

## J. Piaget's theory of intelligence: operational aspect

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*“In the narrow sense, AI is concerned with extending the capacity of machines to perform functions that would be considered intelligent if performed by people. Its goal is to construct machines and, in doing so, it can be thought of as a branch of advanced engineering. But in order to construct such machines, it is usually necessary to reflect not only on the nature of machines but on the nature of the intelligent functions to be performed.”*

*Seymour Papert*

### **Abstract**

The Piaget's theory of intelligence is considered from the point of view of genesis and gradual development of human thinking operations. Attention is given to operational aspects of cognitive structures and knowledge. The significance of the Piaget's theory of intelligence is revealed for modeling conceptual reasoning in the framework of artificial intelligence.

*Keywords: intelligence, thinking operations, cognitive structures, artificial intelligence.*

## **1 Introduction**

The key problem of intelligence (both natural and artificial one) is knowledge construction and reasoning. The traditional methodology of artificial intelligence (AI) does not consider computer as an active source of knowledge. Cognitive structures are installed in computer by

copying directly the experts' knowledge. The concept of "scheme" in AI means a graphical image or graph reflecting some known relationships between the units of knowledge. Computer cognitive structures are declarative, mechanisms of their using are separated from them and, as a rule, these mechanisms are often fixed. In this sense, one can say that an "apriorism" reigns over AI. The source of knowledge is an expert; knowledge is alienated from him (or her) and entered in computer; the "if-then" rules are the final results of the expert's mental activity. Reasoning is reduced to only seeking and using necessary rules. The "apriorism" of cognitive structures ignores their active nature, namely, the mechanisms of their constructions. However it is clear intuitively that the process of reasoning must include not only cognitive structures themselves but also the methods of their creating and recreating. In the process of reasoning there could arise the necessity to correct knowledge due to impossibility to explain new facts through the known ones or to reduce a new task to some already familiar sub-tasks. Hence it becomes clear that the process of using knowledge and the process of its creating can not be nothing but two different aspects of a single process. We believe that creating knowledge and reasoning (using) knowledge can not be alienated from one another; they are the identical activities of brain: it is recognized only what is constructed.

Partly the "apriorism" in AI is a consequence of those psychological theories which consider judgement as a received or rejected relationship or a state of thought (the works of Buhler, K. and Selz, O. criticized by Piaget [1]) In these theories, thought appears in the form of the consciousness of a relationship (for example,  $A < B$ ) or in the form of the consciousness of rules "if-then" or in the form of clear formal intention. The psychology of intelligence in AI goes along the way of analyzing the final stages or states of cognitive structures. Operations of thinking as a result of which there appear objects, properties, classes, relations remain to be unused, unclaimed, not called for.

In the 70-80 years, the foundation of knowledge engineering was being created, that's why the attention of the investigators was given firstly to the question how human knowledge could be organized and used in computer intelligent systems. However it is clear that to know

what is the structure of knowledge is insufficient. It is not less important to understand the operational aspect of knowledge, i.e. the mechanisms by the use of which knowledge is generated. We consider it to be very interesting to turn to the operational theory of intelligence which assumes the thought as an interiorized mental action and does not separate thinking from noninteriorized acts in the real world. This theory was created by Piaget J. and his disciples on the base of the long-term experimental investigations of the genesis of intellectual operations in child's mind from the moment of birth until the period when thought takes its perfect form of logical formal operations and cognitive structures take the form of "grouping" [1].

Our primary goal for addressing ourselves to the works of Piaget was to find a foundation for our computer model of conceptual type of data and conceptual reasoning based on the theory of lattices. But we believe that the piagetian theory of intelligence has the enormous methodological significance for solving many key problems of AI. It can be noted that nowadays we can surely say that the theory of Piaget has stood the test on "solidity", and for the proof of it we refer to the fact that the main assertions of Piaget are recognized by the psychologists engaged in cross-cultural investigation [3].

## 2 The nature of intelligence

Piaget believed the intelligence to be a mechanism (both on biological and on cognitive level) by the use of which an organism adapts to the environment, i.e. creates such the schemata of his activity or behavior that allow him to be in an equilibrium with his environment. This equilibrium is a dynamic one, it is achieved by means of two global processes appearing in the different forms depending on the sphere of intellectual activity: assimilation and accommodation.

Assimilation consists in treating the output stimuli by the use of already created schemes of behavior without their changing (because stimuli are familiar to or recognizable by an organism). But when the process of recognizing stimuli fails due to the insolvency of available schemata then the accommodation process comes forward (appears on

the stage). Accommodation consists in changing schemata or inventing new ones to assimilate new stimuli.

Scheme is one of the main concepts of Piaget - it is a cognitive structure (or mental structure) by which the individual intellectually adapts to and organizes his environment. The organization of behavior (sensory-motor, speech, cognitive and so forth) consists in constructing schemata of behavior. Scheme, according to Piaget, is absolutely not identical to the scheme of relation. Scheme is a dynamical union of a relation and operations by means of which this relation is constructed. Scheme determines the possibility of combining operations thus permitting to construct and reconstruct the typical sequences of operations in the typical situations.

Scheme, in any sphere of behavior, tends to the conservation of the organism's identity and thereby to the conservation of the organism's representation of the environment. For example, the sensory-motor intelligence constructs the schemata of real world objects: the individual can do many different operations with objects - he (or she) can hide, throw, turn over an object. But with all these manipulations, an object remains identical to itself conserving its color, shape, size and so on. The sensory-motor operations provide the flexibility for the motor behavior, for example, one can get an object by different ways, can come to it from different places in space. The sensory-motor operations constitute a group which is invariant with respect to the set of all real objects, but any object is also invariant with respect to the group of sensory-motor operations. Piaget believed the mind to have structures much in the same way that the body does. Thus concepts as cognitive structures are invariant with respect to reasoning operations (for example, the multiplication and the addition of concepts (classes)): the different expressions of natural language can correspond to the same concept (for example, expressions "a river on the bench of which a bison has been killed" and "a river in Australia beginning in  $N$  and having the length of 3000 km" could refer to the same river. At the same time, the operations of reasoning are invariant with respect to all concepts a person operates on.

In the sense of the adaptive nature of intelligence we can give the

following definition of reasoning: it is a mental activity which is expressed by means of language and tends to form in mind cognitive structures and to maintain (to conform) them in the equilibrium during the constant interaction with people and objects in reality.

Piaget asserted the existence of balance between the processes of assimilation and accommodation. Probably there is a criteria which does not allow one of these two processes to prevail over the other.

In sum, intelligence, according to Piaget, is not an ability. It is a form of adaptation, leading to an equilibrium to which tend all the interactions between organism and his environment beginning with the set of biological, physiological, sensory-motor adaptations and ending with the highest form of adaptation - thinking. This continuous row of adaptations must be viewed to be constructed during the evolution process and by means of the laws of evolution. Each form of adaptation provides a more stable and widely spreading equilibrium. In the Piaget's theory, the major source of motivation for intellectual development is disequilibrium which can be thought of as "cognitive conflict" when expectations or predictions are not confirmed by experience.

There exists the problem of determining the lowest boundary of intelligence. The numerous investigations move more and more this boundary away from human to animals. For example, it is known nowadays that some birds and primates possess a rudimentary understanding of number. Chimpanzee Sara known due to the works of David Primack [3] was able to numbering objects from one to four and to show an object with the correct number. She has learned to establish one-to-one correspondence between two different sets of objects. However she failed to solve an analogous task but formulated with new stimuli, hence her ability was turned out to be firmly incorporated in the context. Sara Boise, S.T. has shown [3] that when training the actions with numbers is incorporated in the life rich with the interpersonal patterns of interactions and when training arises from the prior established relations based on games then chimpanzee is found to be able not only to understand one-to-one correspondence but she can also learn to add numbers and even to solve the arithmetical tasks with which children of 3 years old have success.

The question arises of determining the peculiarity of the highest form of intelligence – human intelligence (maybe, a particular case of intelligence or intermediate stage of it). Each new form of behavior is connected with the new forms of assimilation and accommodation. People change objects in the reality and create the new material and mental objects (artifacts [3]), using them as a tool for regulating their interactions with the world. Broadly speaking, the artifacts are not only instruments of labour but also they are representations, concepts, operational schemata, i.e. all mental constructions invented and created by people and, without which it would be impossible to create the new objects in the reality. Conversely, the mental activity would be impossible without practical actions.

Creating artifacts is the manifestation of the activity of knowledge, the activity of intelligence. It is important that the development of intelligence proceeds in the social environment, i.e. through the interaction between the persons. Due to the cultural interactions, the artifacts can be improved and assimilated by the individuals. What is invented by one person is not viable until the novation will be communicated to, mastered and assimilated by the others.

Individuals like societies, cultures are active subjects of their development, although their behavior is not fully determined by their own choice in their environment. However people have one principle characteristic distinguishing them from animals – they conceive their mental operations and they are able actively to use, control and improve them (operations) making them more and more perfect. Thereby an individual (like a society) can change himself, he is able to learn and to invent new mental functions using mental operations that have been already assimilated by him. In sum, we can conclude that the highest form of intelligence demonstrates ever more and more increasing changes of the environment in the enormous and more and more increasing scope in time and in space. The communicative structures in the human society develop continuously in their intensity and in their content, they develop in time as well as in space. The possibilities to move, to memorize increase too; the capability of thinking can be improved and increased to a large extent by means of computer [4]. But at the same time, the

highest forms of intelligence are provided with (supported by) all the lowest forms of intelligence.

### 3 The development stage of intelligence

In the piagetian theory of intelligence, the stages of intellectual development play an important role. Piaget defined four main stages [7]:

- 1) sensory-motor stage, i.e. direct action upon environment,
- 2) preoperational/prelogical period during which cognitive behavior is still influenced by perceptual activity. Actions are internalized via representations but thought is still tied to perception; during this stage there proceeds the development of speech and of the ability to operate symbols and signs),
- 3) the stage of operations (logical thought), i.e. in this stage child is capable to reason in a way that is not dependent on immediate perceptual and motor actions (the stage of concrete operations),
- 4) the stage of logical formal operations (complete logical thought independent on context). During this stage operational grouping becomes invariant with respect to concrete objects of reasoning and they can be transferred into new context (in what monkeys fail).

We call the last capability unification. The fourth stage appears due to the awareness by an individual his (or her) mental operations which he (or she) used during the prior stage of concrete mental operations. Once realizing the fact that an operation executed or carried on a pair of objects can be carried on any pair of analogous objects or on any subset of objects with the same properties an individual can be ready to release operations from a context, to control them and to consider them as subjects of mathematical, philosophical, aesthetic reflections.

The stages of intellectual development (it is better to say the spheres) are closely tight. All intellectual capabilities develop consecutively and in parallel with one another, although some of them can

dominate the others at a certain stage. During the sensory-motor stage when the cognitive structures of object, time and space are formed, in parallel with them, the concepts of causality and of goal appear too. Child begins to understand the causal links between an object and its properties as well as between the properties of object and the action he (or she) can make with it.

A child begins to act deliberately using effectively the motions which have been already assimilated by him. In the very beginning of the second year of child's life there appears verbal pointing [3]. E. Bates [3] informs the following observations of a girl of age of 13 months: K. sits in a corridor before the kitchen's door. She looks at the mother and calls her by means of giving a sound of the very high tone "Ha". The mother comes to her. K. continues to look in the direction of the kitchen stretching the shoulders and the high part of her body in this direction. The mother carries her in the kitchen and K. shows her the sink, the mother gives her a glass of water and the girl drinks it with pleasure.

Children begin to understand that people unlike things (inanimate objects) act deliberately: they do not try to look in the direction in which the doll sees. According to Tomasello M. [3], the appearance of the ability to think of the other people as acting deliberately plays an important role in the development of imitative forms of behavior.

Communicative function becomes more complicated as well. For example, if a toy-car unexpectedly has gone upon the floor then a child of age of one year, as a rule, shows the toy-car and after that he will inspect whether the mother sees the toy-car too. About 18-months a child, as a rule, firstly looks at the mother in order to be sure that she sees the toy and after that he shows the toy-car. If a child of this age is alone in the room then he does not point at the car until somebody of the adults enters the room.

The achievement of the higher level of intellectual development does not mean stopping the development of the abilities appeared at the prior stages. On the contrary, these abilities continue to develop. For example, the motor activity becomes more sophisticated. The complex movement such as slalom, juggling, acrobatics, windsurfing could not



occur at the early stage of intellectual development since they request more developed logical operations. Complex movements are learnable and in the learning process, there must be engaged memory, speech, feelings, muscles, visual images coordinated with one another. For example, Ephimov, L.F. [10] investigating the self-reportage of slalom racers in the periods they analyze new route before the competition informs us: slalom racer several times goes along the route by foot, explores the state of snow, steepness of slope, the distance between the obstacles; he determines the speed he can (or need to) achieve at each part of the route, he tries to conserve in mind the route in detail so that to be able to recreate mentally his future movement along the route as a whole. The analogous examples one can find in [8]. Deliberate behavior at the sensory-motor stage is connected with the development of anticipation mechanisms. Anticipation permits to coordinate the actions of an individual with those of the other people.

The parallelism, the development “stage by stage” and the coordination of intellectual actions can partly account for the fact that some relationships and operations formed at the preliminary stages turn out later to be not discerned. For example the adults do not aware the links between object (class) and its properties as the causal relationships.

Modeling cognitive structures in AI presupposes that all their necessary elements (images, representations of objects, concepts, relationships of time and space and so forth) are already available in their final forms ready to using. But we believe modeling intellectual structures to be impossible without taking into account not only the mechanisms of their forming and functioning but also the mechanisms of their gradual development (genetic mechanisms) from their simplest forms up to the more complicated ones.

Knowing the genetic mechanisms allows us to understand the relations of the succession between intellectual structures (or/and operations) at the different stages of their development. For example, pattern recognition surely passes the way from the very simple assimilating forms of “grasping” the similarity between objects or stimuli up to the more complicated forms such as determining the logical identity of concepts or the unification of complex cognitive constructions. We

will have to answer the questions how the more complicated cognitive structures interact with the less complicated but more ancient ones.

## 4 The operationality of intelligence

Any relation has two aspects: declarative and productive ones. How could one assert that  $7 > 2$ ? For this goal we need in an operation that deletes two units from the set of seven units and discovers that the number of units remaining after deleting two units is not equal to zero. In much the same way we prove that "class  $A$  is a subset of class  $B$ " ( $A$  is a  $B$  or  $A \leq B$ ), where  $A, B$  are the names of some classes (concepts). This relation has the following interpretation: " $A$  is a  $B$ " is satisfied if and only if the set  $I(A)$  of all conceivable objects of the class  $A$  is included in to the set  $I(B)$  of all conceivable objects of the class  $B$ . All operations upon concepts are interpreted in the set of all objects (conceivable or real). For example, multiplying two concepts  $A * B$  means finding the largest concept  $C$  which simultaneously belongs to concept  $A$  and to concept  $B$ , i.e.  $I(C) = I(A) \cap I(B)$ ,  $I(C) \subseteq I(A)$ ,  $I(C) \subseteq I(B)$ . The relation " $A$  is a  $B$ " is true if and only if  $I(A) \cap I(B) = I(A)$  is satisfied. For example, FATHER is simultaneously "a person who has a child" and "a person who is a man". The intersection of the sets of "all conceivable parents" and "all conceivable men" gives us with the necessity the set of all conceivable fathers and only this set.

## 5 The concept of grouping

The declarative part of "is a" relation is an expression constructed from the names of concepts by means of the signs of operations or relationships (maybe, syntactic rules). Expressions consist of words, for example, "mammal is an animal". The productive part of this relation is a mental action performed with conceivable sets of objects. This action is acquired from experience, firstly by manipulating with real objects, and later with their representations.

Grouping, according to Piaget, corresponds to algebraic structure

in mathematics. Piaget defined the laws of grouping as follows:

- $A + B = C, B + A = C$  – commutative law,
- $A = C - B, B = C - A$  – reversibility,
- $(A + B) + C = A + (B + C)$  – associative law,
- $A - A = 0$  – identity,
- $A + A = A$  – tautology.

An example of additive grouping is: the set of classes with addition operation. Let  $A, B, C$  be classes (concepts) and  $a, b, c$  – be the properties of classes. A logical class is the union of objects having the same common property. So, class  $A$ , for example, is the set of beads, defined by their brown color “a”, the class  $B$  is the set of beads “not a”, namely the set of beads defined by their white color “b”. To add classes  $A + B = C$  means to define the least class of beads including two classes  $A$  and  $B$  where  $C$  is defined by the common property of classes  $A$  and  $B$ , i.e., in a given example, by the property  $c$  – wooden beads.

It is insufficient to have only addition operation to deal with classes. How could one form the set of objects belonging at the same time to different classes, for example, “water transport”, “mountain landscape”, throat-microphone, “snow-slip”, “tragicomedy” and so forth. We need in multiplication operation. To multiply two classes  $A, B$  is to find the largest class containing objects belonging at the same time to both of these classes, i.e. the result of multiplication operation is the set of objects which possess (union) the properties of the class  $A$  and those of the class  $B$ .  $A * B = C$ . For example, “water transport” is the union of the properties “to float (sail)” and “to be a transport”, i.e.  $c = a \cup b$ .

One of the important aspects of mental operations is their reversibility. Addition operation has subtraction as its reverse operation. ( $A = C - B$ ). Reverse operation with respect to multiplication operation is division ( $A = C \div B$ ). If subtraction is easy to understand (it

is dissociation of class) then for division operation it is not the case. Consider the meaning of division operation. For example, a child saw a fox at the picture but he said that it is a dog. According to a child, a dog and a fox are very similar. However an adult does not agree with a child, he begins to explain: it is not a dog, it does not bike, a fox is wild, it lives in the forest, steals hens, a dog does not do this, it lives at home with people, it guards hens, eats meals of people and so on. Division operation is necessary for dividing two concepts. Let's the concept  $Z$  be equal to  $DOG + FOX$ ,  $z$  be the property common for dog and fox. To divide concepts is to find a property  $y$  such that the union of  $y$  and  $z$  results in the property  $c = y \cup z$  which corresponds (belongs) only to the set of dogs and only to this set:  $I(c) = I(DOG)$ .

The reversibility of operations, according to Piaget, occurs due to a person becomes aware of his own mental operations. Piaget called social interaction one of the variables that facilitates cognitive development and leads to the appearance of logical reversible operations. Coming to look at some thing from another's point of view, questioning one's reasoning and seeking validations from others are all essential acts of accommodation which imply the transition from concrete non-reversible mental operations to mobile and reversible ones.

The operations on concepts have a complex structure. They are the operations with names of classes (concepts): water transport, a child of cow. With the use of these operations, we construct the expressions which can be interpreted on the set of all conceivable objects. For example, the class of "flying frogs" is formed by multiplying two classes: the class of flying objects and the class of frogs. The word "flying" presupposes that the mental generalization had place in the past by acquiring in mind the class of all flying objects. It is important that expressions of natural language are interpreted by means of logical mental constructions which, in their turn, are interpreted by means of mental operations on conceivable (thinkable) sets of objects.

The conceivable sets of objects are formed with the use of representations of objects. What is a representation? We believe that it is necessary to distinguish "representation of object" from "image of object" or "set of objects". Representation contains the knowledge of the

whole set of real objects belonging to some class of them. For example, to know what a cow is means to know that it can be black, white or brown, that it can be multicolored, that it can have the horns and so forth. Each time, by extracting some of the cow's features, one can call in mind for an image of real animal. Representation serves as some generic program that generates an image of concrete object using some given features. Concepts are linked with real objects through representations of objects. It is possible that representation is an intermediate stage on the way to the limit, i.e. to the concept, expressed with the help of a word. We believe that a concept exists because of the transition proceeds from a potentially infinite set of all real objects of a given class to the value of its limit, i.e. to its name. This transition is almost done in representation of object but it is easy to go back from the representation to an image of object. It is easy to differentiate concepts with the use of their representations choosing appropriate features for this differentiation.

Usually in AI, knowledge are separated from data. Object-oriented data bases and object-oriented knowledge bases are divorced from one another. It is clear that the both types of systems must interact since data determines the knowledge and verifies it and, in its turn, the knowledge organizes data. In [11] we tried to analyze the problem of data-knowledge transformation and interaction.

## 6 Coordination of intelligent operations

All intellectual operations are coordinated with one another. This coordination ensures the integrity of an organism. We will give some examples of coordinated intellectual operations. The description of visual-motor coordinated operations are given in [6]. Children of the age of 3-5 years have been asked to draw the direct line on the list of paper and to stop drawing in the moment when a certain vertical line can be achieved. Small children (of 3 years old) fail to do this: vision and hand's movement have been not coordinated in this age. Children of 5-6 years old can do the task and also demonstrate the anticipating behavior. In [7] it is shown that if one's hand is moving then one's

eyes always repeat the same way the hand passes but the vision passes ahead the hand in order to inform the hands about the location of the goal of hand's movement. The eyes and the hand cooperate as "the agents" coordinating their actions.

We will consider the coordination of classifying operations. The following components take part in classification process:

- 1) the set of class's names;
- 2) the set of conceivable objects;
- 3) the set of object's properties.

Conceivable objects are assumed to be connected with their properties by means of causal links. The links between conceivable objects and the names of their classes (concepts) and between the names of classes and the appropriate properties are of the analogous nature. Classes and properties are not distinguished with respect to the nature of their links with objects. The only difference between them is that the class is artifact (constructed by people) but property is an observable physical phenomenon. Classes, being artifacts, are determined by their properties. Each class (or property), through its name, is associated with the unique set of conceivable objects, we say, with its interpretation as follows: the interpretation of class (property) with the name  $A$  is the set of conceivable objects which belong to this class, i.e.

$$A \rightarrow I(A) = o : o \leq A.$$

The operations of addition and multiplication are defined on the set of classes. These operations are interpreted by means of the set-theoretical operations (intersection and union) defined on the set of all subsets of conceivable objects.

Each conceivable object is associated uniquely with the set  $C(o)$  of its properties,  $C(o) = c : o \leq c$ ,  $o \rightarrow C(o)$ . Let  $C(o)$  be the description of the object  $o$ . We define two operations on the set of all descriptions of all conceivable objects – the generalization and the combining of descriptions. The first of them defines for a pair of descriptions their

maximal common part (common description or common property), the second operation defines for a pair of descriptions the minimal description which combines these two descriptions:

$$C(o_i) \cap C(o_j) = C_x, \quad C(o_i) \cup C(o_j) = C_y.$$

The coordination of classifying operations means that the operations on class's names, the operations on conceivable objects and the operations on object's descriptions are performed simultaneously and they are in agreement with one another. For the completeness of operation's definition, we will consider the cases of empty interpretation and of empty description. It is possible that the result of multiplication is a class with empty interpretation. In this case the description of this class is said to be contradictory and to be equal to the special symbol  $F$  – the forbidden description. Also, it is possible that the result of addition is a class with empty description. It means that the objects of the class obtained have no common property. In this case the description of this class is said to be the most common description and to be equal to the special symbol  $G$ .

The coordinated classifying operations generate some set of logical assertions which can be understood if the classification is performed as the system of coordinated operations. First of all the classification is connected with understanding the operations of quantification: “not all  $C$  is  $A$ ”, “all  $B$  are some  $C$ ”, “all  $A$  are some  $C$ ”, “some  $C$  are  $B$ ” and so on (in the case  $C = A + B$ ). The violation of the coordination of classifying operations implies the violation of the truth of reasoning. Piaget has shown that 1) operational reasoning is a result of gradual development of a person, 2) appearing formal operations is connected with spontaneous appearing the ability to coordinate mental operations, 3) a key problem of the development of operational classification in mind is the problem of understanding the inclusion relation. If understanding the inclusion relation is not achieved by a person then it is impossible understanding by him both classifying and quantifying operations.

Piaget and Inhelder have created the special tests in order to study the development of classifying operations for small children of different

age. The results of their investigations are informed in [2]. We will consider some examples. A child is presented by the collection of three red squares, two blue squares and three blue circles. A typical example of the answers of a small child to the questions of an experimenter is the following:

Q.: Are all the red ones squares?

A.: No, because some are blue too.

Q.: Are all the blue ones circles?

A.: Yes.

Q.: Are all the circles blue?

A.: Yes.

Q.: Are all the squares red?

A.: Yes.

The questions were of two forms: 1) are all elements of  $A$  have the property of the whole  $B$ ? 2) are all elements of the whole  $B$  have the property of the part  $A$ ? The question in the form 1) turned out to be more difficult than that in the forms 2). The children's mistakes were connected with their transformation of the question "are all  $X$  some  $Y$ " to the question in the form "are all  $X$  all  $Y$ ?" Thus the small children frequently were referring "all" to the entire collection, in an absolute sense, instead of focusing on the sub-collections.

The problem of forming operational thinking draws much pedagogical attention. How to achieve that a child begins to understand classifying operations or the relation of transitivity? Any grouping is a system of coordinated operations, but in the case of the classification we deal with three such groupings. The understanding can not be partial. It is a difficulty. But if a child realizes the mental coordinated operations then a vast leap proceeds in his thinking. Many activities must be learned together. It is argued in [7] that "reading, writing



and spelling to be isolated from one another in a curriculum, as they typically are, makes no sense at all. The three need to occur together, not as separate subjects”.

## 7 Unification

We believe the mechanisms of recognition to develop much in the same way that all intelligent capabilities do. Assimilation is not possible without recognition. We believe that the simplest form of recognition is based on the similarity of objects. The following and more complicated form of recognition is based on the logical identity or the equivalence of concepts. And the most perfect and complex form of recognition is the process of unification, i.e. the recognition of complex cognitive structures, for example, grouping or even the system of coordinated grouping.

Each previous scheme of recognition is assimilated by the following one. We assume that the results of all earlier proceeded acts of establishing the similarity between objects or the identity between concepts are conserved (maybe in the form of links or rules) and thereby facilitate the passing more complex acts of recognition. If we assume the continuity between perceived stimuli, images, representations and concepts, then we must conclude that once established relations of similarity or identity at the level of perception will form the analogous relations between corresponding images, representations, concepts at the other intellectual levels. Thus, we must consider the coordinated multi-level relations of similarity (distinction) which underlie the complex unification schemata.

We believe that any act of recognition begins with acquiring the similarity of various entities.

The operation which establishes the similarity on a set of entities of the same nature forms a grouping, the properties of which are defined by those of the operation itself: it is symmetrical but not transitive operation. The similarity is measurable; we say that entities are “not familiar at all”, ....“ much familiar”, “identical”. On the one hand, we can construct the set of all entities in pairs familiar to one another

(the transitive closure). On the other hand, we can order entities in accordance with the degree of their similarity.

Hence the relationship of similarity generates two groupings: the first one with the operation establishing similarity of entities, and the second, in the form of the ordered set of entities.

Logical identity of concepts requires the mastered processes of classification. Already the act itself of forming a class is an act of establishing the equivalence between the objects of this class and, at the same time, it is establishing the identity between the class and the common property of objects of this class. The identity has the following logical content: the class  $A$  is equivalent to the property "a" if and only if their interpretations  $I(A)$ ,  $I(a)$  on the set of conceivable objects are equal  $I(A) = I(a)$ . We can define also the relationship of approximation between concepts if the following relation is satisfied between their interpretations:  $I(A) \subseteq I(B)$  and there does not exist a concept  $X$  such that  $I(A) \subseteq I(X) \subseteq I(B)$ .

It is clear that each description (the union of properties), has one and only one interpretation. But the same set of objects can be associated with different but equivalent descriptions having equivalent interpretations. The set of equivalent descriptions for the same class of objects is to be said the different names of this class. The operation that establishes the equivalence of names of classes underlies the processes of common reasoning.

One of the fundamental capability of intelligence consists in the possibility to use the same cognitive scheme for solving the different problems as well as in the possibility to obtain the same result by means of different operational schemata. This phenomenon we connect with carrying over (tuning) the complex operational schemata from one domain of intellectual activity to another one. For example, mechanisms of assimilation and accommodation at the biological level and at the cognitive level are familiar. The syntactic structure of arithmetical expression is analogous to that of logical expression as well as to the syntactic structure of natural language's expression. For example, to find  $x, y$  such that  $x + y = 32$  means to find one of the decompositions of the number 32. Just the same, to find  $a, b$  such that "a + blue \*

b = good weather” means to find one of the descriptions (decompositions) of the concept “good weather”: the sun + blue \* sky = good weather. We consider unification to be the process of transferring the complex operational schemata from one context to another to solve the new tasks in the new domain of thinking. The unification is the most perfect and creative ability of human reasoning.

## 8 Conclusion

The Piaget’s theory of intelligence had an enormous influence on educational philosophy and methods of teaching children. Seymour. Papert, who had working with Piaget in Geneva, used his ideas for creating the LOGO Programming Language designed as a tool for leaning [4]. Thousands of teachers throughout the world became exited by the intellectual and creative potential of LOGO. The kind of learning that children do with computers is called “learning by doing”. Using a computer the child is able to build a model and learn from seeing a complete system in action as opposed to learning by rote, or in fragment.

We believe that knowing the Piaget’s theory of intelligence is important for the specialists in artificial intelligence. During panel discussions at the conferences people very often assert that it is sufficient to collect and to combine many separate programs modeling finite number of intellectual functions in order to trigger thinking computer. Machine learning, data mining are nowadays the principle problems in artificial intelligence.

Traditional application of expert systems (ES) in almost all the areas is restricted by using only hand-encoded expert knowledge as a base for logical inference in computer. Standard ES’s do not have learning capabilities. Mechanisms of logical inference are firmly associated with the knowledge representation form used in ES. Knowledge control mechanism, as a rule, is selected in advance and fixed. This approach is far from satisfying practitioners, it often results in skepticism with respect to expert systems in use. That’s why one of the urgent requests in the direction of improving the methodology of ES (like intelligent systems) creation is to provide computer with the mechanism

of reasoning available for extracting new knowledge from data.

Meanwhile conceptual reasoning implies not only passive navigation along the already constructed knowledge structures, but also their constructing, updating, extracting new knowledge from data in the form of classes, features, dependencies, recognizing and identifying objects, solving the implication and diagnostic problems and so on. All these activities have the common psychological nature and must be applied automatically to any type of concepts independently of the problem domain. Different processes of common reasoning must not be separated from one another as they psychologically constitute a system of coordinated operations. Only the collection of these operations united in a system can really permit to formulate the concept of conceptual type of data and to provide the integrity of conceptual reasoning.

The mathematical theory of conceptual type of data must be the lattice theory. One can come to this conclusion by analyzing both the fundamental researches in the psychological theory of intelligence [22], and the experience of modeling thinking processes in the frame of artificial intelligence. Classification has been considered as idempotent semigroup with unit in [17]. Algebraic model of classification and pattern recognition based on an algebraic lattice with two operations (generalization and refinement) has been advanced in [12]. A lot of experience has been collected on the application of algebraic lattices in machine learning: the works of Finn, V.K. and his disciples [16], [19], conceptual model of Wille, R. [23], the works of the French group of the investigators [18]. The following works are devoted to the application of algebraic lattices for extracting classifications, functional dependencies and implications from data [13], [14], [15], [20], [21]. An advantage of algebraic lattices approach consists in the fact that the algebraic lattice can be defined both as an algebraic structure which is declarative and as a system of dual operations with the use of which the elements of this lattice can be generated. This fact allows to conquer the absence of the operationality in the majority of knowledge models in intellectual computer systems.

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