

## Capitolul 1. Limite de şiruri.

1. a) Utilizând definiția limitei cu "ε" să se arate că şirul numeric  $a_n$  este convergent şi are limita  $a$ .  
b) Să se determine rangul, începând de la care termenul şirului diferă de  $a$  cu mai puțin de 0.001.

$$1.1. \quad a_n = \frac{4n+2}{5n+1}, \quad a = \frac{4}{5}.$$

$$1.2. \quad a_n = \frac{2n+1}{3n-2}, \quad a = \frac{2}{3}.$$

$$1.3. \quad a_n = \frac{1-2n}{n+2}, \quad a = -2.$$

$$1.4. \quad a_n = \frac{3n}{2n-1}, \quad a = \frac{3}{2}.$$

$$1.5. \quad a_n = \frac{7n+1}{1-3n}, \quad a = -\frac{7}{3}.$$

$$1.6. \quad a_n = \frac{2n-1}{7n-3}, \quad a = \frac{2}{7}.$$

$$1.7. \quad a_n = \frac{2-n}{1-2n}, \quad a = \frac{1}{2}.$$

$$1.8. \quad a_n = \frac{6-3n}{2n-1}, \quad a = -\frac{3}{2}.$$

$$1.9. \quad a_n = \frac{2n^2+1}{8n^2-1}, \quad a = \frac{1}{4}.$$

$$1.10. \quad a_n = \frac{2n^2}{3-n^2}, \quad a = -2.$$

$$1.11. \quad a_n = \frac{3n^2+2}{1-4n^2}, \quad a = -\frac{3}{4}.$$

$$1.12. \quad a_n = \frac{1-5n^2}{2-4n^2}, \quad a = \frac{5}{4}.$$

$$1.13. \quad a_n = \frac{4n^3}{2n^3-1}, \quad a = 2.$$

$$1.14. \quad a_n = \frac{8-n^3}{1+2n^3}, \quad a = -\frac{1}{2}.$$

$$1.15. \quad a_n = \frac{2+n^3}{2n^3-1}, \quad a = \frac{1}{2}.$$

$$1.16. \quad a_n = \frac{1-n^3}{1+n^3}, \quad a = -1.$$

2. a) Să se arate că şirurile date sunt convergente:

$$2.1. \quad a_n = \frac{2n+3}{3n-2}.$$

$$2.2. \quad a_n = \frac{n-1}{n+1}.$$

$$2.3. \quad a_n = \frac{2n+1}{n+3}.$$

$$2.4. \quad a_n = \frac{1-2n}{n+1}.$$

$$2.5. \quad a_n = \frac{1-3n}{1-4n}.$$

$$2.6. \quad a_n = \frac{2n-1}{n+2}.$$

$$2.7. \quad a_n = \sum_{k=1}^n \frac{1}{k(k+1)}.$$

$$2.8. \quad a_n = \sum_{k=2}^n \frac{1}{k(k-1)}.$$

$$2.9. \quad a_n = \sum_{k=1}^n \frac{2}{(2k-1)(2k+1)}.$$

$$2.10. \quad a_n = \sum_{k=1}^n \frac{1}{k^2}.$$

$$2.11. \quad a_n = \underbrace{\sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}}}_{n \text{ rădăcini}}.$$

$$2.12. \quad a_n = \underbrace{\sqrt[3]{6 + \sqrt[3]{6 + \dots + \sqrt[3]{6}}}}_{n \text{ rădăcini}}.$$

$$2.13. \quad a_n = \sum_{k=1}^n \frac{\sin k}{3^k}.$$

$$2.14. \quad a_n = \sum_{k=1}^n \frac{\sin k!}{k(k+1)}.$$

$$2.15. \quad a_n = 1 + \frac{1}{2!} + \dots + \frac{1}{n!}.$$

$$2.16. \quad a_n = \frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \dots + \frac{(-1)^{n-1}}{n(n+1)}.$$

**b) Să se arate că:**

$$2.17. \quad \lim_{n \rightarrow \infty} \sin \frac{\pi}{2} n \neq 1.$$

$$2.18. \quad \lim_{n \rightarrow \infty} \cos \pi n \neq 1.$$

$$2.19. \quad \lim_{n \rightarrow \infty} \frac{2n-1}{n+1} \neq 1.$$

$$2.20. \quad \lim_{n \rightarrow \infty} \frac{3n-1}{2n+1} \neq 2.$$

$$2.21. \quad \lim_{n \rightarrow \infty} n^2 \sin \frac{\pi n}{4} \neq 0.$$

$$2.22. \quad \lim_{n \rightarrow \infty} \frac{n}{n+1} \cos \frac{2\pi n}{3} \neq 1.$$

**3. Să se calculeze următoarele limite:**

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|--|---|
| 3.1. $\lim_{n \rightarrow \infty} \frac{(n+2)^2 + (n-1)^2}{(2n-1)^2 + (n+1)^2}.$ | 3.2. $\lim_{n \rightarrow \infty} \frac{(n+1)^2 - (n-4)^2}{(3n+1)^2 + (n-1)^2}.$  |
| 3.3. $\lim_{n \rightarrow \infty} \frac{(2-n)^2 - (1+n)^2}{(n-3)^2 - (n+2)^2}.$  | 3.4. $\lim_{n \rightarrow \infty} \frac{(2n-1)^2 - (n-1)^2}{(n+1)^2 + (n-1)^2}.$  |
| 3.5. $\lim_{n \rightarrow \infty} \frac{(1+2n)^3 - 8n^3}{(1-3n)^2 - 3n^2}.$      | 3.6. $\lim_{n \rightarrow \infty} \frac{(n+3)^3 + (n-1)^3}{2n^3 + 3n}.$           |
| 3.7. $\lim_{n \rightarrow \infty} \frac{(n+5)^2 + (n+2)^2}{(n+2)^3 - (n+1)^3}.$  | 3.8. $\lim_{n \rightarrow \infty} \frac{(2n+1)^2 + (1-3n)^2}{(n-2)^3 - (n-1)^3}.$ |
| 3.9. $\lim_{n \rightarrow \infty} \frac{(n+2)^4 - (n-2)^4}{(n+3)^2 + (n-3)^2}.$  | 3.10. $\lim_{n \rightarrow \infty} \frac{(n+1)^4 - (n-1)^4}{(n+1)^3 + (n-1)^3}.$  |
| 3.11. $\lim_{n \rightarrow \infty} \frac{(2n+1)! + (2n+2)!}{(2n+3)!}.$           | 3.12. $\lim_{n \rightarrow \infty} \frac{n! + (n+2)!}{(n-1)! + (n+2)!}.$          |
| 3.13. $\lim_{n \rightarrow \infty} \frac{(n+3)! - (n+1)!}{(n+2)!}.$              | 3.14. $\lim_{n \rightarrow \infty} \frac{(2n-1)! + (2n+1)!}{(2n)!(n+1)}.$         |
| 3.15. $\lim_{n \rightarrow \infty} \frac{(3n)! + (3n-2)!}{(3n-1)!(2n+1)}.$       | 3.16. $\lim_{n \rightarrow \infty} \frac{(n-1)! + (n-2)!}{(n-3)!(3n^2-1)}.$       |

4. Să se calculeze limitele:

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|---|--|
| 4.1. $\lim_{n \rightarrow \infty} \sqrt{n+1} \left( \sqrt{n+3} - \sqrt{n+2} \right).$                       | 4.2. $\lim_{n \rightarrow \infty} \left( \sqrt{(n-1)(n+4)} - n \right).$   |
| 4.3. $\lim_{n \rightarrow \infty} \left( \sqrt{n^2 + 3n + 2} - n \right).$                                  | 4.4. $\lim_{n \rightarrow \infty} \left( n + \sqrt[3]{n^2 - n^3} \right).$   |
| 4.5. $\lim_{n \rightarrow \infty} \left( \sqrt{n^2 + 4n - 2} - \sqrt{n^2 - 2} \right).$                     | 4.6. $\lim_{n \rightarrow \infty} \sqrt{n-1} \left( \sqrt{n+1} - \sqrt{n-3} \right).$  |
| 4.7. $\lim_{n \rightarrow \infty} \left( n\sqrt{n} - \sqrt{n(n^2 - 1)} \right).$                            | 4.8. $\lim_{n \rightarrow \infty} n \left( \sqrt[3]{2 + 8n^3} - 2n \right).$   |
| 4.9. $\lim_{n \rightarrow \infty} \sqrt[3]{n} \left( \sqrt[3]{n^2} - \sqrt[3]{n(n+1)} \right).$             | 4.10. $\lim_{n \rightarrow \infty} n^2 \left( \sqrt[3]{n^3 + 7} - \sqrt[3]{n^3 + 1} \right).$                                |
| 4.11. $\lim_{n \rightarrow \infty} \left( \frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n-1}{n^2} \right).$ | 4.12. $\lim_{n \rightarrow \infty} \left( \frac{2 + 4 + \dots + 2n}{n+2} - n \right).$                                       |
| 4.13. $\lim_{n \rightarrow \infty} \left( \frac{n+2}{1+2+\dots+n} - \frac{3}{2} \right).$                   | 4.14. $\lim_{n \rightarrow \infty} \frac{1+3+5+\dots+2n-1}{2+4+6+\dots+2n}.$   |
| 4.15. $\lim_{n \rightarrow \infty} \frac{5+10+\dots+5n}{n^2+1}.$  | 4.16. $\lim_{n \rightarrow \infty} \frac{1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1)}{n^3}.$                                      |
| 4.17. $\lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + \dots + (2n-1)^2}{n^3}.$                               | 4.18. $\lim_{n \rightarrow \infty} \left( \frac{1^2 + 2^2 + \dots + n^2}{n^2} - \frac{n}{3} \right).$                        |
| 4.19. $\lim_{n \rightarrow \infty} \frac{3^n - 5^{n+1}}{3^{n+1} + 5^{n+2}}.$                                | 4.20. $\lim_{n \rightarrow \infty} \left( \frac{7}{10} + \frac{29}{100} + \dots + \frac{2^n + 5^n}{10^n} \right).$           |
| 4.21. $\lim_{n \rightarrow \infty} \frac{4^n + 7^n}{4^n - 7^{n-1}}.$  | 4.22. $\lim_{n \rightarrow \infty} \left( \frac{3}{4} + \frac{5}{16} + \dots + \frac{1+2^n}{4^n} \right).$                   |
| 4.23. $\lim_{n \rightarrow \infty} \frac{3^n + 5^{-n}}{3^{-n} + 5^n}.$                                      | 4.24. $\lim_{n \rightarrow \infty} \frac{1 + \frac{1}{5} + \dots + \frac{1}{5^n}}{1 + \frac{1}{7} + \dots + \frac{1}{7^n}}.$ |

5. Să se calculeze limitele:

$$5.1. \quad \lim_{n \rightarrow \infty} \left( \frac{2n+3}{2n-1} \right)^n.$$

$$5.2. \quad \lim_{n \rightarrow \infty} \left( \frac{n+2}{n+1} \right)^{1-n}.$$

$$5.3. \quad \lim_{n \rightarrow \infty} \left( \frac{3n-1}{3n+2} \right)^{2n+1}.$$

$$5.4. \quad \lim_{n \rightarrow \infty} \left( \frac{2n+1}{2n-5} \right)^{\frac{n}{6}}.$$

$$5.5. \quad \lim_{n \rightarrow \infty} \left( \frac{n^2-1}{n^2+1} \right)^{2n-1}.$$

$$5.6. \quad \lim_{n \rightarrow \infty} \left( \frac{3n^2+2}{3n^2-1} \right)^{\frac{n}{3}}.$$

$$5.7. \quad \lim_{n \rightarrow \infty} \left( \frac{3n+1}{3n} \right)^{1-n^2}.$$

$$5.8. \quad \lim_{n \rightarrow \infty} \left( \frac{2n^2+1}{2n^2-3} \right)^{1-n^3}.$$

$$5.9. \quad \lim_{n \rightarrow \infty} \left( \frac{2n^2+2}{2n^2+1} \right)^{n^2}.$$

$$5.10. \quad \lim_{n \rightarrow \infty} \left( \frac{n^2+n+1}{n^2+n-1} \right)^{n^2-1}.$$

$$5.11. \quad \lim_{n \rightarrow \infty} \left( \frac{3n^2+2}{3n^2-1} \right)^{n^2+1}.$$

$$5.12. \quad \lim_{n \rightarrow \infty} \left( \frac{2n^2+n+2}{2n^2-2n+3} \right)^{2n}.$$

$$5.13. \quad \lim_{n \rightarrow \infty} \left( \frac{3n^2+6n+7}{3n^2+6n+4} \right)^{6n^2-5n+4}.$$

$$5.14. \quad \lim_{n \rightarrow \infty} \left( \frac{n^2+1}{n^2-1} \right)^{2n^2}.$$

$$5.15. \quad \lim_{n \rightarrow \infty} \left( \frac{n^2+2n+3}{n^2+3n+4} \right)^{2n-1}.$$

$$5.16. \quad \lim_{n \rightarrow \infty} \left( \frac{4n^2+2}{4n^2-2} \right)^{n^2}.$$

$$5.17. \quad \lim_{n \rightarrow \infty} \left( \frac{n^2+n-1}{3n^2-n+1} \right)^{\frac{1}{n}}.$$

$$5.18. \quad \lim_{n \rightarrow \infty} \left( \frac{1+2n}{3n-1} \right)^n.$$

$$5.19. \quad \lim_{n \rightarrow \infty} \left( \frac{n+1}{n-1} \right)^{\frac{2n^2+1}{2n^2-1}}.$$

$$5.20. \quad \lim_{n \rightarrow \infty} \left( \frac{n^2-1}{2n^2+1} \right)^{\frac{2n}{n+1}}.$$