The Mathematical School founded by Constantin Sibirschi

Constantin Sibirschi (1930-1991) worked in Dynamical Systems, an area of mathematics which is the study of systems which evolve in time such as for example the solar system. An old difficult problem in dynamical systems, which interested many mathematicians, was the problem of the stability of the solar system. This is the problem of determining whether in the very long distant future, the solar system will preserve its present state or whether one of the planets could escape from the system or collision of planets could occur. In the simpler case of the sun and only one planet, the equations of motion are easy to integrate. But this problem is very difficult in the general case of all planets. Poincaré was interested in this problem and because the differential equations obtained from Newton's law of gravitation are very difficult to study, he decided to concentrate on much simpler equations, the planar polynomial differential systems. Poincaré founded their qualitative theory and more generally the qualitative theory of dynamical systems. Unlike his predecessors, Poincaré was interested in studying the mutual relations among the various solutions and even the totality of solutions by geometric, qualitative rather than quantitative methods. Poincaré published several papers on these systems and stated two problems on them which are still open today.

One of the main contributions of Sibirschi was the development of a theory of algebraic invariants for planar polynomial differential systems. The classical invariant theory was a very active area of research in the second part of the nineteenth century. This theory was about polynomial invariants, under the action of the linear group $GL(n, \mathbb{R})$, of n-forms of degree m. The main theorems of classical invariant theory were proved by David Hilbert (1862-1943) in the early 1890's.

Sibirchi's original idea was to construct an analogous theory in which n-forms are replaced by degree n polynomial differential systems. He published many papers and four books on this topic (in MathSciNet he is listed with 96 publications) and two of his books have been translated in English. Sibirschi's work was the first systematic study of algebraic invariants in the theory of polynomial differential systems. This work was not always done by Sibirschi alone but with students whom he initiated in his theory. Some of these students later became his collaborators who in turn had their own students. A third generation of mathematicians works, along the second generation, on the algebraic invariant theory of these systems. Thus the second most important contribution of Sibirschi was as a builder of a school which resisted the passing of time after the death of its founder and which today is thriving with activity.

This school has acquired an international reputation. Work in invariant theory of polynomial differential equations is now done for example in Brazil, Canada, France, Slovenia, Spain and USA, and the influence of the school continues to grow. Collaborations of members of the school with scientists in these countries led to results published in very good mathematical journals.

The problems stated by Poincaré on these systems have a global content. Another problem with a global content which has not been solved, was formulated by Hilbert in 1900. Global problems are hard because we are not concerned with the study of individual systems but with whole classes of systems. A class of systems may depend on several parameters and in general cannot be studied in a single presentation (normal form) of the systems involved, but in several such presentations. The results however need to be integrated, so the different presentations need to be glued together in a unique whole. It is in this process that the algebraic invariants play an essential role. The passing from one presentation to another is done by changes of coordinates which are assembled in groups such as for example the group of linear transformations $GL(n, \mathbb{R})$. In this gluing process, the invariant polynomials with respect to these groups accomplish the translation from one chart to another so that we have a full map of the whole class of systems. Another role of the algebraic invariants is to distinguish objects which are not isomorphic.

Thus the invariant theory of polynomial differential systems built by the school of Sibirschi and applied by other scientists in the world, plays an important role in obtaining global results, independent of the specific coordinate systems chosen in a presentation.

Dana Schlomiuk
Professor of Mathematics
Département de Mathématiques et de Statistique, Université de Montéal,
Member of the CRM (Centre de Recherches Mathématiques)
Montréal, CANADA