About an algebraic study of the center-focus problem

M. N. Popa, V. V. Pricop

Abstract. Consider the nonlinear differential system

$$\dot{x} = \sum_{i=0}^{\ell} P_{m_i}(x, y), \ \dot{y} = \sum_{i=0}^{\ell} Q_{m_i}(x, y),$$

where P_{m_i} and Q_{m_i} are homogeneous polynomials of degree $m_i \geq 1$ in relation to x and y, and $m_0 = 1$. The set $\{1, m_i\}_{i=1}^{\ell}$ consists of a finite number $(l < \infty)$ of distinct integer numbers. It is shown that the maximal number of algebraically independent focal quantities which participate in solving the center-focus problem for given differential system with $m_0 = 1$, having at the origin of coordinates a singular point of the second type (center or focus), do not exceed

$$\varrho = 2(\sum_{i=1}^{\ell} m_i + \ell) + 3.$$

There is an assumption that the number ω of essential conditions of center which solves the center-focus problem for given differential system with $m_0 = 1$, having at the origin of coordinates a singular point of the second type (center or focus), do not exceed ϱ i. e. $\omega \leq \varrho$.