H_i$, we have $x \circ y = \{x\} \subset H_i$. Hence each $H_i$ is H-absorbing. □

**Corollary 1.** *Let $H = \oplus_{i\in\Omega} H_i$ (hyper K-algebra). Then $H \overset{\mathrm{H}}{=} \oplus_{i\in\Omega} H_i$ (closed set), moreover $H$ is (closed set)-CUD*

*Proof.* Since a hyper K-ideal is closed set, the proof follows from Theorem 5. □

Now we show that there exists a (hyper K-ideal)-decomposable hyper K-algebra such that it is not a union of any family of hyper K-algebras. Hence the converse of Theorem 5, is not true in general. In the next section we show that if $H$ is (hyper BCK-ideal)-decomposable, then it is a union of a family of hyper BCK-algebras.

**Example 3.** Let $H = \{0, 1, 2, 3\}$ and consider the following table:

| $\circ$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | {0} | {0} | {0} | {0} |
| 1 | {1} | {0,1} | {0,1} | {1,2} |
| 2 | {2} | {2} | {0,2} | {2} |
| 3 | {3} | {3} | {3} | {0,3} |

Then $(H, \circ, 0)$ is a hyper K-algebra and $H = \{0, 1, 2\} \oplus \{0, 3\}$. It is clear that $\{0, 1, 2\}$ and $\{0, 3\}$ are hyper K-subalgebras of $H$, but $H$ is not the union of $H_1 = \{0, 1, 2\}$ and $H_2 = \{0, 3\}$ because $1 \circ 3 = \{1, 2\} \neq \{1\}$.

## 3. Union hyper BCK-algebras

In this section we show that a hyper BCK-algebra is (closed set)-decomposable if and only if it is the union of a family of hyper BCK-algebras.

**Lemma 1.** *Let $(H, \circ, 0)$ be a hyper BCK-algebra. If $0 \in (x \circ u) \circ 0$ where $x, u \in H$, then $x \ll u$.*

*Proof.* Suppose $0 \in (x \circ u) \circ 0$, then there is an element $t \in x \circ u$ in which $0 \in t \circ 0$, that is, $t \ll 0$. Since $0 \ll t$ and HK4 holds, we conclude that $t = 0$. Since $t \in x \circ u$, thus $0 \in x \circ u$ or $x \ll u$. □

**Theorem 6.** *In any hyper BCK-algebra* 0 *is a scalar.*

*Proof.* In any hyper BCK-algebra we have $0 \circ x = \{0\}$, i.e., 0 is a left scalar. Now we show that 0 is a right scalar, i.e., $x \circ 0 = \{x\}$. We know $x \in x \circ 0$. Now let $u \in x \circ 0$. Then we show that $u = x$. From $0 \in u \circ u$ and HK2 we get that $0 \in (x \circ 0) \circ u = (x \circ u) \circ 0$. Hence by Lemma 1, $x \ll u$. On the other hand we have $x \circ 0 \ll x$, so $u \ll x$. By considering HK4, from $u \ll x$ and $x \ll u$ we conclude that $u = x$. Thus $x \circ 0 = \{x\}$. □

**Theorem 7.** *Let* $(H_i, \circ_i, 0), i \in \Omega$ *be hyper BCK-algebras such that* $H_i \cap H_j = \{0\}$, $i \neq j \in \Omega$. *Then* $(H, \circ, 0)$, *where* $H = \cup_{i\in\Omega} H$ *and*

$$x \circ y := \begin{cases} x \circ_i y & \text{for } x, y \in H_i \\ \{x\} & \text{for } x \in H_i,\ y \notin H_i, \end{cases}$$

*is a hyper BCK-algebra. We denote it by* $H = \oplus_{i\in\Omega} H_i$ (*hyper BCK-algebra*).

*Proof.* By Lemma 6, the element 0 is a scalar in any hyper BCK-algebra, hence the proof follows from Theorem 3. □

**Theorem 8.** *Let* $H = \oplus_{i\in\Omega} H_i$ (*hyper BCK-algebra*). *Then each* $H_i$, $i \in \Omega$ *is weak implicative* (*implicative, strong implicative*) *if and only if* $H$ *is weak implicative* (*implicative, strong implicative*).

*Proof.* Since 0 is a scalar, the proof follows from Theorem 4. □

**Lemma 2.** *Any closed subset of a hyper BCK-algebra is a hyper BCK-subalgebra.* □

**Theorem 9.** ***(Main Theorem)*** *Let* $(H, \circ, 0)$ *be a hyper BCK-algebra. Then* $H = \oplus_{i\in\Omega} A_i$ (*closed set*) *if and only if* $H = \oplus_{i\in\Omega} A_i$ (*hyper BCK-algebra*).

*Proof.* ($\Rightarrow$) Let $H = \oplus_{i\in\Omega} A_i$ (closed set). Then by Lemma 2, $A_i$ is a hyper BCK-subalgebra for any $i \in \Omega$. Now, suppose $0 \neq x \in A_i$ and $y \notin A_i$. In view of Theorem 7, we must show $x \circ y = \{x\}$. We assume that $y \in A_j$ where $i \neq j \in \Omega$. If $0 \in x \circ y$, i.e., $x \ll y$ then $x \in A_j$, because $y \in A_j$ and $A_j$ is a closed set. This is a contradiction to $A_i \cap A_j = \{0\}$. So, let $0 \notin x \circ y$. We know $x \circ y \ll x$. Let $u \in x \circ y$. Then $u \ll x$, and $0 \in (x \circ y) \circ u = (x \circ u) \circ y$. Therefore $0 \in (x \circ u) \circ y$. Hence there exists $t \in x \circ u$ in which, $0 \in t \circ y$, i.e., $t \ll y$. Since $y \in A_j$ and $A_j$ is a closed set we get that $t \in A_j$. By Theorem 1, as $A_i$ is H-absorbing, we have $t \in x \circ u \subseteq A_i$ and consequently

$t \in A_i \cap A_j$. This implies that $t = 0$ and hence $0 \in x \circ u$, i.e., $x \ll u$. On the other hand we had $u \ll x$, these imply that $x = u$. Therefore $x \circ y = \{x\}$.

($\Leftarrow$) The proof follows from Corollary 1. □

**Corollary 2.** *Let* $(H, \circ, 0)$ *be a hyper BCK-algebra. Then* $H = \oplus_{i\in\Omega} H_i$ (*hyper BCK-algebra*) *if and only if* $H = \oplus_{i\in\Omega} H_i$ (*hyper BCK-ideal*). □

**Corollary 3.** *Let* $(H, \circ, 0)$ *be a hyper BCK-algebra. Then* $H = \oplus_{i\in\Omega} H_i$ (*hyper BCK-ideal*) *if and only if* $H = \oplus_{i\in\Omega} H_i$ (*closed set*). □

## References

[1] **R. A. Borzooei, A. Hasankhani, M. M. Zahedi, and Y.B. Jun**: *On hyper K-algebras*, Math. Japonicae **52** (2000), 113 – 121.

[2] **Y. Imai and K. Iséki**: *On axiom systems of propositional calculi xiv*, Proc. Japan Academy **42** (1966), 19 – 22.

[3] **Y. B. Jun, X. L. Xin, E. H. Roh, and M. M. Zahedi**: *Strong on hyper BCK-ideals of hyper BCK-algebras*, Math. Japonicae **51** (2000), 493 – 498.

[4] **Y. B. Jun, M. M. Zahedi, X. L. Xin, and R. A. Borzooei**: *On hyper BCK-algebra*, Italian J. Pure and Appl Math. **10** (2000), 127 – 136.

[5] **M. A. Nasr-Azadani and M. M. Zahedi**: *Quasi union on hyper K-algebras*, Quasigroups and Related Systems **16** (2008), 65 – 74.

[6] **M. A. Nasr-Azadani and M. M. Zahedi**: *S-absorbing set and (P)-decommposition in hyper algebras*, Italian J. Pure and Appl. Math. (2008), (to appear).

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