Universitatea Academiei de Ştiinţe a Moldovei

Andrei Corlat, Sergiu Corlat

Culegere de probleme de calcul diferenţial şi integral

Material didactic la disciplina Analiza matematică

Chişinău – 2012

CZU

Andrei Corlat, Sergiu Corlat. Culegere de probleme de calcul diferenţial şi integral. (Material didactic la disciplina Analiza matematică).
Chişinău, 2012.

Recomandat de Senatul Universităţii Academiei de Ştiinţe a Moldovei

Descrierea CIP a Camerei Naţionale a Cărţii

Responsabilitatea asupra conţinutului
revine în exclusivitate autorului

Cuprins

Capitolul 1. LIMITE DE ŞIRURI

1. a) Utilizând definiţia limitei cu "ε" să se arate că şirul numeric a_n este convergent şi are limita a.

b) Să se determine rangul, începând de la care termenul şirului diferă de a cu mai puţin de 0.001.

1.1. $a_n = \dfrac{4n+2}{5n+1}, \quad a = \dfrac{4}{5}.$

1.2. $a_n = \dfrac{2n+1}{3n-2}, \quad a = \dfrac{2}{3}.$

1.3. $a_n = \dfrac{1-2n}{n+2}, \quad a = -2.$

1.4. $a_n = \dfrac{3n}{2n-1}, \quad a = \dfrac{3}{2}.$

1.5. $a_n = \dfrac{7n+1}{1-3n}, \quad a = -\dfrac{7}{3}.$

1.6. $a_n = \dfrac{2n-1}{7n-3}, \quad a = \dfrac{2}{7}.$

1.7. $a_n = \dfrac{2-n}{1-2n}, \quad a = \dfrac{1}{2}.$

1.8. $a_n = \dfrac{6-3n}{2n-1}, \quad a = -\dfrac{3}{2}.$

1.9. $a_n = \dfrac{2n^2+1}{8n^2-1}, \quad a = \dfrac{1}{4}.$

1.10. $a_n = \dfrac{2n^2}{3-n^2}, \quad a = -2.$

1.11. $a_n = \dfrac{3n^2+2}{1-4n^2}, \quad a = -\dfrac{3}{4}.$

1.12. $a_n = \dfrac{1-5n^2}{2-4n^2}, \quad a = \dfrac{5}{4}.$

1.13. $a_n = \dfrac{4n^3}{2n^3-1}, \quad a = 2.$

1.14. $a_n = \dfrac{8-n^3}{1+2n^3}, \quad a = -\dfrac{1}{2}.$

1.15. $a_n = \dfrac{2+n^3}{2n^3-1}, \quad a = \dfrac{1}{2}.$

1.16. $a_n = \dfrac{1-n^3}{1+n^3}, \quad a = -1.$

2. a) Să se arate că şirurile date sunt convergente:

2.1. $a_n = \dfrac{2n+3}{3n-2}.$

2.2. $a_n = \dfrac{n-1}{n+1}.$

2.3. $a_n = \dfrac{2n+1}{n+3}.$

2.4. $a_n = \dfrac{1-2n}{n+1}.$

2.5. $a_n = \dfrac{1-3n}{1-4n}.$

2.6. $a_n = \dfrac{2n-1}{n+2}.$

2.7. $a_n = \sum_{k=1}^{n} \frac{1}{k(k+1)}$.

2.8. $a_n = \sum_{k=2}^{n} \frac{1}{k(k-1)}$.

2.9. $a_n = \sum_{k=1}^{n} \frac{2}{(2k-1)(2k+1)}$.

2.10. $a_n = \sum_{k=1}^{n} \frac{1}{k^2}$.

2.11. $a_n = \underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{n \text{ rădăcini}}$.

2.12. $a_n = \underbrace{\sqrt[3]{6+\sqrt[3]{6+\ldots+\sqrt[3]{6}}}}_{n \text{ rădăcini}}$.

2.13. $a_n = \sum_{k=1}^{n} \frac{\sin k}{3^k}$.

2.14. $a_n = \sum_{k=1}^{n} \frac{\sin k!}{k(k+1)}$.

2.15. $a_n = 1 + \frac{1}{2!} + \cdots + \frac{1}{n!}$.

2.16. $a_n = \frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \cdots + \frac{(-1)^{n-1}}{n(n+1)}$.

b) Să se arate că:

2.17. $\lim_{n\to\infty} \sin \frac{\pi}{2} n \neq 1$.

2.18. $\lim_{n\to\infty} \cos \pi n \neq 1$.

2.19. $\lim_{n\to\infty} \frac{2n-1}{n+1} \neq 1$.

2.20. $\lim_{n\to\infty} \frac{3n-1}{2n+1} \neq 2$.

2.21. $\lim_{n\to\infty} n^2 \sin \frac{\pi n}{4} \neq 0$.

2.22. $\lim_{n\to\infty} \frac{n}{n+1} \cos \frac{2\pi n}{3} \neq 1$.

3. Să se calculeze următoarele limite:

3.1. $\lim_{n\to\infty} \frac{(n+2)^2+(n-1)^2}{(2n-1)^2+(n+1)^2}$.

3.2. $\lim_{n\to\infty} \frac{(n+1)^2-(n-4)^2}{(3n+1)^2+(n-1)^2}$.

3.3. $\lim_{n\to\infty} \frac{(2-n)^2-(1+n)^2}{(n-3)^2-(n+2)^2}$.

3.4. $\lim_{n\to\infty} \frac{(2n-1)^2-(n-1)^2}{(n+1)^2+(n-1)^2}$.

3.5. $\lim_{n\to\infty} \frac{(1+2n)^3-8n^3}{(1-3n)^2-3n^2}$.

3.6. $\lim_{n\to\infty} \frac{(n+3)^3+(n-1)^3}{2n^3+3n}$.

3.7. $\lim_{n\to\infty} \frac{(n+5)^2+(n+2)^2}{(n+2)^3-(n+1)^3}$.

3.8. $\lim_{n\to\infty} \frac{(2n+1)^2+(1-3n)^2}{(n-2)^3-(n-1)^3}$.

3.9. $\lim_{n\to\infty} \frac{(n+2)^4-(n-2)^4}{(n+3)^2+(n-3)^2}$.

3.10. $\lim_{n\to\infty} \frac{(n+1)^4-(n-1)^4}{(n+1)^3+(n-1)^3}$.

3.11. $\lim_{n\to\infty} \frac{(2n+1)!+(2n+2)!}{(2n+3)!}$.

3.12. $\lim_{n\to\infty} \frac{n!+(n+2)!}{(n-1)!+(n+2)!}$.

3.13. $\lim\limits_{n\to\infty} \dfrac{(n+3)! - (n+1)!}{(n+2)!}$.

3.14. $\lim\limits_{n\to\infty} \dfrac{(2n-1)! + (2n+1)!}{(2n)!(n+1)}$.

3.15. $\lim\limits_{n\to\infty} \dfrac{(3n)! + (3n-2)!}{(3n-1)!(2n+1)}$.

3.16. $\lim\limits_{n\to\infty} \dfrac{(n-1)! + (n-2)!}{(n-3)!(3n^2-1)}$.

4. Să se calculeze limitele:

4.1. $\lim\limits_{n\to\infty} \sqrt{n+1}\left(\sqrt{n+3} - \sqrt{n+2}\right)$.

4.2. $\lim\limits_{n\to\infty} \left(\sqrt{(n-1)(n+4)} - n\right)$.

4.3. $\lim\limits_{n\to\infty} \left(\sqrt{n^2+3n+2} - n\right)$.

4.4. $\lim\limits_{n\to\infty} \left(n + \sqrt[3]{n^2 - n^3}\right)$.

4.5. $\lim\limits_{n\to\infty} \left(\sqrt{n^2+4n-2} - \sqrt{n^2-2}\right)$.

4.6. $\lim\limits_{n\to\infty} \sqrt{n-1}\left(\sqrt{n+1} - \sqrt{n-3}\right)$.

4.7. $\lim\limits_{n\to\infty} \left(n\sqrt{n} - \sqrt{n(n^2-1)}\right)$.

4.8. $\lim\limits_{n\to\infty} n\left(\sqrt[3]{2+8n^3} - 2n\right)$.

4.9. $\lim\limits_{n\to\infty} \sqrt[3]{n}\left(\sqrt[3]{n^2} - \sqrt[3]{n(n+1)}\right)$.

4.10. $\lim\limits_{n\to\infty} n^2\left(\sqrt[3]{n^3+7} - \sqrt[3]{n^3+1}\right)$.

4.11. $\lim\limits_{n\to\infty} \left(\dfrac{1}{n^2} + \dfrac{2}{n^2} + \cdots + \dfrac{n-1}{n^2}\right)$.

4.12. $\lim\limits_{n\to\infty} \left(\dfrac{2+4+\ldots+2n}{n+2} - n\right)$.

4.13. $\lim\limits_{n\to\infty} \left(\dfrac{n+2}{1+2+\ldots+n} - \dfrac{3}{2}\right)$.

4.14. $\lim\limits_{n\to\infty} \dfrac{1+3+5+\ldots+2n-1}{2+4+6+\ldots+2n}$.

4.15. $\lim\limits_{n\to\infty} \dfrac{5+10+\ldots+5n}{n^2+1}$.

4.16. $\lim\limits_{n\to\infty} \dfrac{1\cdot 2 + 2\cdot 3 + \ldots + n(n+1)}{n^3}$.

4.17. $\lim\limits_{n\to\infty} \dfrac{1^2+3^2+\ldots+(2n-1)^2}{n^3}$.

4.18. $\lim\limits_{n\to\infty} \left(\dfrac{1^2+2^2+\ldots+n^2}{n^2} - \dfrac{n}{3}\right)$.

4.19. $\lim\limits_{n\to\infty} \dfrac{3^n - 5^{n+1}}{3^{n+1} + 5^{n+2}}$.

4.20. $\lim\limits_{n\to\infty} \left(\dfrac{7}{10} + \dfrac{29}{100} + \cdots + \dfrac{2^n+5^n}{10^n}\right)$.

4.21. $\lim\limits_{n\to\infty} \dfrac{4^n + 7^n}{4^n - 7^{n-1}}$.

4.22. $\lim\limits_{n\to\infty} \left(\dfrac{3}{4} + \dfrac{5}{16} + \cdots + \dfrac{1+2^n}{4^n}\right)$.

4.23. $\lim\limits_{n\to\infty} \dfrac{3^n + 5^{-n}}{3^{-n} + 5^n}$.

4.24. $\lim\limits_{n\to\infty} \dfrac{1 + \frac{1}{5} + \cdots + \frac{1}{5^n}}{1 + \frac{1}{7} + \cdots + \frac{1}{7^n}}$.

5. Să se calculeze limitele:

5.1. $\lim\limits_{n\to\infty} \left(\dfrac{2n+3}{2n-1}\right)^n$.

5.2. $\lim\limits_{n\to\infty} \left(\dfrac{n+2}{n+1}\right)^{1-n}$.

5.3. $\lim\limits_{n\to\infty}\left(\dfrac{3n-1}{3n+2}\right)^{2n+1}$.

5.4. $\lim\limits_{n\to\infty}\left(\dfrac{2n+1}{2n-5}\right)^{\frac{n}{6}}$.

5.5. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-1}{n^2+1}\right)^{2n-1}$.

5.6. $\lim\limits_{n\to\infty}\left(\dfrac{3n^2+2}{3n^2-1}\right)^{\frac{n}{3}}$.

5.7. $\lim\limits_{n\to\infty}\left(\dfrac{3n+1}{3n}\right)^{1-n^2}$.

5.8. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+1}{2n^2-3}\right)^{1-n^3}$.

5.9. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+2}{2n^2+1}\right)^{n^2}$.

5.10. $\lim\limits_{n\to\infty}\left(\dfrac{n^2+n+1}{n^2+n-1}\right)^{n^2-1}$.

5.11. $\lim\limits_{n\to\infty}\left(\dfrac{3n^2+2}{3n^2-1}\right)^{n^2+1}$.

5.12. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+n+2}{2n^2-2n+3}\right)^{2n}$.

5.13. $\lim\limits_{n\to\infty}\left(\dfrac{3n^2+6n+7}{3n^2+6n+4}\right)^{6n^2-5n+4}$.

5.14. $\lim\limits_{n\to\infty}\left(\dfrac{n^2+1}{n^2-1}\right)^{2n^2}$.

5.15. $\lim\limits_{n\to\infty}\left(\dfrac{n^2+2n+3}{n^2+3n+4}\right)^{2n-1}$.

5.16. $\lim\limits_{n\to\infty}\left(\dfrac{4n^2+2}{4n^2-2}\right)^{n^2}$.

5.17. $\lim\limits_{n\to\infty}\left(\dfrac{n^2+n-1}{3n^2-n+1}\right)^{\frac{1}{n}}$.

5.18. $\lim\limits_{n\to\infty}\left(\dfrac{1+2n}{3n-1}\right)^{n}$.

5.19. $\lim\limits_{n\to\infty}\left(\dfrac{n+1}{n-1}\right)^{\frac{2n^2+1}{2n^2-1}}$.

5.20. $\lim\limits_{n\to\infty}\left(\dfrac{n^2-1}{2n^2+1}\right)^{\frac{2n}{n+1}}$.

Capitolul 2. LIMITE DE FUNCŢII

1. Să se calculeze următoarele limite:

1.1. $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2+x-6}$.

1.2. $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2+6x-7}$.

1.3. $\lim\limits_{x\to 3} \dfrac{x^2+2x-15}{x^2-x-6}$.

1.4. $\lim\limits_{x\to 4} \dfrac{x^2-7x+12}{x^2-6x+8}$.

1.5. $\lim\limits_{x\to 0} \dfrac{x^3-6x^2+7x}{x^2+x}$.

1.6. $\lim\limits_{x\to -2} \dfrac{x^2+x-2}{x^2-x-6}$.

1.7. $\lim\limits_{x\to 1} \dfrac{x^3-1}{x^2-1}$.

1.8. $\lim\limits_{x\to 1} \dfrac{x^m-1}{x^n-1},\ m,n\in\mathbb{N}$.

1.9. $\lim\limits_{x\to 0} \dfrac{(x+2)(1-x)(2x+1)-2}{x^2+x}$.

1.10. $\lim\limits_{x\to 2} \dfrac{x^4-5x^2+4}{x^4-3x^2-4}$.

1.11. $\lim\limits_{x\to -2} \dfrac{x^3+2x^2-x-2}{x^3-7x-6}$.

1.12. $\lim\limits_{x\to -1} \dfrac{x^4+x^2-2}{x^4-1}$.

1.13. $\lim\limits_{x\to -1} \dfrac{x^3+2x+3}{x^3+1}$.

1.14. $\lim\limits_{x\to 2} \dfrac{x^4-2x^3-3x^2+4x+4}{x^4-6x^3+13x^2-12x+4}$.

1.15. $\lim\limits_{x\to 1} \dfrac{2x^4-x^2-1}{x^4-1}$.

1.16. $\lim\limits_{x\to 0} \dfrac{(x+1)^3-(3x+1)}{2x^4+x^2}$.

1.17. $\lim\limits_{x\to -1} \dfrac{x^3+3x^2+7x+5}{x^3-x^2-x+1}$.

1.18. $\lim\limits_{x\to 2} \dfrac{x^3-3x-2}{x^2-4}$.

1.19. $\lim\limits_{x\to -1} \dfrac{(x^3-2x-1)^2}{x^4-2x^2+1}$.

1.20. $\lim\limits_{x\to 0} \dfrac{(x+2)^3-8}{(x+1)^4-(1+2x)}$.

2. Să se calculeze următoarele limite:

2.1. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x^2+x-2}$.

2.2. $\lim\limits_{x\to 2} \dfrac{x^2-5x+6}{\sqrt{x+2}-2}$.

2.3. $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+x+4}-2}{\sqrt{1-x+x^2}-1}$.

2.4. $\lim\limits_{x\to 4} \dfrac{\sqrt{x}-2}{\sqrt{4+3x}-4}$.

2.5. $\lim\limits_{x\to 0} \dfrac{\sqrt{4-x+x^2}-(2+x)}{x^2+x}$.

2.6. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{2+x}-\sqrt{2-x}}$.

2.7. $\lim\limits_{x\to 5} \dfrac{\sqrt{x+4}-\sqrt{2x-1}}{x^2-25}$.

2.8. $\lim\limits_{x\to 0} \dfrac{3\sqrt{x^2+x+1}-(3+x)}{x^2+3x}$.

2.9. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1}$.

2.10. $\lim\limits_{x\to 4} \dfrac{\sqrt[3]{16x}-4}{\sqrt{x+4}-\sqrt{2x}}$.

2.11. $\lim\limits_{x\to 2} \dfrac{\sqrt[3]{x-1}-1}{x^3-8}$.

2.12. $\lim\limits_{x\to -2} \dfrac{x^3+8}{\sqrt[3]{x-6}+2}$.

2.13. $\lim\limits_{x\to 1} \dfrac{\sqrt{x+2}-\sqrt{3x}}{\sqrt[3]{x}-1}$.

2.14. $\lim\limits_{x\to 8} \dfrac{\sqrt[3]{x}-2}{\sqrt{x+1}-3}$.

2.15. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{2+x}-\sqrt[3]{2-x}}{\sqrt{2+x}-\sqrt{2-x}}$.

2.16. $\lim\limits_{x\to 1} \dfrac{\sqrt{x}+\sqrt{x-1}-1}{\sqrt{x^2-1}}$.

2.17. $\lim\limits_{x\to 8} \dfrac{\sqrt[3]{x}-2}{x-8}$.

2.18. $\lim\limits_{x\to -8} \dfrac{\sqrt[3]{15+2x}+1}{\sqrt[3]{9+x}+x+7}$.

2.19. $\lim\limits_{x\to 7} \dfrac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}$.

2.20. $\lim\limits_{x\to 0} \dfrac{\sqrt[5]{2x^2+10x+1}-\sqrt[7]{x^2+10x+1}}{x}$.

3. Să se calculeze următoarele limite:

3.1. $\lim\limits_{x\to\infty} \left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)$.

3.2. $\lim\limits_{x\to\infty} \left(\sqrt{9x^4+3x^2-7}-3x^2\right)$.

.

3.3. $\lim\limits_{x\to\infty} \left(\sqrt{x^2+2x-1}-\sqrt{x^2-2x-1}\right)$.

3.4. $\lim\limits_{x\to\infty} \left(\sqrt{x^4+x^2}-\sqrt{x^4+8x^2+3}\right)$.

3.5. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-3x+2}-x\right)$.

3.6. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x}-\sqrt{x^2+2x+3}\right)$.

3.7. $\lim\limits_{x\to\infty}\left(x\sqrt[3]{8x^3+5}-2x\right)$.

3.8. $\lim\limits_{x\to\infty}\sqrt{x^3+8}\left(\sqrt{x^3+2}-\sqrt[3]{x^3-1}\right)$.

3.9. $\lim\limits_{x\to\infty}x\sqrt{x}\left(x-\sqrt[3]{x^3-5}\right)$.

3.10. $\lim\limits_{x\to\infty}x\sqrt{x}\left(\sqrt{x^4+3}-\sqrt{x^4+2}\right)$.

3.11. $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+2}-\sqrt{x+3}\right)$.

3.12. $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2-x}\right)$.

.

3.13. $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{2x^2-1}-\dfrac{x^2}{2x+1}\right)$.

3.14. $\lim\limits_{x\to1}\left(\dfrac{1}{1-x}-\dfrac{2}{1-x^2}\right)$.

3.15. $\lim\limits_{x\to1}\left(\dfrac{1}{x-1}-\dfrac{3}{x^3-1}\right)$.

3.16. $\lim\limits_{x\to2}\left(\dfrac{1}{(x-2)(x-1)}-\dfrac{2}{x^2-2x}\right)$.

3.17. $\lim\limits_{x\to-1}\left(\dfrac{2}{x+1}-\dfrac{x-3}{x^2-1}\right)$.

3.18. $\lim\limits_{x\to2}\left(\dfrac{1}{x(x-2)^2}-\dfrac{1}{x^2-3x+2}\right)$.

4. Să se calculeze următoarele limite:

4.1. $\lim\limits_{x\to0}\dfrac{\sin5x}{x}$.

4.2. $\lim\limits_{x\to0}\dfrac{\sin8x+\sin6x}{2x}$.

4.3. $\lim\limits_{x\to0}\dfrac{\sin2x}{\sin5x}$.

4.4. $\lim\limits_{x\to0}\dfrac{\cos5x-\cos3x}{4x^2}$.

4.5. $\lim\limits_{x\to0}\dfrac{\sin^2 2x}{\sin^2 3x}$.

4.6. $\lim\limits_{x\to0}\dfrac{1-\cos4x}{1-\cos8x}$.

4.7. $\lim\limits_{x\to0}\dfrac{1-\cos2x}{x^2}$.

4.8. $\lim\limits_{x\to0}\dfrac{1-\cos3x}{2x\sin x}$.

4.9. $\lim\limits_{x\to\pi}\dfrac{\sin2x}{\sin3x}$.

4.10. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{1-2\sin x}{\pi-6x}$.

4.11. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\operatorname{tg}5x}{\operatorname{tg}3x}$.

4.12. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\operatorname{tg}^2 x}{\sqrt{2}\cos x-1}$.

4.13. $\lim\limits_{x\to\pi}\dfrac{\sin2x}{\operatorname{tg}3x}$.

4.14. $\lim\limits_{x\to-\frac{\pi}{4}}\dfrac{1+\sin2x}{1+\cos4x}$.

4.15. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}-2\cos x}{\pi-4x}$.

4.16. $\lim\limits_{x\to0}\left(\dfrac{1}{\sin x}-\operatorname{ctg}x\right)$.

4.17. $\lim\limits_{x\to0}\dfrac{\operatorname{tg}x-\sin x}{2x^3}$.

4.18. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{6\sin^2 x-5\sin x+1}{4\sin^2 x-1}$.

4.19. $\lim\limits_{x\to0}\dfrac{\sqrt{1+x\sin x}-1}{x^2}$.

4.20. $\lim\limits_{x\to\pi}\dfrac{\sqrt{1-\operatorname{tg}x}-\sqrt{1+\operatorname{tg}x}}{\sin2x}$.

4.21. $\lim\limits_{x\to0}\dfrac{\operatorname{tg}(\sin x)-\sin(\operatorname{tg}x)}{x^3}$.

4.22. $\lim\limits_{x\to0}\dfrac{\operatorname{tg}(\operatorname{tg}x)-\sin(\sin x)}{\operatorname{tg}x-\sin x}$.

5. Să se calculeze următoarele limite:

5.1. $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x-3}\right)^{2x-1}$.

5.2. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+4}{x^2-4}\right)^{x^2}$.

5.3. $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{2x+3}\right)^{\frac{x}{2}}$.

5.4. $\lim\limits_{x\to\infty}\left(\dfrac{\sqrt{x}+3}{\sqrt{x}+2}\right)^{\frac{1-x}{1-\sqrt{x}}}$.

5.5. $\lim\limits_{x\to 0}(1+5x)^{\frac{1}{x}}$.

5.6. $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{\sin 2x}}$.

5.7. $\lim\limits_{x\to 0}(1+2\operatorname{tg}^2 x)^{\operatorname{ctg}^2 x}$.

5.8. $\lim\limits_{x\to 0}(\cos 2x)^{\frac{1}{x^2}}$.

5.9. $\lim\limits_{x\to 0}(\cos x+\sin x)^{\frac{1}{x}}$.

5.10. $\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$.

5.11. $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{\operatorname{tg}^2 x}$.

5.12. $\lim\limits_{x\to\frac{\pi}{2}}(1+\operatorname{ctg} x)^{\operatorname{tg} x}$.

5.13. $\lim\limits_{x\to\frac{\pi}{2}}\left(\operatorname{ctg}\dfrac{x}{2}\right)^{\frac{1}{\cos x}}$.

5.14. $\lim\limits_{x\to 1}(2-x)^{\operatorname{tg}\frac{\pi x}{2}}$.

5.15. $\lim\limits_{x\to 0}\left(4-\dfrac{3}{\cos x}\right)^{\operatorname{tg}^2 x}$.

5.16. $\lim\limits_{x\to 0}\left[\operatorname{tg}\left(\dfrac{\pi}{4}-x\right)\right]^{\operatorname{ctg} x}$.

6. Să se calculeze limitele:

6.1. $\lim\limits_{x\to 0}\dfrac{\ln(1+2x^2)}{\sqrt{1+x^2}-1}$.

6.2. $\lim\limits_{x\to 0}\dfrac{\ln(1+\sin 2x)}{\sin 4x-\sin 2x}$.

6.3. $\lim\limits_{x\to 0}\dfrac{3^x-1}{\ln(1+2x)}$.

6.4. $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{\operatorname{arctg} 4x}$.

6.5. $\lim\limits_{x\to 0}\dfrac{\ln(1+2x)}{\operatorname{arctg} 3x}$.

6.6. $\lim\limits_{x\to 0}\dfrac{3^x-2^x}{2x-\operatorname{arctg} x}$.

6.7. $\lim\limits_{x\to 0}\dfrac{2^{3x}-3^{2x}}{2\arcsin x-\sin x}$.

6.8. $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{2x}}{x+\sin x^2}$.

6.9. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x\sin x}-1}{e^{x^2}-1}$.

6.10. $\lim\limits_{x\to 2}\dfrac{x^2-4}{\ln(x-1)}$.

6.11. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$.

6.12. $\lim\limits_{x\to -1}\dfrac{3-\sqrt{10+x}}{\sin 3\pi x}$.

6.13. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{2^{\cos^2 x}-1}{\ln\sin x}$.

6.14. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\ln\sin 3x}{(6x-\pi)^2}$.

6.15. $\lim\limits_{x\to 0}\dfrac{\operatorname{tg} 2x-3\arcsin x}{\sin 6x-6\operatorname{arctg} 2x}$.

6.16. $\lim\limits_{x\to\infty}x\left(2^{\frac{1}{x}}-1\right)$.

6.17. $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\ln\sin 5x}{\ln\sin 9x}$.

6.18. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{1+x}-1-\sin x}{\ln(1+x)}$.

6.19. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{\cos x}-\sqrt[4]{\cos 2x}}{1-\cos 12x}$.

6.20. $\lim\limits_{x\to\frac{1}{4}} \dfrac{1-\operatorname{ctg}\pi x}{\ln\operatorname{tg}\pi x}$.

6.21. $\lim\limits_{x\to 0} (\cos 2x)^{-\frac{1}{x^2}}$.

6.22. $\lim\limits_{x\to 0} \dfrac{e^{x^2}-\cos x}{\sin^2 x}$.

Capitolul 3. DERIVABILITATE

1. Să se calculeze derivata funcţiei:

1.1. $f(x) = x^3 + x^2 - x + 1.$

1.2. $f(x) = \frac{1}{4}\, x^4 - \frac{1}{3}\, x^3 + 2x^2 - 1.$

1.3. $f(x) = 2x^5 - x^{-2} + 3x.$

1.4. $f(x) = \frac{1}{x} - \frac{4}{x^2} - \frac{1}{x^3} + x.$

1.5. $f(x) = x^{\frac{1}{2}} + x^{\frac{2}{3}} - x^{-\frac{1}{3}}.$

1.6. $f(x) = \sqrt[3]{x} - \frac{1}{\sqrt[3]{x}} + 1.$

1.7. $f(x) = e^x \sin x.$

1.8. $f(x) = \operatorname{tg} x \ln x.$

1.9. $f(x) = 2^x \operatorname{ctg} x.$

1.10. $f(x) = x \arcsin x.$

1.11. $f(x) = (x^2 + 1) \operatorname{arctg} x.$

1.12. $f(x) = x^2 \ln x.$

1.13. $f(x) = \cos x \ln x.$

1.14. $f(x) = x \operatorname{arcctg} x.$

1.15. $f(x) = \frac{x}{x^2 - 1}.$

1.16. $f(x) = \frac{x^2 - 1}{x^2 + 1}.$

1.17. $f(x) = \frac{\sin x}{\ln x}.$

1.18. $f(x) = \frac{\operatorname{arctg} x}{e^x}.$

1.19. $f(x) = \frac{\sin x - \cos x}{\sin x + \cos x}.$

1.20. $f(x) = \frac{1 - \sin x}{1 + \sin x}.$

1.21. $f(x) = \ln 3 - \cos 2.$

1.22. $f(x) = \arcsin x + \arccos x.$

2. Să se calculeze derivata funcţiei:

2.1. $f(x) = (x^2 + 1)^{10}.$

2.2. $f(x) = \frac{1}{(x^2 + 2x + 3)^3}.$

2.3. $f(x) = \sqrt{x^2 - x + 7}.$

2.4. $f(x) = \frac{1}{\sqrt[3]{x^3 + x^2 + 1}}.$

2.5. $f(x) = \sin^2 x$.

2.6. $f(x) = \ln^2 x$.

2.7. $f(x) = \sin 3x$.

2.8. $f(x) = \sin(\ln x)$.

2.9. $f(x) = \cos 2x$.

2.10. $f(x) = \cos(e^x)$.

2.11. $f(x) = \operatorname{tg} 3x$.

2.12. $f(x) = \operatorname{tg} 2^x$.

2.13. $f(x) = \operatorname{ctg} \frac{x}{2}$.

2.14. $f(x) = \operatorname{ctg}(x^2 + x + 1)$.

2.15. $f(x) = e^{\sin x}$.

2.16. $f(x) = e^{-x}$.

2.17. $f(x) = 2^{\operatorname{tg} x}$.

2.18. $f(x) = 3^{\sqrt{x}}$.

2.19. $f(x) = \ln(\sin x)$.

2.20. $f(x) = \ln(\operatorname{arctg} x)$.

2.21. $f(x) = \operatorname{arctg} \sqrt{x}$.

2.22. $f(x) = \operatorname{arctg}\ e^x$.

2.23. $f(x) = \arcsin \sqrt{x}$.

2.24. $f(x) = \arcsin e^{-x}$.

3. Să se calculeze derivata funcţiei:

3.1. $f(x) = \ln \operatorname{tg} \dfrac{x}{2}$.

3.2. $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$.

3.3. $f(x) = \ln \sqrt[4]{\dfrac{1 - \sin x}{1 + \sin x}}$.

3.4. $f(x) = \ln \dfrac{x^2 - 1}{x^2 + 1}$.

3.5. $f(x) = \ln \sin \dfrac{2x + 4}{x + 1}$.

3.6. $f(x) = \ln \operatorname{tg}\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right)$.

3.7. $f(x) = \operatorname{arctg} \sqrt{4x - 1}$.

3.8. $f(x) = \sqrt{x} - \operatorname{arctg} \sqrt{x}$.

3.9. $f(x) = \operatorname{arctg} \dfrac{1+x}{1-x}$.

3.10. $f(x) = \operatorname{arctg} \dfrac{x}{1+\sqrt{1+x^2}}$.

3.11. $f(x) = \arcsin \dfrac{1-x}{\sqrt{2}}$.

3.12. $f(x) = \arcsin \sqrt{1-x^2}$.

3.13. $f(x) = \arccos \dfrac{1-x^2}{1+x^2}$.

3.14. $f(x) = \cos(2 \arccos x)$.

3.15. $f(x) = e^{\sqrt{\frac{1+x}{1-x}}}$.

3.16. $f(x) = e^{\operatorname{tg} \frac{1}{x}}$.

3.17. $f(x) = \operatorname{tg}^2 x + \ln \cos^2 x$.

3.18. $f(x) = \operatorname{arcctg}\left(\operatorname{ctg}^2 x\right)$.

3.19. $f(x) = \sqrt{2x^2 + \sqrt{x^2+1}}$.

3.20. $f(x) = \sqrt{2+x^2}\sqrt[3]{3+x^3}$.

4. Să se calculeze derivata funcţiei:

4.1. $f(x) = \ln\left(2x - 3 + \sqrt{4x^2 - 12x + 10}\right) - \operatorname{arctg}(2x-3)\sqrt{4x^2 - 12x + 10}$.

4.2. $f(x) = x^2\sqrt{x^4+1} + \ln\left(x^2 + \sqrt{x^4+1}\right)$.

4.3. $f(x) = x + e^{-x} \operatorname{arctg} e^x - \ln\sqrt{1+e^{2x}}$.

4.4. $f(x) = \sqrt{49x^2+1} \operatorname{arctg} 7x - \ln\left(7x + \sqrt{49x^2+1}\right)$.

4.5. $f(x) = \arcsin e^{-2x} + \ln\left(e^{2x} + \sqrt{e^{4x}-1}\right)$.

4.6. $f(x) = \dfrac{3-\sin x}{2}\sqrt{\cos^2 x - 2\sin x} + 2\arcsin\dfrac{1+\sin x}{\sqrt{2}}$.

4.7. $f(x) = \operatorname{arctg}\sqrt{e^x} + e^x \arcsin\sqrt{\dfrac{e^x}{e^x+1}} - \sqrt{e^x}$.

4.8. $f(x) = 2\sqrt{3} \operatorname{arctg}\dfrac{\sqrt{3}}{1-2x^2} + \ln\dfrac{x^4 - x^2 + 1}{x^4 + 2x^2 + 1}$.

4.9. $f(x) = \ln\dfrac{2(x^2+2x+2)}{2x^2+2x+1} + 4\operatorname{arctg}(x+1) - \operatorname{arctg}(2x+1)$.

4.10. $f(x) = \dfrac{5x+2}{x^2+x+1} + \ln\sqrt[3]{\dfrac{(x-1)^2}{x^2+x+1}} + \dfrac{8}{\sqrt{3}} \operatorname{arctg}\dfrac{2x+1}{\sqrt{3}}$.

4.11. $f(x) = x \ln\left(\sqrt{1-x} + \sqrt{1+x}\right) + \dfrac{1}{2}(\arcsin x - x).$

4.12. $f(x) = (3x-2)^4 \arcsin \dfrac{1}{3x-2} + \left(3x^2 - 4x + 2\right)\sqrt{9x^2 - 12x + 3}.$

4.13. $f(x) = e^{2\arcsin x}\left[\cos(2\arcsin x) + \sin(2\arcsin x)\right].$

4.14. $f(x) = \sqrt{1 + \sqrt[3]{1 + \sqrt[4]{1+x^4}}}.$

4.15. $f(x) = \dfrac{2}{3x-2}\sqrt{12x - 9x^2 - 3} + \ln \dfrac{1 + \sqrt{12x - 9x^2 - 3}}{3x-2}.$

4.16. $f(x) = x\left(2x^2+5\right)\sqrt{x^2+1} + 3\ln\left(x + \sqrt{x^2+1}\right).$

4.17. $f(x) = \sqrt{x^2 + 5x + 4} + 3\ln\left(\sqrt{x+4} + \sqrt{x+1}\right).$

4.18. $f(x) = \dfrac{x \arcsin 2x}{\sqrt{1-4x^2}} + \ln\sqrt{1-4x^2}.$

4.19. $f(x) = \dfrac{1}{4\sqrt{3}} \ln \dfrac{\sqrt{x^2+2} - x\sqrt{3}}{\sqrt{x^2+2} + x\sqrt{3}} + \dfrac{1}{2}\operatorname{arctg}\dfrac{\sqrt{x^2+2}}{x}.$

4.20. $f(x) = \dfrac{\cos x}{3(2+\sin x)} + \dfrac{4}{3\sqrt{3}}\operatorname{arctg}\dfrac{2\operatorname{tg}\frac{x}{2}+1}{\sqrt{3}}.$

4.21. $f(x) = \dfrac{1}{\cos x} + \dfrac{1}{3\cos^3 x} - \dfrac{1}{2}\ln\dfrac{1+\cos x}{1-\cos x}.$

4.22. $f(x) = 2\sqrt{1-x^2}\arcsin x - 2x + x(\arcsin x)^2.$

4.23. $f(x) = \dfrac{\ln(1+\sin x)}{\operatorname{tg} x} + x - \ln\operatorname{tg}\dfrac{x}{2}.$

4.24. $f(x) = \log_{\frac{1}{2}}\left(x + \dfrac{1}{2}\right)^2 + \log_2\sqrt{4x^2+4x+1}.$

4.25. $f(x) = x^x.$

4.26. $f(x) = \sin x^{\cos x}.$

4.27. $f(x) = x + x^x + x^{x^x}.$

4.28. $f(x) = x^{e^x}.$

4.29. $f(x) = x^{e^{\sin x}}.$

4.30. $f(x) = x^{3^x}2^x.$

5. Să se studieze derivabilitatea următoarelor funcţii:

5.1. $f : \mathbb{R} \longrightarrow \mathbb{R}, \qquad f(x) = \left|x^3 - 4x\right|.$

5.2. $f : \left(-\frac{1}{3}, \infty\right) \longrightarrow \mathbb{R}, \quad f(x) = \begin{cases} \ln(1+3x), & \text{dacă} \quad -\dfrac{1}{3} < x \leq 0 \\ 3x, & \text{dacă} \quad x > 0. \end{cases}$

5.3. $f : \mathbb{R} \longrightarrow \mathbb{R}, \qquad f(x) = \begin{cases} \sin^3 x \operatorname{sgn} x, & \text{dacă} \quad |x| \leq \frac{\pi}{4} \\ \dfrac{3\sqrt{2}}{4} x \operatorname{sgn} x - \dfrac{\sqrt{2}(3\pi - 4)}{4}, & \text{dacă} \quad |x| > \frac{\pi}{4}. \end{cases}$

5.4. $f : \mathbb{R} \longrightarrow \mathbb{R}, \qquad f(x) = \begin{cases} \operatorname{tg}\left(x^3 + x^2 \sin \dfrac{2}{x}\right), & x \neq 0 \\ 0, & x = 0. \end{cases}$

5.5. $f : \mathbb{R} \longrightarrow \mathbb{R}, \qquad f(x) = \begin{cases} \sqrt[3]{1 - 2x^3 \sin \dfrac{5}{x}} - 1 + x, & x \neq 0 \\ 0, & x = 0. \end{cases}$

5.6. $f : \mathbb{R} \longrightarrow \mathbb{R}, \qquad f(x) = \dfrac{|x+1| - |4-x|}{|x| + |x-5|}.$

5.7. $f : \mathbb{R} \longrightarrow \mathbb{R}, \qquad f(x) = |\cos x|.$

5.8. $f : \mathbb{R} \longrightarrow \mathbb{R}, \qquad f(x) = \begin{cases} x, & \text{dacă} \quad x \in \mathbb{Q} \\ 0, & \text{dacă} \quad x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$

5.9. $f : \mathbb{R} \longrightarrow \mathbb{R}, \qquad f(x) = \begin{cases} \operatorname{arctg} ax, & \text{dacă} \quad |x| \leq 1, a \in \mathbb{R} \\ b \operatorname{sgn} x + \dfrac{x-1}{2}, & \text{dacă} \quad |x| > 1, b \in \mathbb{R}. \end{cases}$

5.10. $f:\mathbb{R}\longrightarrow\mathbb{R}$, $f(x)=\begin{cases} 2^{\frac{1}{x-1}}, & \text{dacă } x<1 \\ 0, & \text{dacă } x=1 \\ \ln(x^2-2x+2), & \text{dacă } x>1. \end{cases}$

6. Să se calculeze derivatele de ordinul n ($n\in\mathbb{Z}$, $n\geq 1$) ale funcţiilor următoare:

6.1. $f(x)=xe^{2x}$.

6.2. $f(x)=\dfrac{12x-1}{6x-1}$.

6.3. $f(x)=\sin 3x+\cos(x+2)$.

6.4. $f(x)=\ln(x+3)$.

6.5. $f(x)=(x-1)^n(x-2)^n$.

6.6. $f(x)=x^ne^{-x}$.

6.7. $f(x)=\sin x$.

6.8. $f(x)=\cos x$.

6.9. $f(x)=\sin^2 x$.

6.10. $f(x)=\sin^4 x+\cos^4 x$.

6.11. $f(x)=\dfrac{x}{x^2-4x-12}$.

6.12. $f(x)=\dfrac{3}{x^2-x-2}$.

6.13. $f(x)=x\sin x$.

6.14. $f(x)=\operatorname{arctg} x$.

6.15. $f(x)=\dfrac{1}{\sqrt{x-1}}$.

6.16. $f(x)=e^x\sin x$.

6.17. $f(x)=e^x\cos 2x$.

6.18. $f(x)=\dfrac{\ln x}{x}$.

6.19. $f(x)=\dfrac{2x+1}{3x+2}$.

6.20. $f(x)=\sqrt[3]{e^{2x-1}}$.

7. Utilizând diferenţiale, să se calculeze cu aproximaţie:

7.1. $f(x)=x^5$, $x=3,01$.

7.2. $f(x)=x^6$, $x=1,997$.

7.3. $f(x)=\sqrt[3]{x^2}$, $x=1,029$.

7.4. $f(x)=\sqrt{3+x+\cos x}$, $x=0,01$.

7.5. $f(x)=\sqrt{\dfrac{3-x}{1+x}}$, $x=-0,85$.

7.6. $f(x)=\dfrac{1}{\sqrt{3x+2}}$, $x=0,668$.

7.7. $f(x)=\arcsin x$, $x=0,08$.

7.8. $f(x)=\operatorname{arctg} x$, $x=1,03$.

7.9. $f(x) = \sin x\ ,\ x = 31^\circ$.

7.10. $f(x) = \ln \operatorname{tg} x\ ,\ x = 48^\circ$.

7.11. $f(x) = \dfrac{x + \sqrt{10 - x^2}}{2}\ ,\ x = 0,99$.

7.12. $f(x) = \sqrt{x^2 + 12}\ ,\ x = 1,98$.

8. Să se calculeze derivata y'_x:

8.1. $\begin{cases} x = \sin^2 t, \\ y = \cos^2 t. \end{cases}$

8.2. $\begin{cases} x = e^{-t}, \\ y = t^2. \end{cases}$

8.3. $\begin{cases} x = \sqrt{t}, \\ y = \sqrt[3]{t}. \end{cases}$

8.4. $\begin{cases} x = e^t, \\ y = \arcsin t. \end{cases}$

8.5. $\begin{cases} x = \dfrac{3at}{1+t^3}, \\ y = \dfrac{3at^2}{1+t^3}. \end{cases}$

8.6. $\begin{cases} x = \dfrac{1}{t+1}, \\ y = \dfrac{t}{t+1}. \end{cases}$

8.7. $\begin{cases} x = \operatorname{arctg} e^t, \\ y = \sqrt{e^{2t} + 1}. \end{cases}$

8.8. $\begin{cases} x = \operatorname{arctg} t, \\ y = \ln \dfrac{1+t^2}{t+1}. \end{cases}$

8.9. $\begin{cases} x = \dfrac{t}{1-t^2} \arcsin t + \ln \sqrt{1-t^2}, \\ y = \dfrac{t}{\sqrt{1-t^2}}. \end{cases}$

8.10. $\begin{cases} x = \ln \operatorname{tg} t, \\ y = \operatorname{cosec}^2 t. \end{cases}$

8.11. $\begin{cases} x = \dfrac{5t^2+2}{5t^3}, \\ y = \sin\left(\dfrac{1}{3}t^3 + t\right). \end{cases}$

8.12. $\begin{cases} x = \sqrt{4-t^2}, \\ y = \operatorname{tg} \sqrt{2+t}. \end{cases}$

8.13. $\begin{cases} x = e^t \cos t, \\ y = e^t \sin t. \end{cases}$

8.14. $\begin{cases} x = a(\sin t - t\cos t), \\ y = a(\cos t + t \sin t). \end{cases}$

8.15. $xy + \ln y = 1$.

8.16. $\sqrt{x} + \sqrt{y} = 1$.

8.17. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$.

8.18. $e^y + xy = 2e$.

8.19. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

8.20. $y^5 + y^3 + y - x = 0$.

8.21. $\operatorname{arctg} \dfrac{y}{x} = \ln \sqrt{x^2 + y^2}$.

8.22. $y^2 = 2px$.

8.23. $x^2 + y^2 - 6x + 10y - 2 = 0$.

8.24. $x^2 y + \operatorname{arctg} \dfrac{y}{x} = 0$.

9. Să se scrie ecuaţiile tangentelor la graficele funcţiilor în punctele specificate:

9.1. $f(x) = x^2 - x - 12\ ,\ x = 3$.

9.2. $f(x) = \dfrac{1}{3}(3x - x^3)\ ,\ x = 2$.

9.3. $f(x) = \dfrac{x^3 + 1}{x^3 - 1}\ ,\ x = 0$.

9.4. $f(x) = \dfrac{x}{x^2 + 1}\ ,\ x = -1$.

9.5. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 3x}\ ,\ x = 2$.

9.6. $f(x) = \ln \dfrac{x^2 - 2x + 1}{x^2 + x + e}\ ,\ x = 0$.

9.7. $f(x) = \cos 2x - 2 \sin x\ ,\ x = \dfrac{\pi}{2}$.

9.8. $f(x) = \operatorname{arctg} \dfrac{1}{x}\ ,\ x = 1$.

9.9. $f(x) = \dfrac{x}{\sqrt[3]{x+1}}\ ,\ x = -2$.

9.10. $f(x) = 4 \operatorname{tg} x - \dfrac{\sin x}{\cos^2 x}\ ,\ x = \dfrac{\pi}{4}$.

10. Să se determine în ce puncte şi sub ce unghi se intersectează graficele funcţiilor:

10.1. $f_1(x) = \sin x, \quad f_2(x) = \sqrt{3} \cos x$.

10.2. $f_1(x) = x^2, \quad f_2(x) = x$.

10.3. $f_1(x) = x^3, \quad f_2(x) = x^2$.

10.4. $f_1(x) = (x-2)^2, \quad f_2(x) = 4 - x^2$.

10.5. $f_1(x) = \dfrac{1}{\sqrt[3]{x}}, \quad f_2(x) = x$.

10.6. $f_1(x) = \dfrac{1}{x^3}, \quad f_2(x) = x^2$.

10.7. $f_1(x) = 4x^2 + 2x - 8$,
$f_2(x) = x^3 - x + 10$.

10.8. $f_1(x) = \ln x, \quad f_2(x) = 2 - \dfrac{x}{e}$.

10.9. $f_1(x) = 3x - x^2, \quad f_2(x) = x^2 - x$.

10.10. $f_1(x) = \sin x, \quad f_2(x) = \cos x$.

11. Să se studieze monotonia şi să se determine punctele de extrem pentru fiecare din funcţiile f pe domeniul lor maxim de definiţie:

11.1. $f(x) = x^2 - x - 12$.

11.2. $f(x) = 6x - x^2$.

11.3. $f(x) = 3x^3 - 4x^2 + 1$.

11.4. $f(x) = x^3 - 6x^2 + 2$.

11.5. $f(x) = (x+1)^2 (x-4)^3$.

11.6. $f(x) = x^2 - 8\ln x$.

11.7. $f(x) = \sqrt[3]{(2-x)(1-x)^2}$.

11.8. $f(x) = (x-1)\sqrt{x^2-1}$.

11.9. $f(x) = \ln(1+x) - x + \frac{x^2}{2}$.

11.10. $f(x) = \frac{x^2}{x-1}$.

11.11. $f(x) = \ln\sqrt{1+x^2} + \operatorname{arctg} x$.

11.12. $f(x) = x^2 e^{\frac{1}{x}}$.

11.13. $f(x) = \ln(4x^2+1) - 8\operatorname{arctg} 2x$.

11.14. $f(x) = \frac{x^3}{3} e^{-x}$.

11.15. $f(x) = \ln x + \operatorname{arctg} x$.

10.16 $f(x) = x^2 \ln x$.

11.17. $f(x) = x - 2\operatorname{arctg}(x-1) - 1$.

11.18. $f(x) = \sin^3 x + \cos^3 x$.

11.19. $f(x) = \cos x + \frac{1}{2}\sin 2x$.

11.20. $f(x) = \frac{1}{x-1} + \frac{2(x-1)}{x^2-2x}$.

12. Să se determine intervalele de concavitate, convexitate şi eventualele puncte de inflexiune pentru funcţiile următoare:

12.1. $f(x) = 2x^4 - 3x^2 + 3x - 2$.

12.2. $f(x) = x^4 + 4x^3$.

12.3. $f(x) = 3x^2 - x^3 + 1$.

12.4. $f(x) = x + \cos x$.

12.5. $f(x) = e^{-x^2} + 2x$.

12.6. $f(x) = \ln(1+x^2)$.

12.7. $f(x) = \frac{(x+1)^2}{x^3}$.

12.8. $f(x) = \frac{\ln(x+1)}{\sqrt{x+1}}$.

12.9. $f(x) = \left(\frac{x}{2-x}\right)^4$.

12.10. $f(x) = \sin x + \frac{1}{3}\sin 3x$.

12.11. $f(x) = \sqrt[3]{x-1} - \sqrt[3]{x}$.

12.12. $f(x) = \sin x - \sin^3 x$.

12.13. $f(x) = \sin^4 x - \cos^4 x$.

12.14. $f(x) = x^5 - 10x^2 + 7x$.

12.15. $f(x) = \operatorname{tg} x + \cos x$.

12.16. $f(x) = x + \ln x^2$.

12.17. $f(x) = \ln \dfrac{x}{x-3}$.

12.18. $f(x) = \dfrac{\sqrt{x+1}}{x}$.

12.19. $f(x) = e^x - \dfrac{1}{2}x^2 + 1$.

12.20. $f(x) = 3x + 2\sin\dfrac{x}{2}$.

13. Să se reprezinte grafic următoarele funcţii, $f : D \longrightarrow \mathbb{R}$, D – fiind domeniul maxim de definiţie:

13.1. $f(x) = 3x - x^3$.

13.2. $f(x) = 2 - 3x - x^3$.

13.3. $f(x) = \dfrac{1}{16}x^2(x-4)^2$.

13.4. $f(x) = x^2 - x^4$.

13.5. $f(x) = x(2x^2 + 9x + 12)$.

13.6. $f(x) = (x-1)^2(3-x)^2$.

13.7. $f(x) = \dfrac{3x-2}{x^3}$.

13.8. $f(x) = \dfrac{x^3+4}{x^2}$.

13.9. $f(x) = \left(\dfrac{x}{x-1}\right)^2$.

13.10. $f(x) = \dfrac{3x^4+1}{x^3}$.

13.11. $f(x) = \dfrac{x^3}{x-1}$.

13.12. $f(x) = 3x + \dfrac{6}{x} - \dfrac{1}{x^3}$.

13.13. $f(x) = \dfrac{3}{x+2} - \dfrac{3}{x-2} - 1$.

13.14. $f(x) = \dfrac{\ln(x+1)}{\sqrt{x+1}}$.

13.15. $f(x) = \sin x - \sin^2 x$.

13.16. $f(x) = \cos 3x + 3\cos x$.

13.17. $f(x) = \sin x + \dfrac{1}{2}\sin 2x$.

13.18. $f(x) = \cos x \cos 3x$.

13.19. $f(x) = \arccos\dfrac{2x}{1+x^2}$.

13.20. $f(x) = \arcsin\dfrac{1-x^2}{1+x^2}$.

13.21. $f(x) = \ln x - x + 1$.

13.22. $f(x) = x^2 \ln x$.

13.23. $f(x) = \dfrac{\ln x}{x}$.

13.24. $f(x) = \ln\left(\dfrac{x-5}{x}\right) + 2$.

13.25. $f(x) = x\,\mathrm{arctg}\,x$.

13.26. $f(x) = \mathrm{arctg}\sin x$.

13.27. $f(x) = \ln(\sin x - \cos x)$.

13.28. $f(x) = x^{\frac{2}{3}}e^{-\frac{x^2}{3}}$.

13.29. $f(x) = \dfrac{e^{x+2}}{x+2}$.

13.30. $f(x) = e^{\sin x + \cos x}$.

13.31. $f(x) = \sqrt[3]{x(x^2-1)}$.

13.32. $f(x) = \sqrt[3]{(x-2)(x+1)^2}$.

14. Să se calculeze limitele următoare folosind regula lui l'Hospital:

14.1. $\lim\limits_{x\to 1} \dfrac{x^3-5x^2+4}{2x^3-x^2-1}$.

14.2. $\lim\limits_{x\to 1} \dfrac{x^5-1}{\ln x}$.

14.3. $\lim\limits_{x\to 0} \dfrac{\sin 5x}{2x}$.

14.4. $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\cos 5\pi x}{\cos 3\pi x}$.

14.5. $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} x - x}{\sin x - x}$.

14.6. $\lim\limits_{x\to 0} \dfrac{e^x - e^{-x}}{\ln(1+x)}$.

14.7. $\lim\limits_{x\to \infty} \dfrac{\ln(1+\frac{1}{x^2})}{\pi - 2\operatorname{arctg} x}$.

14.8. $\lim\limits_{x\to 0} \dfrac{\ln\cos 2x}{\ln\cos 3x}$.

14.9. $\lim\limits_{x\to 0} \dfrac{\sin 2x - 2xe^x + 3x^2}{\operatorname{arctg} x - \sin x - \frac{x^3}{6}}$.

14.10. $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\ln \operatorname{tg} x}{\operatorname{ctg} 2x}$.

14.11. $\lim\limits_{x\to \infty} \dfrac{\pi - 2\operatorname{arctg} x}{e^{\frac{2}{x}} - 1}$.

14.12. $\lim\limits_{x\to 1} \dfrac{\ln x - x + 1}{\operatorname{tg}^2(x-1)}$.

14.13. $\lim\limits_{x\to \infty} \dfrac{x^2}{e^x}$.

14.14. $\lim\limits_{x\to \infty} \dfrac{x^4}{e^x}$.

14.15. $\lim\limits_{x\to 1+} \dfrac{\ln(x-1)}{\operatorname{ctg} \pi x}$.

14.16. $\lim\limits_{x\to 0+} \dfrac{\ln x}{\ln \sin x}$.

14.17. $\lim\limits_{x\to 0} x \operatorname{ctg} \pi x$.

14.18. $\lim\limits_{x\to \frac{\pi}{2}} \left(x - \dfrac{\pi}{2}\right) \operatorname{tg} x$.

14.19. $\lim\limits_{x\to 0} (\operatorname{ctg} x \arcsin x)$.

14.20. $\lim\limits_{x\to 0} \sin x \ln(\operatorname{ctg} x)$.

14.21. $\lim\limits_{x\to 2} (x-2) \operatorname{tg} \dfrac{\pi x}{4}$.

14.22. $\lim\limits_{x\to 3} (x-3) \operatorname{ctg} \dfrac{\pi x}{3}$.

14.23. $\lim\limits_{x\to 1} \left(\dfrac{1}{x-1} - \dfrac{1}{\ln x}\right)$.

14.24. $\lim\limits_{x\to 0} \left(\dfrac{1}{x^2} - \operatorname{ctg}^2 x\right)$.

14.25. $\lim\limits_{x\to 3} \left(\dfrac{2x-3}{x^2-7x+12} - \dfrac{1}{(x-2)\ln(x-2)}\right)$.

14.26. $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{\arcsin x}\right)$.

14.27. $\lim\limits_{x\to 1} \left(\dfrac{2}{1-x^2} - \dfrac{3}{1-x^3}\right)$.

14.28. $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{e^{2x}-1}\right)$.

14.29. $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$.

14.30. $\lim_{x\to\infty}\left(\frac{2}{\pi}\operatorname{arctg} x\right)^{x}$.

14.31. $\lim_{x\to 0}(x+3^x)^{\frac{2}{x}}$.

14.32. $\lim_{x\to 0}(x+e^x)^{\frac{1}{x}}$.

14.33. $\lim_{x\to\pi+}(x-\pi)^{\sin x}$.

14.34. $\lim_{x\to 0+}|\ln x|^{\frac{x}{2}}$.

14.35. $\lim_{x\to 0+}\left(\frac{1}{x}\right)^{\sin x}$.

14.36. $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$.

Capitolul 4. INTEGRALA NEDEFINITĂ

1. Să se calculeze:

1.1. $\int (x^3 + x^2 + x - 2)\, dx.$

1.2. $\int (x^5 + x^4 + x^{-2} + x^{-3} + 1)\, dx.$

1.3. $\int \left(\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3}\right) dx.$

1.4. $\int \frac{3 - x - x^2}{x^3}\, dx.$

1.5. $\int \frac{(x^3+1)^3}{x^3}\, dx.$

1.6. $\int \frac{(x^2+1)^3}{x^4}\, dx.$

1.7. $\int (x^{-\frac{1}{3}} + x^{\frac{2}{3}})\, dx.$

1.8. $\int (\sqrt{x} + \sqrt[3]{x})\, dx.$

1.9. $\int \left(\sqrt[3]{x^2} - \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[3]{x^2}} + \sqrt[4]{x^5}\right) dx.$

1.10. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3 dx.$

1.11. $\int \frac{x^2}{x^2+1}\, dx.$

1.12. $\int \frac{x^4}{x^2+1}\, dx.$

1.13. $\int \frac{1 - \cos^3 x}{\cos^2 x}\, dx.$

1.14. $\int \frac{1 + 3x^2}{x^2(1+2x^2)}\, dx.$

1.15. $\int \frac{2\mathrm{tg}^2 x + 3}{\sin^2 x}\, dx.$

1.16. $\int \frac{x^6 - x^2 + 1}{x^2+1}\, dx.$

1.17. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x}\, dx.$

1.18. $\int \frac{dx}{\sin^2 x \cos^2 x}.$

1.19. $\int \sin^2 \frac{x}{2}\, dx.$

1.20. $\int \cos^2 \frac{x}{2}\, dx.$

1.21. $\int \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2 \, dx.$

1.22. $\int \left(\cos^4\frac{x}{2} - \sin^4\frac{x}{2}\right) \, dx.$

1.23. $\int \operatorname{tg}^2 x \, dx.$

1.24. $\int \operatorname{ctg}^2 x \, dx.$

1.25. $\int \left(3^{\frac{x}{2}} + 3^{-\frac{x}{2}}\right)^2 \, dx.$

1.26. $\int e^x \left(1 + \frac{e^{-x}}{\sin^2 x}\right) \, dx.$

1.27. $\int e^x 2^x \, dx.$

1.28. $\int 3^x \left(2 + \frac{3^{-x}}{\sqrt{1-x^2}}\right) \, dx.$

1.29. $\int \frac{3^x + 4^x}{12^x} \, dx.$

1.30. $\int \left(\frac{2}{\sqrt{1-x^2}} - \frac{3}{1+x^2}\right) \, dx.$

2. Să se calculeze:

2.1. $\int (2x+5)^3 \, dx.$

2.2. $\int (3x-7)^5 \, dx.$

2.3. $\int (4-x)^{10} \, dx.$

2.4. $\int \sqrt[3]{(2x-7)^5} \, dx.$

2.5. $\int \sqrt[4]{\left(1-\frac{x}{2}\right)^3} \, dx.$

2.6. $\int \sqrt{3x-7} \, dx.$

2.7. $\int \sin 5x \, dx.$

2.8. $\int \sin(3x-2) \, dx.$

2.9. $\int \cos 4x \, dx.$

2.10. $\int \cos\left(3-\frac{x}{2}\right) \, dx.$

2.11. $\int \frac{dx}{\cos^2 7x}.$

2.12. $\int \frac{dx}{\cos^2\left(\frac{x}{2}-3\right)}.$

2.13. $\int \frac{dx}{\sin^2\frac{x}{3}}.$

2.14. $\int \frac{dx}{\sin^2(3-2x)}.$

2.15. $\int \frac{dx}{\sqrt{1-4x^2}}.$

2.16. $\int \frac{dx}{\sqrt{4-x^2}}.$

2.17. $\int \frac{dx}{1+9x^2}$.

2.18. $\int \frac{dx}{1+\frac{x^2}{4}}$.

2.19. $\int \frac{dx}{x+1}$.

2.20. $\int \frac{dx}{3x-1}$

2.21. $\int e^{-x}\ dx$.

2.22. $\int e^{3x-1}\ dx$.

2.23. $\int 2^{3x+1}\ dx$.

2.24. $\int 5^{1-2x}\ dx$.

3. Să se calculeze:

3.1. $\int \left(x^2+3x+1\right)^{10}(2x+3)\ dx$.

3.2. $\int \sqrt{3-2x+x^2}\,(x-1)\ dx$.

3.3. $\int \sqrt[3]{x^3-8}\ x^2\ dx$.

3.4. $\int \sqrt{x^4+1}\ x^3\ dx$.

3.5. $\int \frac{2x-3}{x^2-3x+7}\ dx$.

3.6. $\int \frac{6x-7}{3x^2-7x+10}\ dx$.

3.7. $\int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\ dx$.

3.8. $\int e^{\sin x}\cos x\ dx$.

3.9. $\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\ dx$.

3.10. $\int \frac{e^{\operatorname{arctg} x}+1}{1+x^2}\ dx$.

3.11. $\int e^{x^3}x^2\ dx$.

3.12. $\int \sin^3 x\cos x\ dx$.

3.13. $\int \cos^5 x\sin x\ dx$.

3.14. $\int \sin^7 x\cos x\ dx$.

3.15. $\int \frac{\sin x}{\cos^5 x}\ dx$.

3.16. $\int \frac{\cos x}{1+2\sin x}\ dx$.

3.17. $\int \frac{dx}{x\ln x}$.

3.18. $\int \frac{dx}{x\left(\ln x+2\right)}$.

3.19. $\int \frac{\sqrt[3]{(1+\ln x)^2}}{x}\ dx$.

3.20. $\int \frac{(\ln x+4)^2}{x}\ dx$.

3.21. $\int \frac{\operatorname{arctg}^2 x}{1+x^2}\ dx$.

3.22. $\int \frac{\operatorname{arctg} 2x+x}{1+4x^2}\ dx$.

3.23. $\int \frac{\arcsin^2 x - 1}{\sqrt{1-x^2}}\, dx.$

3.24. $\int \sqrt{\frac{\arccos x}{x^2-1}}\, dx.$

3.25. $\int \frac{dx}{\sin x}.$

3.26. $\int \frac{dx}{\cos x}.$

3.27. $\int \frac{2^x}{\sqrt{1-4^x}}\, dx.$

3.28. $\int \frac{6x-7}{3x^2-7x+10}\, dx.$

3.29. $\int 3^{2x^2+x-1}\,(4x+1)\, dx.$

3.30. $\int xe^{-x^2}\, dx.$

4. Să se calculeze:

4.1. $\int \frac{1}{x^2} \sin \frac{1}{x}\, dx.$

4.2. $\int \frac{dx}{\sqrt{x} + \sqrt[4]{x}}\ .$

4.3. $\int \frac{dx}{\sin^2 x + 4\cos^2 x}.$

4.4. $\int \frac{dx}{1+\sin^2 x}.$

4.5. $\int \frac{6\operatorname{tg} x}{3\sin 2x + 5\cos^2 x}\, dx.$

4.6. $\int \frac{8+\operatorname{tg} x}{18\sin^2 x + 2\cos^2 x}\, dx.$

4.7. $\int \frac{\ln \arcsin x}{\sqrt{1-x^2}\arcsin x}\, dx.$

4.8. $\int \ln \frac{1+x}{1-x} \cdot \frac{1}{x^2-1}\, dx.$

4.9. $\int \frac{e^{\operatorname{ctg} 2x} + \operatorname{tg} 2x}{\sin^2 2x}\, dx.$

4.10. $\int \frac{\operatorname{arcctg} \sqrt{x}}{(1+x)\sqrt{x}}\, dx.$

4.11. $\int \frac{\cos^3 x}{\sqrt{\sin x}}\, dx.$

4.12. $\int \frac{dx}{x^2\sqrt{1+x^2}}\ .$

4.13. $\int \frac{dx}{\sqrt{-8-6x-x^2}}\ .$

4.14. $\int \frac{dx}{x^2+4x+5}\ .$

4.15. $\int \frac{x+1}{x^2-x+1}\, dx.$

4.16. $\int \frac{3x^2-2x}{x^3-x^2+1}\, dx.$

4.17. $\int \sqrt{\frac{1+x}{1-x}} \cdot \frac{1}{1-x}\, dx.$

4.18. $\int \frac{e^x - 1}{e^x + 1}\, dx.$

4.19. $\int \frac{7x+3}{x^2+4}\, dx.$

4.20. $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}.$

4.21. $\int \frac{3\cos x + 2\sin x}{(3\sin x - 2\cos x)^2}\, dx.$

4.22. $\int \frac{3\cos x - 2\sin x}{(2\cos x + 3\sin x)^3}\, dx.$

5. Să se calculeze:

5.1. $\int x \sin x \, dx.$

5.2. $\int x \cos x \, dx.$

5.3. $\int x^2 \sin x \, dx.$

5.4. $\int (x^2 - x + 1) \cos x \, dx.$

5.5. $\int x e^x \, dx.$

5.6. $\int x^2 e^x \, dx.$

5.7. $\int x e^{3x} \, dx.$

5.8. $\int (x^2 + x - 1) e^x \, dx.$

5.9. $\int x \ln x \, dx.$

5.10. $\int x^2 \ln x \, dx.$

5.11. $\int (x^2 + x + 1) \ln x \, dx.$

5.12. $\int \ln x \, dx.$

5.13. $\int x^n \ln x \, dx \ , \ n \in \mathbb{N}.$

5.14. $\int \ln^2 x \, dx.$

5.15. $\int x \operatorname{arctg} x \, dx.$

5.16. $\int \operatorname{arctg} x \, dx.$

5.17. $\int \arccos^2 x \, dx.$

5.18. $\int \operatorname{arcsin}^2 x \, dx.$

5.19. $\int x^2 3^x \, dx.$

5.20. $\int x 2^x \, dx.$

5.21. $\int e^x \sin x \, dx.$

5.22. $\int e^x \cos x \, dx.$

5.23. $\int \cos(\ln x) \, dx.$

5.24. $\int \sin(\ln x) \, dx.$

5.25. $\int \frac{x}{\sin^2 x} \, dx.$

5.26. $\int \frac{x}{\cos^2 x} \, dx.$

5.27. $\int \frac{\operatorname{arctg} x}{x^2} \, dx.$

5.28. $\int \frac{\arcsin x}{\sqrt{1+x}} \, dx.$

6. Să se calculeze:

6.1. $\int \frac{x-5}{(x-3)(x-4)} \, dx.$

6.2. $\int \frac{2x+5}{(x-1)(x+6)} \, dx.$

6.3. $\int \frac{dx}{(x+2)(x-1)}$.

6.4. $\int \frac{x+1}{(x+2)(x+3)}\,dx$.

6.5. $\int \frac{dx}{(x-3)(x-2)(x+1)}$.

6.6. $\int \frac{x^3+3x^2+3x+1}{x(x+2)(x+3)}\,dx$.

6.7. $\int \frac{x^3-3x^2+3x}{(x-1)(x-2)}\,dx$.

6.8. $\int \frac{2x^4-5x^2-8x-8}{x(x-2)(x+2)}\,dx$.

6.9. $\int \frac{dx}{(x-2)(x+1)(x+2)}$.

6.10. $\int \frac{x^2-x-9}{x^2-x-6}\,dx$.

6.11. $\int \frac{x^2+2x+2}{x(x+2)(x-1)(x+3)}\,dx$.

6.12. $\int \frac{x^2+2x-11}{(x-1)(x+3)(x-5)}\,dx$.

6.13. $\int \frac{dx}{x^2(x+2)}$.

6.14. $\int \frac{x^2+4x+6}{(x+2)^2x}\,dx$.

6.15. $\int \frac{x^5-2x^2+3}{(x-2)^2}\,dx$.

6.16. $\int \frac{2x+1}{(x-1)^3}\,dx$.

6.17. $\int \frac{x^2-2x+3}{x^2(x-2)}\,dx$.

6.18. $\int \frac{x+1}{(x-1)^2(x-3)}\,dx$.

6.19. $\int \frac{5x-1}{(x-1)^2(x-2)}\,dx$.

6.20. $\int \frac{dx}{x^2(x+5)^2}$.

6.21. $\int \frac{dx}{x(x+1)^2(x+2)^3}$.

6.22. $\int \frac{x}{(x+1)^2(x+2)^2(x-1)}\,dx$.

7. Să se calculeze:

7.1. $\int \frac{dx}{x^3+8}$.

7.2. $\int \frac{dx}{x(x^2+1)}$.

7.3. $\int \frac{dx}{(x+1)(x^2+2)}$.

7.4. $\int \frac{x-2}{x(x^2+4)}\,dx$.

7.5. $\int \frac{x}{x^3+1}\,dx$.

7.6. $\int \frac{dx}{(x-2)(x-4)(x^2+2x+2)}$.

7.7. $\int \frac{x^4}{x^4-1}\,dx$.

7.8. $\int \frac{x-1}{x(x^2+1)}\,dx$.

7.9. $\int \frac{x^3+4x^2+3x+2}{(x+1)^2(x^2+1)}\,dx$.

7.10. $\int \frac{x(x^2+2x+10)}{(x+1)^2(x^2-x+1)}\,dx$.

7.11. $\int \frac{dx}{x^2(x^2-2x+2)}$.

7.12. $\int \frac{3x^2-6x+1}{(x+1)^2(3x^2-8x+9)}\,dx$.

7.13. $\int \frac{x-1}{(x^2+1)^2}\,dx.$

7.14. $\int \frac{x^4+2x^2+4}{(x^2+1)^3}\,dx.$

7.15. $\int \frac{x^2+2x+7}{(x-2)(x^2+1)^2}\,dx.$

7.16. $\int \frac{3x+1}{x(1+x^2)^2}\,dx.$

7.17. $\int \frac{5x+8}{(x^2+4)^2}\,dx.$

7.18. $\int \frac{3x+5}{(x^2+2x+2)^2}\,dx.$

7.19. $\int \frac{x^2}{(x+1)^2(x^2-x+1)}\,dx.$

7.20. $\int \frac{2x^4+5x^2-2}{2x^3-x-1}\,dx.$

8. Să se calculeze:

8.1. $\int \frac{dx}{3\sin x+4\cos x}.$

8.2. $\int \frac{dx}{\sin 2x+\cos^2 x}.$

8.3. $\int \frac{dx}{\sin x-\cos x}.$

8.4. $\int \frac{dx}{3\sin x+4\cos x+5}.$

8.5. $\int \frac{3\sin x+2\cos x}{\sin^2 x\cos x+4\cos^3 x}\,dx.$

8.6. $\int \frac{\sin x+3\cos x}{\sin^2 x\cos x+\cos^3 x}\,dx.$

8.7. $\int \frac{\sin x(1+\sin^2 x)}{\cos 2x}\,dx.$

8.8. $\int \frac{\cos^3 x(1+\cos^2 x)}{\sin^2 x(1+\sin^2 x)}\,dx.$

8.9. $\int \cos 2x\cos 4x\,dx.$

8.10. $\int \cos 3x\cos x\cos 5x\,dx.$

8.11. $\int \sin x\sin 3x\,dx.$

8.12. $\int \sin 2x\sin 4x\sin 6x\,dx.$

8.13. $\int \sin x\cos 3x\,dx.$

8.14. $\int \sin 2x\cos 4x\cos 6x\,dx.$

8.15. $\int \sin^2 2x\cos^2 2x\,dx.$

8.16. $\int \sin^4 x\cos^4 x\,dx.$

8.17. $\int \sin^2 x\cos^4 x\,dx.$

8.18. $\int \sin^4 x\cos^2 x\,dx.$

8.19. $\int \sin^2 x\cos^3 x\,dx.$

8.20. $\int \sin^3 x\cos^2 x\,dx.$

8.21. $\int \cos^5 x\,dx.$

8.22. $\int \sin^3 x\cos^3 x\,dx.$

Capitolul 5. INTEGRALA RIEMANN

1. Să se calculeze:

1.1. $\int_{-1}^{2} x^2 \, dx.$

1.2. $\int_{-1}^{2} \sqrt[3]{x} \, dx.$

1.3. $\int_{-1}^{1} (4x^3 - 3x^2 + 2x - 1) \, dx.$

1.4. $\int_{1}^{3} (x^2 + x - 2) \, dx.$

1.5. $\int_{0}^{\frac{\pi}{2}} \sin x \, dx.$

1.6. $\int_{0}^{\pi} \cos x \, dx.$

1.7. $\int_{0}^{\frac{\pi}{4}} \frac{x^2}{1+x^2} \, dx.$

1.8. $\int_{0}^{1} \frac{dx}{\sqrt{x^2+1}}.$

1.9. $\int_{0}^{1} e^x \, dx.$

1.10. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}.$

1.11. $\int_{-\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}.$

1.12. $\int_{\frac{1}{e}}^{e^2} \frac{dx}{x}.$

1.13. $\int_{0}^{1} \frac{dx}{1+x^2}.$

1.14. $\int_{0}^{\frac{\pi}{2}} x \sin x \, dx.$

1.15. $\int_{1}^{e} x \ln x \, dx.$

1.16. $\int_{2}^{3} \frac{x+1}{x^2(x-1)} \, dx.$

1.17. $\int_1^2 \frac{e^{\frac{1}{x^2}}}{x^3}\ dx.$

1.18. $\int_0^1 xe^x\ dx.$

1.19. $\int_{-\pi}^{\pi} \sin^2 x\ dx.$

1.20. $\int_{-\pi}^{\pi} \cos^2 x\ dx.$

1.21. $\int_e^{e^3} \frac{dx}{x \ln x}.$

1.22. $\int_0^{\frac{\pi}{4}} \mathrm{tg}^3 x\ dx.$

2. Să se calculeze ariile plane limitate de curbele:

2.1. $f(x) = 3x - x^2, \quad g(x) = 0.$

2.2. $f(x) = 4x - x^2, \quad g(x) = 0.$

2.3. $f(x) = x^2 + 1, \quad g(x) = 2.$

2.4. $f(x) = x^2, \quad g(x) = 4.$

2.5. $f(x) = x^2, \quad g(x) = x + 2.$

2.6. $f(x) = x^2 - x, \quad g(x) = 3x.$

2.7. $f(x) = 2x - x^2, \quad g(x) = x.$

2.8. $f(x) = (x-1)^2 + 2, \quad g(x) = 3x - 1.$

2.9. $f(x) = x^2, \quad g(x) = 2x - x^2.$

2.10. $f(x) = x^2, \quad g(x) = 3x + 4.$

2.11. $f(x) = x^3, \quad g(x) = \sqrt{x}.$

2.12. $f(x) = \frac{5}{x}, \quad g(x) = 6 - x.$

2.13. $f(x) = x^2, \quad g(x) = \sqrt[3]{x}.$

2.14. $f(x) = x^2, \quad g(x) = 2\sqrt{2x}.$

2.15. $f(x) = -\sqrt{x}, \quad g(x) = \sqrt{x}, \quad x \in [0, 4].$

2.16. $f(x) = e^x, \quad g(x) = e^{-x}, \qquad x \in [0, 1].$

2.17. $f(x) = \ln x, \quad g(x) = \ln^2 x.$

2.18. $f(x) = \frac{1}{4}\left|4 - x^2\right|, \quad g(x) = 7 - |x|.$

2.19. $f(x) = 0, \quad g(x) = -x + 2, \quad h(x) = \sqrt{x}.$

2.20. $f(x) = \frac{1}{x}, \quad g(x) = x, \quad x = 2.$

2.21. $f(x) = \sin x, \quad g(x) = \cos x, \quad x \in \left[0, \frac{\pi}{4}\right].$

2.22. $f(x) = x - \frac{\pi}{2}, \quad g(x) = \cos x, \quad x = 0.$

2.23. $f(x) = \sin^2 x, \quad g(x) = x \sin x, \quad x \in [0, \pi].$

2.24. $f(x) = \sin 2x, \quad g(x) = \sin x, \quad x \in \left[\frac{\pi}{3}, \pi\right].$

2.25. $f(x) = \operatorname{tg} x, \quad g(x) = \frac{2}{3}\cos x, \quad x = 0.$

2.26. $f(x) = \arcsin x, \quad g(x) = \arccos x, \quad h(x) = 0.$

2.27. $f(x) = 2^{x-2} + 1, \quad g(x) = 2^{2-x} + 1, \quad h(x) = \frac{3}{2}.$

2.28. $f(x) = 2 - |2 - x|, \quad g(x) = \frac{6}{|x + 1|}.$

2.29. $f(x) = \left|\lg x\right|, \quad g(x) = 0, \quad x = \frac{1}{10}, \quad x = 10.$

2.30. $f(x) = \ln|1 + x|, \quad g(x) = -xe^{-x}, \quad x = 1.$

3. Să se calculeze ariile plane limitate de curbele:

3.1. $\rho^2 = a^2 \cos 2\varphi.$

3.2. $x = a\cos t, \quad y = b\sin t.$

3.3. $\rho = 4\sin^2\varphi$.

3.4. $x = a\cos^3 t, \quad y = a\sin^3 t$.

3.5. $\rho = a(1+\cos\varphi)$.

3.6. $x = \dfrac{c^2}{a}\cos^3 t, \quad y = \dfrac{c^2}{b}\sin^3 t, \quad c^2 = a^2 - b^2$.

3.7. $\rho = 2 + \cos\varphi$.

3.8. $x = \dfrac{1-t^2}{(1+t^2)^2}, \quad y = \dfrac{2at}{(a+t^2)^2}$.

3.9. $\rho = a\sin 2\varphi$.

3.10. $x = t - t^2, \quad y = t^2 - t^3$.

3.11. $\rho = a\cos\varphi, \quad \rho = a(\cos\varphi + \sin\varphi)$.

3.12. $x = t^2 - 1, \quad y = t^3 - t^2$.

3.13. $\rho = 2 - \cos\varphi, \quad \rho = \cos\varphi$.

3.14. $x = \dfrac{t - t^3}{1+3t^2}, \quad y = \dfrac{4t^2}{1+3t^2}$.

3.15. $\rho = 2\sqrt{3}\cos\varphi, \quad \rho = 2\sin\varphi$.

3.16. $x = \sin 2t, \quad y = \sin t$.

3.17. $\rho = 1 + \sqrt{2}\cos\varphi$.

3.18. $x = 1 + t - t^3, \quad y = 1 - 15t^2$.

3.19. $\rho = 3\sin\varphi, \quad \rho = 5\sin\varphi$.

3.20. $x = 1 + 2\cos t, \quad y = \operatorname{tg} t + 2\sin t$.

4. Să se calculeze lungimile arcelor:

4.1. $f(x) = \dfrac{(x+1)^2}{4} - \dfrac{\ln(x+1)}{2}, \quad x \in [0,1].$

4.2. $f(x) = -\ln\cos x, \quad x \in \left[0, \dfrac{\pi}{6}\right].$

4.3. $f(x) = \ln x, \quad x \in \left[\sqrt{3}, \sqrt{8}\right].$

4.4. $f(x) = \ln(x^2-1), \quad x \in [2,3].$

4.5. $f(x) = \sqrt{2x - x^2} - 1, \quad x \in \left[\dfrac{1}{4}, 1\right].$

4.6. $f(x) = x^2, \quad x \in [0,1].$

4.7. $f(x) = 4\sqrt{x-1}, \quad x \in [1,2].$

4.8. $f(x) = x^2 - \ln\sqrt{x}, \quad x \in [1,2].$

4.9. $f(x) = x\sqrt{x}, \quad x \in [0,9].$

4.10. $f(x) = \ln\sin x, \quad x \in \left[\dfrac{\pi}{3}, \dfrac{2\pi}{3}\right].$

4.11. $x = a\cos^3 t, \quad y = a\sin^3 t, \quad t \in [0, 2\pi].$

4.12. $\rho = 2\sin\varphi.$

4.13. $x = 3(2-t^2), \quad y = 4t^3, \quad x \geqslant 0.$

4.14. $\rho = \cos^3\dfrac{\varphi}{3}.$

4.15. $x = \cos^4 t, \quad y = \sin^4 t, \quad t \in \left[0, \dfrac{\pi}{2}\right].$

4.16. $\rho = a(1 - \cos\varphi).$

4.17. $x = 6\cos^3 t, \quad y = 6\sin^3 t, \quad t \in \left[0, \dfrac{\pi}{3}\right].$

4.18. $\rho = \sin 3\varphi.$

4.19. $x = 2(t - \sin t), \quad y = 2(1 - \cos t), \quad t \in \left[0, \dfrac{\pi}{2}\right].$

4.20. $\rho = \dfrac{1}{2} + \sin\varphi.$

4.21. $x = e^t(\cos t + \sin t), \quad y = e^t(\cos t - \sin t), \quad t \in \left[\dfrac{\pi}{6}, \dfrac{\pi}{4}\right].$

4.22. $\rho = \cos\varphi - \sin\varphi.$

4.23. $x = 2(\cos t + t\sin t), \quad y = 2(\sin t - t\cos t), \quad t \in [0, \pi].$

4.24. $\rho = 2\sin 4\varphi.$

5. Să se calculeze volumul corpului obţinut prin rotaţia în jurul axei OX a suprafeţei mărginite de curbele:

5.1. $f(x) = -x^2 + 7x - 12, \quad g(x) = 0.$

5.2. $f(x) = \dfrac{4}{x}, \quad g(x) = 0, \quad x = 1, \quad x = 4.$

5.3. $f(x) = 2x + \sqrt{2x}, \quad g(x) = 0, \quad x = 2 \quad x = \dfrac{9}{2}.$

5.4. $f(x) = 2x - x^2, \quad g(x) = 2 - x.$

5.5. $f(x) = \arcsin x, \quad x = 0, \quad x = 1.$

5.6. $f(x) = xe^x, \quad g(x) = 0, \quad x = 1.$

5.7. $f(x) = x^2, \quad g(x) = 0, \quad x = 3.$

5.8. $f(x) = (x-2)^2, \quad g(x) = 4.$

5.9. $f(x) = e^{2-x}, \quad g(x) = 0, \quad x = 1, \quad x = 2.$

5.10. $f(x) = e^x, \quad g(x) = 0, \quad x = 0, \quad x = 1.$

5.11. $f(x) = 3\sin x, \quad g(x) = \sin x, \quad x = 0, \quad x = \pi.$

5.12. $f(x) = \sin x, \quad g(x) = 0, \quad x = \dfrac{\pi}{6}, \quad x = \dfrac{\pi}{2}.$

5.13. $f(x) = 4 - x^2, \quad g(x) = 3x, \quad x = -2, \quad x = 0.$

5.14. $f(x) = \sqrt{x}e^{-x}, \quad g(x) = 0, \quad x = 1.$

5.15. $f(x) = \sin^2 x, \quad g(x) = x \sin x, \quad x = 0, \quad x = \pi.$

5.16. $f(x) = \sin 2x, \quad g(x) = 0, \quad x = 0, \quad x = \dfrac{\pi}{4}.$

5.17. $f(x) = 3x - x^2, \quad g(x) = 0.$

5.18. $x = a \cos^3 t, \quad y = a \sin^3 t.$

5.19. $f(x) = x^2, \quad g(x) = \sqrt{x}.$

5.20. $f(x) = x^3, \quad g(x) = x^2.$

Capitolul 6. SERII NUMERICE

1. Să se stabilească natura seriilor următoare, calculând limita şirului sumelor parţiale:

1.1. $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^{n-1}$.

1.2. $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)^{n-1}$.

1.3. $\sum_{n=1}^{\infty} \frac{n}{3^n}$.

1.4. $\sum_{n=1}^{\infty} \frac{2n}{5^n}$.

1.5. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

1.6. $\sum_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)}$.

1.7. $\sum_{n=2}^{\infty} \frac{1}{n^2+n-2}$.

1.8. $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$.

1.9. $\sum_{n=1}^{\infty} \frac{12}{36n^2+12n-35}$.

1.10. $\sum_{n=1}^{\infty} \frac{5}{25n^2-5n-6}$.

1.11. $\sum_{n=1}^{\infty} \frac{7}{49n^2+7n-12}$.

1.12. $\sum_{n=1}^{\infty} \frac{6}{36n^2-24n-5}$.

1.13. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$.

1.14. $\sum_{n=2}^{\infty} \frac{3n-5}{n(n^2-1)}$.

1.15. $\sum_{n=3}^{\infty} \frac{1}{n(n-2)(n+2)}$.

1.16. $\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)(2n+5)}$.

1.17. $\sum_{n=2}^{\infty} \frac{5n+4}{(n-1)n(n+2)}$.

1.18. $\sum_{n=1}^{\infty} \frac{n-1}{n(n+1)(n+2)}$.

1.19. $\sum_{n=1}^{\infty} \frac{3-n}{n(n+1)(n+3)}$.

1.20. $\sum_{n=1}^{\infty} \frac{2-n}{n(n+1)(n+2)}$.

1.21. $\sum_{n=2}^{\infty} \frac{n-\sqrt{n^2-1}}{\sqrt{n(n-1)}}$.

1.22. $\sum_{n=1}^{\infty} \frac{2n-1}{2^n}$.

1.23. $\displaystyle\sum_{n=1}^{\infty}\frac{n2^n}{(n+2)!}$.

1.24. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(n+m)},\ m\in\mathbb{N}$.

2. Folosind criteriul general de convergenţă al lui Cauchy să se stabilească natura seriilor:

2.1. $\displaystyle\sum_{n=1}^{\infty}q^n\sin(2n),\ |q|<1$.

2.2. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$.

2.3. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$.

2.4. $\displaystyle\sum_{n=1}^{\infty}\frac{n+1}{n^2+4}$.

2.5. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos nx}{2^n},\ x\in\mathbb{R}$.

2.6. $\displaystyle\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right)$.

2.7. $\displaystyle\sum_{n=1}^{\infty}\frac{a_n}{10^n},\ |a_n|<10$.

2.8. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos 2^n}{n^2}$.

2.9. $\displaystyle\sum_{n=1}^{\infty}\frac{\sin(n\alpha)}{n(n+1)},\ \alpha\in\mathbb{R}$.

2.10. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+1)}}$.

3. Utilizând condiţia de convergenţă, să se demonstreze divergenţa seriilor:

3.1. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{n^2+1}$.

3.2. $\displaystyle\sum_{n=1}^{\infty}\operatorname{arctg}(n-1)$.

3.3. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n+3}{n+2}$.

3.4. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3n^2+4}{3n^2+2}\right)^{n^2}$.

3.5. $\displaystyle\sum_{n=1}^{\infty}\sqrt{\frac{2n+3}{3n+5}}$.

3.6. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt[3]{n+1}}{\ln^2(n+2)}$.

3.7. $\displaystyle\sum_{n=1}^{\infty}n\operatorname{arctg}\frac{1}{n+1}$.

3.8. $\displaystyle\sum_{n=1}^{\infty}(n^2+1)\ln\frac{n^2+1}{n^2}$.

3.9. $\displaystyle\sum_{n=1}^{\infty}\frac{n^3-1}{n+2}\arcsin\frac{1}{n^2+1}$.

3.10. $\displaystyle\sum_{n=2}^{\infty}\sqrt[n]{0,05}$.

4. Utilizând criteriile de comparaţie, să se studieze natura seriilor:

4.1. $\displaystyle\sum_{n=1}^{\infty}\frac{\sin^2 n\sqrt[3]{n}}{n\sqrt[3]{n}}$.

4.2. $\displaystyle\sum_{n=1}^{\infty}\frac{\ln(n+1)}{-\sqrt[5]{n^9}}$.

4.3. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(\pi n)}{n(n+1)(n+2)}$.

4.4. $\displaystyle\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n-\ln n}$.

4.5. $\displaystyle\sum_{n=2}^{\infty} \frac{\arcsin \frac{(-1)^n n}{n+1}}{n^2+2}$.

4.6. $\displaystyle\sum_{n=1}^{\infty} \frac{3+(-1)^n}{2^{n+2}}$.

4.7. $\displaystyle\sum_{n=2}^{\infty} \frac{\operatorname{arctg}\left[2+(-1)^n\right]}{\ln n}$.

4.8. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+2}{n^3}$.

4.9. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt[n]{\ln n}}$.

4.10. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \sqrt{\ln\left(1+\frac{1}{n}\right)}\right)$.

4.11. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n+n^3}{4^n+\ln^2(n+1)}$.

4.12. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3n+1}{\sqrt{n^6+n^3+1}}$.

4.13. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2n-1}$.

4.14. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+1}{5^n+2}$.

4.15. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{(3n+1)(3n+2)}}$.

4.16. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n+2}-\sqrt{n-2}}{\sqrt{n+1}}$.

4.17. $\displaystyle\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}}-1\right) \sin \frac{1}{\sqrt{n+2}}$.

4.18. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \operatorname{arctg} \frac{1}{\sqrt{n}}$.

4.19. $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n^2+3}{n^2+2}$.

4.20. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}} \arcsin \frac{1}{\sqrt[3]{n^2}}$.

5. Utilizând criteriul D'Alembert, să se stabilească natura următoarelor serii:

5.1. $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{3^n n!}$.

5.2. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{3^n n!}$.

5.3. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{n^n}$.

5.4. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

5.5. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$.

5.6. $\displaystyle\sum_{n=1}^{\infty} \frac{(3n)!}{(n!)^3 4^{3n}}$.

5.7. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!(2n+3)!}{(3n+3)!}$.

5.8. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!!}{n!} \operatorname{arctg} \frac{1}{5^n}$.

5.9. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{\ln 2}}{(\ln 2)^n}$.

5.10. $\displaystyle\sum_{n=2}^{\infty} n \operatorname{tg} \frac{\pi}{2^n}$.

5.11. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1}}{n! + (n+2)!}$.

5.12. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^{n+1}}$.

5.13. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}}$.

5.14. $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n^2-1}}{3^{n^2}\sqrt{n}}$.

5.15. $\displaystyle\sum_{n=1}^{\infty} n^2 \sin \frac{\pi}{2^n}$.

5.16. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{2^n n!}$.

5.17. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{(n+3)!}$.

5.18. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n \sqrt[3]{n}}{(n+1)!}$.

5.19. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+2)}{n!} \sin \frac{2}{5^n}$.

5.20. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{(n+1)^n}$.

6. Utilizând criteriul radical Cauchy, să se stabilească natura seriilor:

6.1. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n+1}{4n+3}\right)^{n^2}$.

6.2. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{7n+6}\right)^{n^2}$.

6.3. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n-1}{n+2}\right)^{n^2}$.

6.4. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+2}\right)^{\frac{n}{2}}$.

6.5. $\displaystyle\sum_{n=1}^{\infty} n^3 \sin^n \frac{\pi}{2n}$.

6.6. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1}}{n^n}$.

6.7. $\displaystyle\sum_{n=1}^{\infty} \frac{n3^n}{5^n}$.

6.8. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^{n^2} \frac{1}{3^n}$.

6.9. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^{n^2} \frac{1}{2^n}$.

6.10. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n2^n}$.

6.11. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^n}{(3n+1)^n}$.

6.12. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} 4^n$.

6.13. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+2}\right)^{\sqrt{n^3+3n+1}}$.

6.14. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\sqrt{n+1}+1}{\sqrt{n+1}+2}\right)$.

6.15. $\sum_{n=1}^{\infty} 3^{n-1}e^{-2n}$.

6.16. $\sum_{n=1}^{\infty} \left(\frac{4n+1}{5n+6}\right)^{n^3}$.

6.17. $\sum_{n=1}^{\infty} \left(\frac{3n^2+2n+1}{5n^2+3n+2}\right)^{n}$.

6.18. $\sum_{n=1}^{\infty} \left[2+(0,1)^{n-1}\right]$.

6.19. $\sum_{n=1}^{\infty} \frac{2^n}{\left(1+\frac{1}{n}\right)^{n^2}}$.

6.20. $\sum_{n=1}^{\infty} 2^{n+1}e^{-n}$.

7. Utilizând criteriul integral Cauchy, să se studieze natura următoarelor serii:

7.1. $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}},\ \alpha \in \mathbb{R}$.

7.2. $\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln^2(n+1)}$.

7.3. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$.

7.4. $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$.

7.5. $\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n+1}}}{\sqrt{n+1}}$.

7.6. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

7.7. $\sum_{n=1}^{\infty} \frac{1}{(9n-1)\ln(9n-1)}$.

7.8. $\sum_{n=2}^{\infty} \frac{1}{n\ln^p n}$.

7.9. $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^p(\ln\ln n)^q}$.

7.10. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} \ln\frac{n+1}{n-1}$.

8. Să se calculeze suma seriei cu exactitatea α:

8.1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3},\ \alpha = 0,01$.

8.2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)^2},\ \alpha = 0,01$.

8.3. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!},\ \alpha = 0,01$.

8.4. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{n}{2^n},\ \alpha = 0,01$.

8.5. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!3^n},\ \alpha = 0,001$.

8.6. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{2^n}{(n+1)^n},\ \alpha = 0,001$.

8.7. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{5^n},\ \alpha = 0,0001$.

8.8. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)!!},\ \alpha = 0,0001$.

8.9. $\sum_{n=0}^{\infty} \frac{\cos \pi n}{3^n(n+1)},\ \alpha = 0,001$.

8.10. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n^2+3)},\ \alpha = 0,01$.

9. Utilizând criteriul lui Leibniz, să se demonstreze natura seriilor:

9.1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

9.2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\ln(n+1)}$.

9.3. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+1}{n(n+1)}$.

9.4. $\sum_{n=3}^{\infty} \frac{(-1)^{n}}{(n+1)\ln n}$.

9.5. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n-1}{3n}$.

9.6. $\sum_{n=1}^{\infty} (-1)^{n+1}$.

9.7. $\sum_{n=1}^{\infty} (-1)^{n+1} [2 + (0,1)^n]$.

9.8. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n-3}{2n-1}$.

9.9. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$.

9.10. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln^2 n}{n}$.

9.11. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{\sqrt{n}}$.

9.12. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n+1)^{n+1}}{n^{n+2}}$.

9.13. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n\sqrt{n}-1}$.

9.14. $\sum_{n=1}^{\infty} (-1)^{n+1} \sin \frac{1}{n}$.

9.15. $\sum_{n=1}^{\infty} (-1)^{n+1} \operatorname{tg} \frac{2}{n}$.

9.16. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\alpha}}$, $\alpha \in \mathbb{R}$.

9.17. $\sum_{n=1}^{\infty} (-1)^{\frac{n(n-1)}{2}} \cdot \frac{2^n + n^2}{3^n + n^3}$.

9.18. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$.

9.19. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!}$.

9.20. $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{\pi}{2} + \pi n\right)}{n^3 + 1}$.

10. Folosind criteriul lui Dirichlet sau criteriul lui Abel, să se demonstreze convergenţa seriilor următoare:

10.1. $\sum_{n=1}^{\infty} \frac{\sin nx}{n}$, $x \in \mathbb{R} \setminus \{2k\pi, k \in \mathbb{Z}\}$.

10.2. $\sum_{n=1}^{\infty} \frac{\sin n \sin n^2}{\sqrt{n}}$.

10.3. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \operatorname{arctg} n$.

10.4. $\sum_{n=1}^{\infty} \int_0^1 x \cos(nx)\, dx$.

10.5. $\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(n^2 x\right) \sin(nx),\ x \in \mathbb{R}.$

10.6. $\sum_{n=1}^{\infty} \frac{1}{n} \cos n \sin(nx),\ x \in \mathbb{R}.$

10.7. $\sum_{n=1}^{\infty} \frac{1}{n} \cos\left(n^2 x\right) \sin(nx),\ x \in \mathbb{R}.$

10.8. $\sum_{n=1}^{\infty} \frac{\sin n\alpha}{\ln\ln(n+2)} \cos\frac{1}{n}.$

10.9. Să se demonstreze, că dacă şirul numeric $\{a_n\}$ converge monoton la zero, atunci seria $\sum_{n=1}^{\infty} a_n \sin n\alpha$ converge pentru orice $\alpha \in \mathbb{R}$, iar seria $\sum_{n=1}^{\infty} a_n \cos n\alpha$ converge pentru orice $\alpha \in \mathbb{R}\backslash\{2\pi m,\ m \in \mathbb{Z}\}$.

11. Să se studieze convergenţa absolută sau semiconvergenţa seriilor următoare:

11.1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}.$

11.2. $\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{\ln\ln n}.$

11.3. $\sum_{n=1}^{\infty} (-1)^{n+1} \sin\frac{\pi}{\sqrt{n}}.$

11.4. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}.$

11.5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\ln(n+1)\ln\ln(n+2)}.$

11.6. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt[3]{n}}{\sqrt{n-1}+2}.$

11.7. $\sum_{n=1}^{\infty} \frac{(n+1)\sin 2n}{n^2 - \ln n}.$

11.8. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n-1)}{n\sqrt{n+1}} \operatorname{tg}\frac{1}{\sqrt{n}}.$

11.9. $\sum_{n=1}^{\infty} \frac{\cos n \cos\frac{1}{n}}{\sqrt[4]{n}}.$

11.10. $\sum_{n=1}^{\infty} \frac{\cos n}{n^\alpha},\ \alpha > 0.$

Capitolul 7. SERII DE PUTERI

1. Să se determine raza de convergenţă pentru următoarele serii de puteri:

1.1. $\sum_{n\geqslant 0} \frac{n+1}{n+2} x^n$.

1.2. $\sum_{n\geqslant 0} 10^n x^n$.

1.3. $\sum_{n\geqslant 0} (-1)^{n+1} \frac{x^n}{n}$.

1.4. $\sum_{n\geqslant 1} \frac{x^n}{n \cdot 5^{n-1}}$.

1.5. $\sum_{n\geqslant 0} n! x^n$.

1.6. $\sum_{n\geqslant 0} \frac{\ln(n+1)}{n+1} x^{n+1}$.

1.7. $\sum_{n\geqslant 1} n^n x^n$.

1.8. $\sum_{n\geqslant 1} 3^{n^2} x^n$.

1.9. $\sum_{n\geqslant 0} \frac{x^n}{n!}$.

1.10. $\sum_{n\geqslant 1} \frac{x^n}{n^2}$.

1.11. $\sum_{n\geqslant 1} \frac{2n+1}{3n^2+2} (x-1)^n$.

1.12. $\sum_{n\geqslant 0} \frac{2^{n+1}(x+1)^{n+1}}{(n+1)\ln^2(n+2)}$.

1.13. $\sum_{n\geqslant 1} \frac{3^n n}{n^n} (x-1)^{2n}$.

1.14. $\sum_{n\geqslant 0} \frac{1}{n!} \left(\frac{nx}{e}\right)^n$.

1.15. $\sum_{n\geqslant 1} \frac{1}{\sqrt{n} 3^n} (x-1)^n$.

1.16. $\sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{n+1} x^n$.

1.17. $\sum_{n=1}^{\infty} \frac{\left(2+(-1)^n\right)^n}{n} (x-1)^n$.

1.18. $\sum_{n\geqslant 1} \left(\frac{2n-3}{3n+1}\right)^n (x+1)^n$.

1.19. $\sum_{n\geqslant 1} \left(\frac{n+1}{2n+3}\right)^n (x-2)^n$.

1.20. $\sum_{n\geqslant 1} \left(\frac{n+3}{n+6}\right)^{n^2} x^n$.

2. Să se determine mulţimile de convergenţă pentru seriile următoare:

2.1. $\sum_{n\geqslant 1} \frac{(x-1)^n}{n\sqrt{n}}$.

2.2. $\sum_{n\geqslant 1} \left(\frac{2n+1}{3n+5}\right)^n (x-2)^n$.

2.3. $\sum_{n\geqslant 1} \frac{(-1)^n}{2n-1} x^n$.

2.4. $\sum_{n\geqslant 1} \frac{1}{3^n n^3}(x-1)^{2n}$.

2.5. $\sum_{n\geqslant 2} 2^n \left(1-\frac{1}{n}\right)^{2n^2} (x-1)^n$.

2.6. $\sum_{n\geqslant 1} \frac{(n!)^2}{(2n)!}(x-2)^n$.

2.7. $\sum_{n\geqslant 1} \left(1-\frac{1}{n}\right)^{n^2} (x-1)^n$.

2.8. $\sum_{n\geqslant 1} \frac{(x-1)^n}{n\sqrt{n}}$.

2.9. $\sum_{n\geqslant 1} \frac{(-1)^n}{3^n\sqrt{n}}(x+1)^n$.

2.10. $\sum_{n\geqslant 1} \frac{2^n \cdot n!}{(2n)!} x^{2n}$.

2.11. $\sum_{n\geqslant 1} \frac{(x+7)^{3n}}{n^2}$.

2.12. $\sum_{n\geqslant 1} (-3)^n x^{2n}$.

2.13. $\sum_{n\geqslant 0} (-1)^{n+1} \frac{(x-4)^{2n+1}}{2n+1}$.

2.14. $\sum_{n\geqslant 1} \frac{n^3}{(n+1)!}(x-5)^{2n+1}$.

2.15. $\sum_{n\geqslant 0} \frac{(x-2)^n}{2^n(n+1)(n+2)}$.

2.16. $\sum_{n\geqslant 1} \frac{2^n}{(2n-1)^2\sqrt{5^{n-1}}} x^n$.

2.17. $\sum_{n\geqslant 1} \frac{n!}{n^n}(x-3)^n$.

2.18. $\sum_{n\geqslant 1} \frac{1}{\ln^n(n+1)}(x-1)^n$.

2.19. $\sum_{n\geqslant 1} \frac{3^n}{\sqrt{2^n}}(x-1)^n$.

2.20. $\sum_{n\geqslant 1} \frac{2^{n^2-1}}{n} x^{n^2}$.

3. Să se dezvolte în serie MacLaurin funcţiile:

3.1. $f(x) = e^{-x^2}$.

3.2. $f(x) = \frac{x^2}{(1+x^2)^2}$.

3.3. $f(x) = \frac{1}{(1-x^3)^2}$.

3.4. $f(x) = e^{-x}$.

3.5. $f(x) = \operatorname{tg} x$.

3.6. $f(x) = x \operatorname{ctg} x$.

3.7. $f(x) = \operatorname{ch} x$.

3.8. $f(x) = \operatorname{sh} x$.

3.9. $f(x) = \sqrt{1 - x^2}$.

3.10. $f(x) = (1 + x^2) \operatorname{arctg} x$.

3.11. $f(x) = e^x \sin x$.

3.12. $f(x) = \ln(1 - x)$.

3.13. $f(x) = \dfrac{\arcsin x}{\sqrt{1 - x^2}}$.

3.14. $f(x) = \arcsin x$.

3.15. $f(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

3.16. $f(x) = \ln \sqrt{1 - x}$.

3.17. $f(x) = \dfrac{5x + 1}{x + 3}$.

3.18. $f(x) = \dfrac{3x + 1}{x^2 + x - 6}$.

3.19. $f(x) = \dfrac{1}{(1 - x^2)(x^2 + 4)}$.

3.20. $f(x) = \dfrac{x}{(x + 1)(x^2 - 1)}$.

Capitolul 8. INTEGRALE IMPROPRII

1. Să se cerceteze care din următoarele integrale improprii sunt convergente:

1.1. $\int_0^{+\infty} \frac{dx}{1+x^2}$.

1.2. $\int_0^1 \ln x \ dx$.

1.3. $\int_1^{+\infty} \frac{dx}{x^\alpha}, \ \alpha \in \mathbb{R}$.

1.4. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

1.5. $\int_1^{+\infty} \frac{1+\ln x}{x} \ dx$.

1.6. $\int_0^1 \frac{dx}{x^\alpha}, \ \alpha \in \mathbb{R}$.

1.7. $\int_{-\infty}^0 xe^x \ dx$.

1.8. $\int_0^1 \frac{dx}{\sqrt{16-x^2}}$.

1.9. $\int_{-\infty}^0 \operatorname{arctg} x \ dx$.

1.10. $\int_{-1}^1 \frac{\arccos x}{\sqrt{1-x^2}} \ dx$.

1.11. $\int_2^{+\infty} \frac{dx}{x^2-1}$.

1.12. $\int_{-2}^0 \frac{\arcsin 2x}{\sqrt{4-x^2}} \ dx$.

1.13. $\int_{-\infty}^{+\infty} \frac{dx}{x^2+4x+5}$.

1.14. $\int_{-1}^0 e^{\frac{2}{x}} \frac{dx}{x^3}$.

1.15. $\int_0^{+\infty} \sin 5x \ dx$.

1.16. $\int_0^e \frac{dx}{e^x-1}$.

1.17. $\int_{-\infty}^0 \frac{x+2}{x^2+1} \ dx$.

1.18. $\int_0^{\frac{\pi}{2}} \frac{dx}{\sin x}$.

1.19. $\int\limits_e^{+\infty} \frac{dx}{x \ln x}$.

1.20. $\int\limits_0^5 \frac{dx}{(x-5)^3}$.

1.21. $\int\limits_1^{+\infty} \frac{dx}{x^3\,(1+x^3)}$.

1.22. $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{tg} x \; dx$.

1.23. $\int\limits_1^{+\infty} e^{-x^2} \; dx$.

1.24. $\int\limits_0^{\frac{\pi}{2}} (\sin x)^p (\cos x)^q \; dx, \;\; \{p, q\} \subset \mathbb{R}$.

1.25. $\int\limits_{-\infty}^{+\infty} \frac{dx}{x^2 - 5x + 14}$.

1.26. $\int\limits_{-1}^{1} \frac{x \; dx}{\left|\sqrt{4-x} - \sqrt{4+x}\right|}$.

1.27. $\int\limits_1^{+\infty} \frac{2x+1}{x^2(x+1)} \; dx$.

1.28. $\int\limits_a^b \frac{dx}{(b-x)^\alpha}, \;\; \alpha \in \mathbb{R}$.

1.29. $\int\limits_e^{+\infty} \frac{dx}{x\sqrt[3]{\ln x}}$.

1.30. $\int\limits_0^1 \frac{dx}{(1-x)\sqrt{x}}$.

1.31. $\int\limits_0^{+\infty} e^{-x} \sin x \; dx$.

1.32. $\int\limits_1^{+\infty} \frac{dx}{x\sqrt{x^2+x+1}}$.

2. Să se cerceteze natura integralelor improprii:

2.1. $\int\limits_1^{+\infty} \frac{x+2}{\sqrt{x^3}} \; dx$.

2.2. $\int\limits_2^{+\infty} \frac{dx}{\sqrt{x(x+1)(x-1)}}$.

2.3. $\int\limits_0^{+\infty} \frac{x^2-1}{x^4+x^2+3} \; dx$.

2.4. $\int\limits_0^2 \frac{dx}{\sqrt[3]{4-x^2}}$.

2.5. $\int\limits_0^{+\infty} \frac{x^3-2x^2+3}{x^4+1} \; dx$.

2.6. $\int\limits_1^2 \frac{dx}{\ln x}$.

2.7. $\int\limits_1^{+\infty} \frac{\sin^2 x}{x^2} \; dx$.

2.8. $\int\limits_0^1 \frac{\cos \frac{1}{x}}{\sqrt[3]{x}}$.

2.9. $\int\limits_1^{+\infty} \frac{dx}{\sqrt{9x+\ln x}}$.

2.10. $\int\limits_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx$.

2.11. $\int\limits_1^{+\infty} \frac{x^2\,dx}{x^4+\sin^2 x}$.

2.12. $\int\limits_0^1 \frac{dx}{\operatorname{tg} x - x}$.

2.13. $\int\limits_2^{+\infty} \frac{dx}{x^p \ln^q x},\ \{p,q\} \subset \mathbb{R}$.

2.14. $\int\limits_0^1 \frac{\ln\left(1+\sqrt[3]{x^2}\right)}{e^x-1}\,dx$.

2.15 $\int\limits_0^{+\infty} \frac{\ln(1+x^2)}{\sqrt{x+\sqrt{x}}}\,dx$.

2.16. $\int\limits_0^1 \frac{\sqrt{x}\,dx}{e^{\sin x}-1}$.

2.17. $\int\limits_e^{+\infty} \frac{dx}{x\ln^\alpha x},\ \alpha \in \mathbb{R}$.

2.18. $\int\limits_0^1 \frac{dx}{e^x-\cos x}$.

2.19. $\int\limits_2^{+\infty} \frac{e^{\alpha x}}{(x-1)^\alpha \ln x}\,dx,\ \alpha \in \mathbb{R}$.

2.20. $\int\limits_0^2 \frac{dx}{\ln x}$.

2.21. $\int\limits_1^{+\infty} \frac{(x+\sqrt{x+2})}{x^2+3\sqrt[5]{x^4+2}}\,dx$.

2.22. $\int\limits_0^1 \frac{\ln x}{1-x^2}\,dx$.

2.23. $\int\limits_1^{+\infty} \ln\frac{e^{\frac{1}{x}}+(n-1)}{n}\,dx,\ n>0$.

2.24. $\int\limits_0^1 \frac{dx}{e^{\sqrt{x}}-1}$.

2.25. $\int\limits_1^{+\infty} \frac{dx}{x\sqrt[3]{x^2+1}}$.

2.26. $\int\limits_0^1 \frac{dx}{\sqrt{1-x^4}}$.

2.27. $\int\limits_0^{+\infty} \frac{\sin^2 x}{1+x^2}\,dx$.

2.28. $\int\limits_0^{\pi} \frac{dx}{\sqrt{\sin x}}$.

2.29. $\int\limits_0^{+\infty} x^{p-1}e^{-x}\,dx,\ p \in \mathbb{R}$.

2.30. $\int\limits_0^1 x^{p-1}(1-x)^{q-1}\,dx,\ \{p,q\} \subset \mathbb{R}$.

3. Să se cerceteze la convergenţă absolută sau semiconvergenţă integralele:

3.1. $\int_1^{+\infty} \frac{\sin x}{x}\, dx.$

3.2. $\int_0^{+\infty} \frac{\sqrt{x}\cos x}{x+10}\, dx.$

3.3. $\int_0^1 (1-x)\sin\frac{\pi}{1-x}\, dx.$

3.4. $\int_0^1 \frac{1}{1-x}\sin\frac{\pi}{1-x}\, dx.$

3.5. $\int_0^1 \frac{x^2}{x^2+1}\sin\frac{1}{x}\, dx.$

3.6. $\int_0^1 \frac{1}{x\,(x^2+1)}\sin\frac{1}{x}\, dx.$

3.7. $\int_0^{\frac{1}{2}} \left(\frac{x}{1-x}\right)\cos\frac{1}{x^2}\, dx.$

3.8. $\int_0^{\frac{1}{2}} \left(\frac{1-x}{x}\right)^2\cos\frac{1}{x^2}\, dx.$

3.9. $\int_0^1 \frac{\sin x^2}{x^2}\, dx.$

3.10. $\int_0^1 \frac{\sin\frac{1}{x^2}}{x^2}\, dx.$

4. Să se calculeze:

4.1. V.P. $\int_1^6 \frac{dx}{4-x}.$

4.2. V.P. $\int_{-\infty}^{+\infty} \frac{1+x}{1+x^2}\, dx.$

4.3. V.P. $\int_{-\frac{1}{2}}^1 \frac{dx}{(x+1)\ln(x+1)}.$

4.4. V.P. $\int_0^{+\infty} \frac{dx}{x^2-x-2}.$

4.5. V.P. $\int_{-2}^2 \frac{dx}{x}.$

4.6. V.P. $\int_0^{+\infty} \frac{dx}{x^2-4x+3}.$

4.7. V.P. $\int_{-1}^3 \frac{dx}{(x-2)^3}.$

4.8. V.P. $\int_{-\infty}^{+\infty} \operatorname{arctg} x\, dx.$

4.9. V.P. $\int_0^{\frac{\pi}{2}} \frac{dx}{1-2\sin x}.$

4.10. V.P. $\int_0^{+\infty} \frac{dx}{1-x^2}.$

Capitolul 9. FUNCŢII DE MAI MULTE VARIABILE

1. Să se determine şi să se reprezinte domeniile de definiţie ale următoarelor funcţii:

1.1. $u = \sqrt{x} + y$.

1.2. $u = \sqrt{xy}$.

1.3. $u = \sqrt{4 - x^2 - y^2}$.

1.4. $u = \sqrt{x^2 + y^2 - 1}$.

1.5. $u = \sqrt{\dfrac{x^2}{9} + \dfrac{y^2}{4} - 1}$.

1.6. $u = \sqrt{(x^2 + y^2 - 4)\,(9 - x^2 - y^2)}$.

1.7. $u = \dfrac{1}{\sqrt{x^2 + y^2 - 16}}$.

1.8. $u = \dfrac{1}{\sqrt{9 - x^2 - y^2}}$.

1.9. $u = \sqrt{4 - x^2 - y^2} + \sqrt{x^2 + y^2 - 1}$.

1.10. $u = y\sqrt{1 - \cos x}$.

1.11. $u = \sqrt{\dfrac{x^2 + y^2 - x}{2x - x^2 - y^2}}$.

1.12. $u = \sqrt{\dfrac{x^2 + y^2 - y}{2y - x^2 - y^2}}$.

1.13. $u = \ln\left(1 - \dfrac{x^2}{9} - \dfrac{y^2}{16}\right)$.

1.14. $u = \ln(x + y)$.

1.15. $u = \sqrt{\ln(x^2 + y^2)}$.

1.16. $u = \lg\left(y^2 - 4x + 8\right)$.

1.17. $u = \dfrac{\sqrt{4x - y^2}}{\ln(1 - x^2 - y^2)}$.

1.18. $u = \arcsin\dfrac{x}{y}$.

1.19. $u = \arccos\dfrac{y}{x + y}$.

1.20. $u = \arcsin\dfrac{x - 1}{y}$.

1.21. $u = \arcsin\dfrac{x}{y^2} + \arccos(1 - y)$.

1.22. $u = \operatorname{ctg}[\pi(x + y)]$.

1.23. $u = \sqrt{\sin[\pi\,(x^2 + y^2)]}$.

1.24. $u = \lg x - \ln\cos y$.

2. Să se studieze existenţa limitelor:

2.1. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2 - y^2}{x^2 + y^2}$.

2.2. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x - y}{x + y}$.

2.3. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2y^2}{x^2y^2+(y-x)^2}$.

2.4. $\lim\limits_{\substack{x\to 0\\ y\to 0}} y\sin\dfrac{1}{x}$.

2.5. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2+y^2}{|x|+|y|}$.

2.6. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{2xy}{x^2+y^2}$.

2.7. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x}{x+y}$.

2.8. $\lim\limits_{\substack{x\to 3\\ y\to 0}} \dfrac{\tan xy}{y}$.

3. Să se calculeze:

3.1. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{xy}{2-\sqrt{xy+4}}$.

3.2. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{\sqrt{xy+1}-1}{2xy}$.

3.3. $\lim\limits_{\substack{x\to 0\\ y\to 2}} \dfrac{\sin xy}{x}$.

3.4. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^4y^2+x^2y^4}{1-\cos\left(x^2+y^2\right)}$.

3.5. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \left(1+x^2+y^2\right)^{\frac{2}{x^2+y^2}}$.

3.6. $\lim\limits_{\substack{x\to\infty\\ y\to\infty}} \left(x^2+y^2\right)\sin\dfrac{1}{x^2+y^2}$.

3.7. $\lim\limits_{\substack{x\to\infty\\ y\to 0}} \left(1+\dfrac{1}{x}\right)^{\frac{x^2}{x+y}}$.

3.8. $\lim\limits_{\substack{x\to\infty\\ y\to\infty}} \dfrac{x^2+y^2}{e^{x+y}}$.

3.9. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^3+y^3}{x^2+y^2}$.

3.10. $\lim\limits_{\substack{x\to\infty\\ y\to\infty}} \left(\dfrac{xy}{x^2+y^2}\right)^{y^2}$.

3.11. $\lim\limits_{\substack{x\to 1\\ y\to 0}} \dfrac{\ln^2(x+y)}{\sqrt{x^2+y^2-2x+1}}$.

3.12. $\lim\limits_{\substack{x\to\infty\\ y\to 3}} \left(1+\dfrac{1}{x}\right)^{\frac{2x^2}{x+y}}$.

4. Să se studieze continuitatea funcţiilor următoare în punctul $(0,0)$:

4.1. $f(x,y)=\begin{cases} \dfrac{xy}{\left(x^2+y^2\right)^2}, & x^2+y^2\neq 0,\\ 0, & x=y=0.\end{cases}$

4.2. $f(x,y)=\begin{cases} \dfrac{x-y}{(x+y)^3}, & x^2+y^2\neq 0,\\ 0, & x=y=0.\end{cases}$

4.3. $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & x^2+y^2\neq 0,\\ 0, & x=y=0.\end{cases}$

4.4. $f(x,y)=\begin{cases}xy^2\cdot\dfrac{x^2-y^2}{x^2+y^2}, & x^2+y^2\neq 0,\\ 0, & x=y=0.\end{cases}$

4.5. $f(x,y)=\begin{cases}(x^2+y^2)\ln(x^2+y^2), & x^2+y^2\neq 0,\\ 0, & x=y=0.\end{cases}$

4.6. $f(x,y)=\begin{cases}\dfrac{2x^2y}{x^4+3y^2}, & x^2+y^2\neq 0,\\ 0, & x=y=0.\end{cases}$

4.7. $f(x,y)=\begin{cases}\dfrac{\sqrt{x^2+y^2}}{\sin xy}, & x^2+y^2\neq 0,\\ 0, & x=y=0.\end{cases}$

4.8. $f(x,y)=\begin{cases}\sin\dfrac{1}{x^2+y^2}, & x^2+y^2\neq 0,\\ 3, & x=y=0.\end{cases}$

4.9. $f(x,y)=\begin{cases}3-x-y, & x^2+y^2\neq 0,\\ 5, & x=y=0.\end{cases}$

4.10. $f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x^4+y^2}, & x^2+y^2\neq 0,\\ 0, & x=y=0.\end{cases}$

5. Să se calculeze derivatele parţiale de primul ordin ale următoarelor funcţii:

5.1. $f(x,y)=x^2-2xy+y^2+1.$

5.2. $f(x,y)=x^3-3x^2y+2xy^2+y^3.$

5.3. $f(x,y)=\dfrac{xy}{y-x}.$

5.4. $f(x,y)=\dfrac{x-y}{x+y}.$

5.5. $f(x,y)=\dfrac{x}{y}$.

5.6. $f(x,y)=\operatorname{arctg}\dfrac{x}{y}$.

5.7. $f(x,y)=\ln(x^2+y^2)$.

5.8. $f(x,y)=x^2\cos y$.

5.9. $f(x,y)=e^{x^2y}$.

5.10. $f(x,y)=\ln\left(\sqrt{x}+\sqrt[3]{y}\right)$.

5.11. $f(x,y)=x^y$.

5.12. $f(x,y)=xy+\dfrac{y}{x}$.

5.13. $f(x,y)=ye^{-xy}$.

5.14. $f(x,y)=\dfrac{x}{y}+\dfrac{y}{x}$.

5.15. $f(x,y)=\ln\dfrac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}$.

5.16. $f(x,y)=\ln\left(y+\sqrt{x^2+y^2}\right)$.

5.17. $f(x,y)=\operatorname{arctg}\dfrac{x+y}{x-y}$.

5.18. $f(x,y)=\arcsin\dfrac{x+y}{xy}$.

5.19. $f(x,y)=\left(x^2+y^2\right)\operatorname{arctg}\dfrac{x}{y}$.

5.20. $f(x,y)=\arccos\dfrac{y}{\sqrt{x^2+y^2}}$.

5.21. $f(x,y)=\operatorname{arctg}\dfrac{x+y}{1-xy}$.

5.22. $f(x,y)=x^{y^2}$.

5.23. $f(x,y)=e^x\ln y+\sin y\ln x$.

5.24. $f(x,y)=\ln(x^2+y^2+3)$.

5.25. $f(x,y,z)=(\cos x)^{yz}$.

5.26. $f(x,y,z)=xy+yz+xz$.

5.27. $f(x,y,z)=\sqrt{x^2+y^2+z^2}$.

5.28. $f(x,y,z)=y^{\frac{x}{z}}$.

5.29. $f(x,y,z)=\ln(1+x+y^2+z^3)$.

5.30. $f(x,y,z)=\sin x\cos(yz)$.

6. Să se calculeze derivatele parţiale de ordinul doi pentru următoarele funcţii:

6.1. $f(x,y)=x^3+y^3-2x^2y+3xy^2$.

6.2. $f(x,y)=xy+\dfrac{y}{x}$.

6.3. $f(x,y)=x^4-x^3y+xy^2-y^4$.

6.4. $f(x,y)=\dfrac{x}{\sin y^2}$.

6.5. $f(x,y)=y\cos(x-y)$.

6.6. $f(x,y)=y^x$.

6.7. $f(x,y)=\operatorname{arctg}\dfrac{x+y}{1-xy}$.

6.8. $f(x,y)=\dfrac{x+y}{x-y}$.

6.9. $f(x,y)=\arccos(xy)$.

6.10. $f(x,y)=\ln(e^x+e^y)$.

6.11. $f(x,y) = \ln\left(x^2+y^2\right)$.

6.12. $f(x,y) = \sqrt[3]{x^2} + \sqrt[4]{y^3}$.

6.13. $f(x,y) = \operatorname{arctg}\dfrac{x+y}{y}$.

6.14. $f(x,y) = ye^x$.

6.15. $f(x,y) = e^y(\cos x + y\sin x)$.

6.16. $f(x,y) = \dfrac{y^2}{1-2x}$.

6.17. $f(x,y) = \arcsin\dfrac{y}{\sqrt{x^2+y^2}}$.

6.18. $f(x,y) = e^{x^2y}$.

6.19. $f(x,y) = \operatorname{arcctg}\dfrac{y}{x}$.

6.20. $f(x,y) = \sqrt{x^2+y^2}$.

6.21. $f(x,y) = y\ln\dfrac{x}{y}$.

6.22. $f(x,y) = e^{x^2+y}$.

6.23. $f(x,y) = \left(x^2+y^2\right)\operatorname{arctg}\dfrac{y}{x}$.

6.24. $f(x,y) = xe^y + ye^x$.

6.25. $f(x,y) = \operatorname{arcctg}\dfrac{x+y}{1-xy}$.

6.26. $f(x,y) = \arcsin\dfrac{x}{\sqrt{x^2+y^2}}$.

6.27. $f(x,y) = e^{\frac{x}{y}}\ln\dfrac{x}{y}$.

6.28. $f(x,y) = e^{\frac{x}{y}}\ln\dfrac{y}{x}$.

6.29. $f(x,y) = \arccos\dfrac{\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}$.

6.30. $f(x,y) = (\cos x)^{\sin y}$.

7. Să se arate că funcţiile următoare verifică relaţiile indicate, în ipoteza că ele sunt diferenţiabile de ordinul cerut de relaţiile respective:

7.1. $f(x,y) = e^x\cos y$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$.

7.2. $f(x,y) = \dfrac{xy}{x-y}$ verifică $\dfrac{\partial^2 f}{\partial x^2} + 2\dfrac{\partial^2 f}{\partial x\partial y} + \dfrac{\partial^2 f}{\partial y^2} = \dfrac{2}{x-y}$.

7.3. $f(x,y) = \ln\left(e^x+e^y\right)$ verifică $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} = 1$.

7.4. $f(x,y) = \ln\left(e^x+e^y\right)$ verifică $\dfrac{\partial^2 f}{\partial x^2}\cdot\dfrac{\partial^2 f}{\partial y^2} = \left(\dfrac{\partial^2 f}{\partial x\partial y}\right)^2$.

7.5. $f(x,y) = \ln\left(x^2+y^2\right)$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$.

7.6. $f(x,y) = e^x\,(x\cos y - y\sin y)$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0.$

7.7. $f(x,y) = \ln\left(x^2 + xy + y^2\right)$ verifică $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = 2.$

7.8. $f(x,y) = \ln\sqrt{(x-a)^2 + (y-b)^2}\ ,\ \{a,b\} \subset \mathbb{R}$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0.$

7.9. $f(x,y) = x^y y^x$ verifică $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = (x + y + \ln f(x,y))\, f(x,y).$

7.10. $f(x,y,z) = (x-y)(y-z)(z-x)$ verifică $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z} = 0.$

7.11. $f(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2} = 0.$

7.12. $f(x,y,z) = \dfrac{1}{x-y} + \dfrac{1}{y-z} + \dfrac{1}{z-x}$

verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2} + 2\left(\dfrac{\partial^2 f}{\partial x \partial y} + \dfrac{\partial^2 f}{\partial y \partial z} + \dfrac{\partial^2 f}{\partial z \partial x}\right) = 0.$

7.13. $f(x,y,z) = \ln\left(e^x + e^y + e^z\right)$ verifică $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z} = 1.$

7.14. $f(x,y,z,t) = \dfrac{x-y}{z-t} + \dfrac{t-x}{y-z}$ verifică $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z} + \dfrac{\partial f}{\partial t} = 0.$

8. Să se calculeze $\dfrac{\partial f}{\partial t}$, unde $f = f(x,y),\ x = \varphi(t),\ y = \psi(t)$:

8.1. $f(x,y) = x^2y^3,\quad x = t,\ y = t^2.$

8.2. $f(x,y) = x^2 - xy + y^2,\quad x = \cos t,\ y = \sin t.$

8.3. $f(x,y) = xy^2 - x^2y,\quad x = \sin t,\ y = \cos t.$

8.4. $f(x,y) = e^{xy}\ln(x+y),\quad x = 1-t^3,\ y = t^3.$

8.5. $f(x,y) = e^{x-2y},\quad x = \sin t,\ y = t^3.$

8.6. $f(x,y) = \ln(e^x + e^y),\quad x = t^2,\ y = 1-t^2.$

8.7. $f(x,y) = x^2 + xy + y^2,\quad x = t^3,\ y = t^2.$

8.8. $f(x,y) = e^{2\left(x^2-y^2\right)},\quad x = \cos t,\ y = \sin t.$

8.9. $f(x,y) = \ln\sin\dfrac{x}{\sqrt{y}},\quad x = 3t^2,\ y = \sqrt{t^2+1}.$

8.10. $f(x,y) = x^y,\quad x = \cos x,\ y = 2x.$

9. Să se calculeze $\dfrac{\partial f}{\partial x}$ şi $\dfrac{\partial f}{\partial y}$, dacă $f = f(u,v)$, $u = \varphi(x,y)$, $v = \psi(x,y)$:

9.1. $f(u,v) = u^2\ln v,\quad u = \dfrac{y}{x},\ v = x+2y.$

9.2. $f(u,v) = u^2 - v^2,\quad u = x\sin y,\ v = x\cos y.$

9.3. $f(u,v) = u^2 + \sqrt{uv},\quad u = x+y,\ v = \dfrac{x}{y}.$

9.4. $f(u,v) = \sqrt[3]{u} + \dfrac{1}{\cos v},\quad u = xy,\ v = x-y.$

9.5. $f(u,v) = uv\operatorname{arctg} uv,\quad u = t^3,\ v = t^2+1.$

9.6. $f(u,v) = u\sin v + v\cos u,\quad u = \dfrac{x}{y},\ v = xy.$

9.7. $f(u,v) = \operatorname{arctg}\dfrac{v}{u},\quad u = x\cos y,\ v = x\sin y.$

9.8. $f(u,v) = u^v,\quad u = y\sin x,\ v = x\cos y.$

9.9. $f(u,v) = u^2 + v^2, \quad u = \dfrac{2y}{x+y}, \ v = x^2 - 3y.$

9.10. $f(u,v) = \ln\left(u^2 + v^2 + 1\right), \quad u = \sin\dfrac{x}{y}, \ v = \sqrt{\dfrac{x}{y}}.$

10. Să se calculeze diferenţiala de ordinul I pentru funcţiile următoare:

10.1. $f(x,y) = x^3y^2 + xy^3 + 2.$

10.2. $f(x,y) = xye^{\frac{x}{y}}.$

10.3. $f(x,y) = x^2 + \sin 3y.$

10.4. $f(x,y) = \dfrac{x+y}{2x-3y}.$

10.5. $f(x,y) = \ln\left(x + y^2\right).$

10.6. $f(x,y) = \ln \operatorname{tg}\dfrac{x}{y}.$

10.7. $f(x,y) = x^2y + xy^3 + y^3.$

10.8. $f(x,y) = \sin x \cos y.$

10.9. $f(x,y) = x\sqrt{y} + \dfrac{y}{\sqrt{x}}.$

10.10. $f(x,y) = \left(x^2 + y^2\right)^5.$

10.11. $f(x,y) = \dfrac{y^2}{x^3}.$

10.12. $f(x,y) = e^{x^2+y^2}.$

10.13. $f(x,y) = \cos 2x + \sin 2x.$

10.14. $f(x,y) = y\cos x^2 + x \sin y^2.$

10.15. $f(x,y) = x^2 + y^2 + \sin xy.$

10.16. $f(x,y) = \sqrt[3]{x^2 + y^2}.$

10.17. $f(x,y,z) = xyz.$

10.18. $f(x,y,z) = x^{y^z}.$

10.19. $f(x,y,z) = \sin(x + y + z).$

10.20. $f(x,y,z) = \arcsin\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}.$

11. Să se scrie diferenţialele de ordinul II pentru funcţiile:

11.1. $f(x,y) = x^3 - x^2y + 2y^3 + 3x - 2y + 5.$

11.2. $f(x,y) = e^{xy}.$

11.3. $f(x,y) = 5x^2y + 3xy + y^2 + 3$.

11.4. $f(x,y) = \dfrac{x}{y}e^{xy}$.

11.5. $f(x,y) = \sqrt{1 + 2xy + y^2}$.

11.6. $f(x,y) = e^y \sin x$.

11.7. $f(x,y) = \ln\left(x^2 + y\right)$.

11.8. $f(x,y) = \left(x^3 + y^2\right)^2$.

11.9. $f(x,y) = x^2 + y^2 + \cos xy$.

11.10. $f(x,y) = \dfrac{1}{\sqrt[3]{x^2 + y^2}}$.

11.11. $f(x,y) = \dfrac{x}{y} - \dfrac{y}{x}$.

11.12. $f(x,y) = y \ln \dfrac{x}{y}$.

11.13. $f(x,y) = \operatorname{arcctg} \dfrac{y}{x+y}$.

11.14. $f(x,y) = e^x \operatorname{tg} y$.

11.15. $f(x,y) = \arcsin \dfrac{x}{\sqrt{x^2 + y^2}}$.

11.16. $f(x,y) = e^{xy^2}$.

11.17. $f(x,y) = xe^y + ye^x$.

11.18. $f(x,y) = (\sin x)^{\cos y}$.

11.19. $f(x,y) = \sqrt[3]{x^4} + \sqrt{y^3}$.

11.20. $f(x,y) = \left(x^2 + y^2\right) \operatorname{arctg} \dfrac{x}{y}$.

12. Utilizând diferenţiala, să se calculeze cu aproximaţie:

12.1. $\sqrt{1,01^3 + 1,98^3}$.

12.2. $(3,01)^{2,03}$.

12.3. $\sqrt[3]{(5,02)^2 + (1,41)^2}$.

12.4. $(2,02)^{3,01}$.

12.5. $\sin 29^\circ \cos 62^\circ$.

12.6. $\sin 31^\circ \operatorname{tg} 46^\circ$.

12.7. $\operatorname{arctg}\left(\dfrac{1,98}{1,03} - 1\right)$.

12.8. $\operatorname{arcctg}\left(\dfrac{1,97}{1,01} - 1\right)$.

12.9. $\ln\left(\sqrt[3]{1,02} + \sqrt[4]{0,98} - 1\right)$.

12.10. $\dfrac{1,02^{3,01}}{\sqrt[3]{0,99\sqrt[4]{1,03^5}}}$.

13. Să se scrie formula Taylor (până la termenii de gradul III inclusiv) corespunzătoare următoarelor funcţii în punctele indicate:

13.1. $f(x,y) = xy^3 + 2xy - 2x^2 + 3x + y - 2, \quad (-1,2).$

13.2. $f(x,y) = x^3 - 3xy^2 + y^3 + 2x - 3y + 1, \quad (1,2).$

13.3. $f(x,y) = x^3 - 5x^2 - xy + y^2 + 10x + 5y + 10, \quad (1,-1).$

13.4. $f(x,y) = \sqrt[3]{x+y}, \quad (0,1).$

13.5. $f(x,y) = \ln(1+x+y), \quad (1,0).$

13.6. $f(x,y) = e^x \sin y, \quad \left(0, \frac{\pi}{2}\right).$

13.7. $f(x,y) = e^{2y} \ln(1+x), \quad (0,0).$

13.8. $f(x,y) = x^y, \quad (1,1).$

13.9. $f(x,y) = e^y \cos x, \quad (0,\pi).$

13.10. $f(x,y) = \ln(1+x)\ln(1+y), \quad (1,1).$

14. Să se determine valorile maxime şi minime ale următoarelor funcţii $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$:

14.1. $f(x,y) = x^3 + y^3 - 9xy + 18.$

14.2. $f(x,y) = x^4 + y^4 - 4xy + 2.$

14.3. $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 2.$

14.4. $f(x,y) = -x^2 - xy - y^2 + x + y.$

14.5. $f(x,y) = x^2 + xy + y^2 + \dfrac{1}{x} + \dfrac{1}{y}.$

14.6. $f(x,y) = x^3 + y^3 - 6xy.$

14.7. $f(x,y) = 3x^2 - x^3 + 3y^2 + 4y.$

14.8. $f(x,y) = x^3 + 3xy^2 - 15x - 12y + 8.$

14.9. $f(x,y) = \left(2x^2 + y^2\right) e^{-\left(x^2+y^2\right)}.$

14.10. $f(x,y) = 3 - \sqrt[3]{x^2 + y^2}.$

14.11. $f(x,y) = xy + \dfrac{20}{x} + \dfrac{20}{y}.$

14.12. $f(x,y) = x^2 y e^{y-x}.$

14.13. $f(x,y) = 1 - \sqrt{x^2 + y^2}.$

14.14. $f(x,y) = \dfrac{x+y}{\sqrt{x^2 + y^2 + 1}}.$

14.15. $f(x,y) = x + y + 4\sin x \sin y.$

14.16. $f(x,y) = ye^{x + y\sin x}.$

14.17. $f(x,y) = x^3 + y^2 - 3x + 4\sqrt{y^5}.$

14.18. $f(x,y) = x\sqrt{y} - x^2 - y + 6x + 1.$

14.19. $f(x,y) = \left(x + y^2\right)\sqrt{e^x}.$

14.20. $f(x,y) = (x-y)^2 + (x-1)^3.$

15. Să se determine extremele condiţionate ale funcţiilor $f(x,y)$ cu legătura $F(x,y) = 0$:

15.1. $f(x,y) = xy, \qquad F(x,y) = x^2 + y^2 - 1.$

15.2. $f(x,y) = \cos 2x + \cos 2y, \qquad F(x,y) = x - y - \dfrac{\pi}{4}.$

15.3. $f(x,y) = xy, \qquad F(x,y) = x + y - 1.$

15.4. $f(x,y) = x + 2y, \qquad F(x,y) = x^2 + y^2 - 5.$

15.5. $f(x,y) = x^2 + y^2 - xy + x + y - 4, \qquad F(x,y) = x + y + 3.$

15.6. $f(x,y) = xy, \qquad F(x,y) = x^3 + y^3 - xy.$

15.7. $f(x,y) = e^{xy}, \qquad F(x,y) = x + y - 1.$

15.8. $f(x,y) = x - y - 4, \qquad F(x,y) = x^2 + y^2 - 1.$

15.9. $f(x,y) = x^2 y, \qquad F(x,y) = 2x + y - 1.$

15.10. $f(x,y) = \dfrac{x}{2} + \dfrac{y}{3}, \qquad F(x,y) = x^2 + y^2 - 1.$

16. Să se determine extremele globale ale funcţiilor pe domeniile D:

16.1. $f(x,y) = x^2 - y^2 + 2, \quad D(x,y) = \left\{(x,y) \,\middle|\, x^2 + y^2 \leqslant 1\right\}.$

16.2. $f(x,y) = x^3 + y^3 - 9xy + 27, \quad D(x,y) = \{(x,y) \,|\, 0 \leqslant x \leqslant 4,\ 0 \leqslant y \leqslant 4\}.$

16.3. $f(x,y) = x^3 + y^3 - 3xy, \quad D(x,y) = \{(x,y) \,|\, 0 \leqslant x \leqslant 2,\ -1 \leqslant y \leqslant 2\}.$

16.4. $f(x,y) = 2x - y + 3, \quad D(x,y) = \{(x,y) \,|\, x \geqslant 0,\ y \geqslant 0,\ x + y \leqslant 2\}.$

16.5. $f(x,y) = x - 2y + 5, \quad D(x,y) = \{(x,y) \,|\, x \leqslant 0,\ y \geqslant 0,\ y - x \leqslant 1\}.$

16.6. $f(x,y) = x^2 + y^2 - xy - x - y, \quad D(x,y) = \{(x,y) \,|\, x \geqslant 0,\ y \geqslant 0,\ x + y \leqslant 3\}.$

16.7. $f(x,y) = 2xy, \quad D(x,y) = \left\{(x,y) \,\middle|\, x^2 + y^2 \leqslant 4\right\}.$

16.8. $f(x,y) = x^2 y, \quad D(x,y) = \left\{(x,y) \,\middle|\, x^2 + y^2 \leqslant 1\right\}.$

16.9. $f(x,y) = x^3 + 4x^2 + y^2 - 2xy$, $D -$ domeniul închis, mărginit de curbele $y = x^2,\ y = 4.$

16.10. $f(x,y) = xy(4 - x - y), \quad D(x,y) = \{(x,y) \,|\, x \geqslant 0,\ y \geqslant 0,\ x + y \leqslant 6\}.$

Bibliografie

[1] Кудрявцев Л.Д., Кутасов А.Д., Чехлов В.И., Шабунин М.И. Сборник задач по математическому анализу. Т. 1, 2, 3. – Москва: Физматлит, 2003.

[2] Демидович Б.П. Сборник задач и упражнений по математическому анализу. – Москва: Наука, 1977.

[3] Сборник задач по математике для втузов. / Под ред. А.В. Ефимова и Б.П. Демидовича. Т. 1, 2, 3. – Москва: Наука, 1984.

[4] Данко П.Е., Попов А.Г., Кожевникова Т.Я. Высшая математика в упражнениях и задачах. Т. 1, 2. – Москва: Высшая школа, 1997.

[5] Дороговцев А.Я. Математический анализ. Сборник задач. – Киев: Вища школа, 1987.

[6] Radomir I., Fulga A. Analiza Matematică. Culegere de probleme. – Cluj-Napoca: Casa de Editură Albastră, 2000.

[7] Petrică I., Constantinescu E., Petre D. Probleme la analiză matematică. Vol. 1, 2. – Bucureşti: Petrion, 1997.

[8] Popa C., Hiris V., Megan M. Introducere în analiza matematică prin exerciţii şi probleme. – Timişoara: Ed. Facla, 1976.

[9] Donciu N., Flondor D. Algebră şi analiză matematică. – Bucureşti: Ed. ALL, 1993.