

Capitolul 2. Limite de funcții.

1. Să se calculeze următoarele limite:

$$1.1. \quad \lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + x - 6}.$$

$$1.2. \quad \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 + 6x - 7}.$$

$$1.3. \quad \lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - x - 6}.$$

$$1.4. \quad \lim_{x \rightarrow 4} \frac{x^2 - 7x + 12}{x^2 - 6x + 8}.$$

$$1.5. \quad \lim_{x \rightarrow 0} \frac{x^3 - 6x^2 + 7x}{x^2 + x}.$$

$$1.6. \quad \lim_{x \rightarrow -2} \frac{x^2 + x - 2}{x^2 - x - 6}.$$

$$1.7. \quad \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}.$$

$$1.8. \quad \lim_{x \rightarrow 1} \frac{x^m - 1}{x^n - 1}, \quad m, n \in \mathbb{N}.$$

$$1.9. \quad \lim_{x \rightarrow 0} \frac{(x+2)(1-x)(2x+1) - 2}{x^2 + x}.$$

$$1.10. \quad \lim_{x \rightarrow 2} \frac{x^4 - 5x^2 + 4}{x^4 - 3x^2 - 4}.$$

$$1.11. \quad \lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - x - 2}{x^3 - 7x - 6}.$$

$$1.12. \quad \lim_{x \rightarrow -1} \frac{x^4 + x^2 - 2}{x^4 - 1}.$$

$$1.13. \quad \lim_{x \rightarrow -1} \frac{x^3 + 2x + 3}{x^3 + 1}.$$

$$1.14. \quad \lim_{x \rightarrow 2} \frac{x^4 - 2x^3 - 3x^2 + 4x + 4}{x^4 - 6x^3 + 13x^2 - 12x + 4}.$$

$$1.15. \quad \lim_{x \rightarrow 1} \frac{2x^4 - x^2 - 1}{x^4 - 1}.$$

$$1.16. \quad \lim_{x \rightarrow 0} \frac{(x+1)^3 - (3x+1)}{2x^4 + x^2}.$$

$$1.17. \quad \lim_{x \rightarrow -1} \frac{x^3 + 3x^2 + 7x + 5}{x^3 - x^2 - x + 1}.$$

$$1.18. \quad \lim_{x \rightarrow 2} \frac{x^3 - 3x - 2}{x^2 - 4}.$$

$$1.19. \quad \lim_{x \rightarrow -1} \frac{(x^3 - 2x - 1)^2}{x^4 - 2x^2 + 1}.$$

$$1.20. \quad \lim_{x \rightarrow 0} \frac{(x+2)^3 - 8}{(x+1)^4 - (1+2x)}.$$

2. Să se calculeze următoarele limite:

$$2.1. \quad \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^2 + x - 2}.$$

$$2.2. \quad \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{\sqrt{x + 2} - 2}.$$

$$2.3. \quad \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + x + 4} - 2}{\sqrt{1 - x + x^2} - 1}.$$

$$2.4. \quad \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{\sqrt{4 + 3x} - 4}.$$

$$2.5. \quad \lim_{x \rightarrow 0} \frac{\sqrt{4 - x + x^2} - (2 + x)}{x^2 + x}.$$

$$2.6. \quad \lim_{x \rightarrow 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{\sqrt{2 + x} - \sqrt{2 - x}}.$$

$$2.7. \quad \lim_{x \rightarrow 5} \frac{\sqrt{x + 4} - \sqrt{2x - 1}}{x^2 - 25}.$$

$$2.8. \quad \lim_{x \rightarrow 0} \frac{3\sqrt{x^2 + x + 1} - (3 + x)}{x^2 + 3x}.$$

$$2.9. \quad \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}.$$

$$2.10. \quad \lim_{x \rightarrow 4} \frac{\sqrt[3]{16x} - 4}{\sqrt{x + 4} - \sqrt{2x}}.$$

$$2.11. \quad \lim_{x \rightarrow 2} \frac{\sqrt[3]{x - 1} - 1}{x^3 - 8}.$$

$$2.12. \quad \lim_{x \rightarrow -2} \frac{x^3 + 8}{\sqrt[3]{x - 6} + 2}.$$

$$2.13. \quad \lim_{x \rightarrow 1} \frac{\sqrt{x + 2} - \sqrt{3x}}{\sqrt[3]{x} - 1}.$$

$$2.14. \quad \lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - 2}{\sqrt{x + 1} - 3}.$$

$$2.15. \quad \lim_{x \rightarrow 0} \frac{\sqrt[3]{2 + x} - \sqrt[3]{2 - x}}{\sqrt{2 + x} - \sqrt{2 - x}}.$$

$$2.16. \quad \lim_{x \rightarrow 1} \frac{\sqrt{x} + \sqrt{x - 1} - 1}{\sqrt{x^2 - 1}}.$$

$$2.17. \quad \lim_{x \rightarrow 8} \frac{\sqrt[3]{x} - 2}{x - 8}.$$

$$2.18. \quad \lim_{x \rightarrow -8} \frac{\sqrt[3]{15 + 2x} + 1}{\sqrt[3]{9 + x} + x + 7}.$$

$$2.19. \quad \lim_{x \rightarrow 7} \frac{\sqrt{x + 2} - \sqrt[3]{x + 20}}{\sqrt[4]{x + 9} - 2}.$$

$$2.20. \quad \lim_{x \rightarrow 0} \frac{\sqrt[5]{2x^2 + 10x + 1} - \sqrt[7]{x^2 + 10x + 1}}{x}.$$

3. Să se calculeze următoarele limite:

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| 3.1. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - 1}) .$ | 3.2. $\lim_{x \rightarrow \infty} (\sqrt{9x^4 + 3x^2 - 7} - 3x^2) .$ |
| 3.3. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x - 1} - \sqrt{x^2 - 2x - 1}) .$ | 3.4. $\lim_{x \rightarrow \infty} (\sqrt{x^4 + x^2} - \sqrt{x^4 + 8x^2 + 3}) .$ |
| 3.5. $\lim_{x \rightarrow \infty} (\sqrt{x^2 - 3x + 2} - x) .$ | 3.6. $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - \sqrt{x^2 + 2x + 3}) .$ |
| 3.7. $\lim_{x \rightarrow \infty} (x\sqrt[3]{8x^3 + 5} - 2x) .$ | 3.8. $\lim_{x \rightarrow \infty} \sqrt{x^3 + 8} (\sqrt{x^3 + 2} - \sqrt[3]{x^3 - 1}) .$ |
| 3.9. $\lim_{x \rightarrow \infty} x\sqrt{x} (x - \sqrt[3]{x^3 - 5}) .$ | 3.10. $\lim_{x \rightarrow \infty} x\sqrt{x} (\sqrt{x^4 + 3} - \sqrt{x^4 + 2}) .$ |
| 3.11. $\lim_{x \rightarrow \infty} \sqrt{x} (\sqrt{x + 2} - \sqrt{x + 3}) .$ | 3.12. $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 - x}) .$ |
| 3.13. $\lim_{x \rightarrow \infty} \left(\frac{x^3}{2x^2 - 1} - \frac{x^2}{2x + 1} \right) .$ | 3.14. $\lim_{x \rightarrow 1} \left(\frac{1}{1 - x} - \frac{2}{1 - x^2} \right) .$ |
| 3.15. $\lim_{x \rightarrow 1} \left(\frac{1}{x - 1} - \frac{3}{x^3 - 1} \right) .$ | 3.16. $\lim_{x \rightarrow 2} \left(\frac{1}{(x - 2)(x - 1)} - \frac{2}{x^2 - 2x} \right) .$ |
| 3.17. $\lim_{x \rightarrow -1} \left(\frac{2}{x + 1} - \frac{x - 3}{x^2 - 1} \right) .$ | 3.18. $\lim_{x \rightarrow 2} \left(\frac{1}{x(x - 2)^2} - \frac{1}{x^2 - 3x + 2} \right) .$ |

4. Să se calculeze următoarele limite:

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| 4.1. $\lim_{x \rightarrow 0} \frac{\sin 5x}{x} .$ | 4.2. $\lim_{x \rightarrow 0} \frac{\sin 8x + \sin 6x}{2x} .$ |
| 4.3. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x} .$ | 4.4. $\lim_{x \rightarrow 0} \frac{\cos 5x - \cos 3x}{4x^2} .$ |
| 4.5. $\lim_{x \rightarrow 0} \frac{\sin^2 2x}{\sin^2 3x} .$ | 4.6. $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 8x} .$ |
| 4.7. $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} .$ | 4.8. $\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{2x \sin x} .$ |
| 4.9. $\lim_{x \rightarrow \pi} \frac{\sin 2x}{\sin 3x} .$ | 4.10. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{\pi - 6x} .$ |
| 4.11. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{tg} 5x}{\operatorname{tg} 3x} .$ | 4.12. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \operatorname{tg}^2 x}{\sqrt{2} \cos x - 1} .$ |

$$\begin{array}{ll}
4.13. \quad \lim_{x \rightarrow \pi} \frac{\sin 2x}{\operatorname{tg} 3x}. & 4.14. \quad \lim_{x \rightarrow -\frac{\pi}{4}} \frac{1 + \sin 2x}{1 + \cos 4x}. \\
4.15. \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - 2 \cos x}{\pi - 4x}. & 4.16. \quad \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \operatorname{ctg} x \right). \\
4.17. \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{2x^3}. & 4.18. \quad \lim_{x \rightarrow \frac{\pi}{6}} \frac{6 \sin^2 x - 5 \sin x + 1}{4 \sin^2 x - 1}. \\
4.19. \quad \lim_{x \rightarrow 0} \frac{\sqrt{1 + x \sin x} - 1}{x^2}. & 4.20. \quad \lim_{x \rightarrow \pi} \frac{\sqrt{1 - \operatorname{tg} x} - \sqrt{1 + \operatorname{tg} x}}{\sin 2x}. \\
4.21. \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg}(\sin x) - \sin(\operatorname{tg} x)}{x^3}. & 4.22. \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg}(\operatorname{tg} x) - \sin(\sin x)}{\operatorname{tg} x - \sin x}.
\end{array}$$

5. Să se calculeze următoarele limite:

$$\begin{array}{ll}
5.1. \quad \lim_{x \rightarrow \infty} \left(\frac{x+2}{x-3} \right)^{2x-1}. & 5.2. \quad \lim_{x \rightarrow \infty} \left(\frac{x^2+4}{x^2-4} \right)^{x^2}. \\
5.3. \quad \lim_{x \rightarrow \infty} \left(\frac{2x+1}{2x+3} \right)^{\frac{x}{2}}. & 5.4. \quad \lim_{x \rightarrow \infty} \left(\frac{\sqrt{x}+3}{\sqrt{x}+2} \right)^{\frac{1-x}{1-\sqrt{x}}}. \\
5.5. \quad \lim_{x \rightarrow 0} (1+5x)^{\frac{1}{x}}. & 5.6. \quad \lim_{x \rightarrow 0} (1+\sin x)^{\frac{1}{\sin 2x}}. \\
5.7. \quad \lim_{x \rightarrow 0} (1+2 \operatorname{tg}^2 x)^{\operatorname{ctg}^2 x}. & 5.8. \quad \lim_{x \rightarrow 0} (\cos 2x)^{\frac{1}{x^2}}. \\
5.9. \quad \lim_{x \rightarrow 0} (\cos x + \sin x)^{\frac{1}{x}}. & 5.10. \quad \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}}. \\
5.11. \quad \lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\operatorname{tg}^2 x}. & 5.12. \quad \lim_{x \rightarrow \frac{\pi}{2}} (1 + \operatorname{ctg} x)^{\operatorname{tg} x}. \\
5.13. \quad \lim_{x \rightarrow \frac{\pi}{2}} \left(\operatorname{ctg} \frac{x}{2} \right)^{\frac{1}{\cos x}}. & 5.14. \quad \lim_{x \rightarrow 1} (2-x)^{\operatorname{tg} \frac{\pi x}{2}}. \\
5.15. \quad \lim_{x \rightarrow 0} \left(4 - \frac{3}{\cos x} \right)^{\operatorname{tg}^2 x}. & 5.16. \quad \lim_{x \rightarrow 0} \left[\operatorname{tg} \left(\frac{\pi}{4} - x \right) \right]^{\operatorname{ctg} x}.
\end{array}$$

6. Să se calculeze limitele:

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| 6.1. $\lim_{x \rightarrow 0} \frac{\ln(1 + 2x^2)}{\sqrt{1 + x^2} - 1}.$ | 6.2. $\lim_{x \rightarrow 0} \frac{\ln(1 + \sin 2x)}{\sin 4x - \sin 2x}.$ |
| 6.3. $\lim_{x \rightarrow 0} \frac{3^x - 1}{\ln(1 + 2x)}.$ | 6.4. $\lim_{x \rightarrow 0} \frac{\arcsin 2x}{\operatorname{arctg} 4x}.$ |
| 6.5. $\lim_{x \rightarrow 0} \frac{\ln(1 + 2x)}{\operatorname{arctg} 3x}.$ | 6.6. $\lim_{x \rightarrow 0} \frac{3^x - 2^x}{2x - \operatorname{arctg} x}.$ |
| 6.7. $\lim_{x \rightarrow 0} \frac{2^{3x} - 3^{2x}}{2 \arcsin x - \sin x}.$ | 6.8. $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{2x}}{x + \sin x^2}.$ |
| 6.9. $\lim_{x \rightarrow 0} \frac{\sqrt{1 + x \sin x} - 1}{e^{x^2} - 1}.$ | 6.10. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\ln(x - 1)}.$ |
| 6.11. $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt[4]{x} - 1}.$ | 6.12. $\lim_{x \rightarrow -1} \frac{3 - \sqrt{10 + x}}{\sin 3\pi x}.$ |
| 6.13. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{2^{\cos^2 x} - 1}{\ln \sin x}.$ | 6.14. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\ln \sin 3x}{(6x - \pi)^2}.$ |
| 6.15. $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x - 3 \arcsin x}{\sin 6x - 6 \operatorname{arctg} 2x}.$ | 6.16. $\lim_{x \rightarrow \infty} x \left(2^{\frac{1}{x}} - 1 \right).$ |
| 6.17. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln \sin 5x}{\ln \sin 3x}.$ | 6.18. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1 + x} - 1 - \sin x}{\ln(1 + x)}.$ |
| 6.19. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos x} - \sqrt[4]{\cos 2x}}{1 - \cos 12x}.$ | 6.20. $\lim_{x \rightarrow \frac{1}{4}} \frac{1 - \operatorname{ctg} \pi x}{\ln \operatorname{tg} \pi x}.$ |
| 6.21. $\lim_{x \rightarrow 0} (\cos 2x)^{-\frac{1}{x^2}}.$ | 6.22. $\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{\sin^2 x}.$ |