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Confluent-Functional solving systems

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Abstract

The paper proposes a statistical knowledge-acquisition approach. The solving systems are considered, which are able to find unknown structural dependences between situational and transforming variables on the basis of statistically analyzed input information. Situational variables describe features, states and relations between environment objects. Transforming variables describe transforming influences, exerted by a goal-oriented system onto an environment. Unknown environment rules are simulated by a structural equations system, associating situational and transforming variables.

Keywords: environment, situational variables, goal variables, goal-oriented systems, confluent systems, functional models, confluent models.

1 Introduction

The paper considers previously introduced goal-oriented solving systems [1], able to select optimally a needed goal situation on the basis of an information in an initial situation. The main attention is paid to the ability of these systems to find unknown environment regulations on the basis of statistical analysis of input information, distorted by random noises, as well as to the ability to use these regulations in order to solve concrete problems of transformation of the initial situation into the goal one.

Environment description variables are usually related. These relations may be described by a set of some unknown interrelations between variables to be found. There are very many relations of this type in nature and in human society. The main point is to propose efficient ways in which such dependencies are modelled when real practical problems are solved. The paper supposes that the information a priori about a form of these relations is known. In this case, unknown functional relations between variables are expressed by known functions with unknown parameters.

The environment [2], in which the problem is defined, means such a triple $< V, K, A >$, where V is a set of objects; K is a set of features, states and relations between objects from V; and A is a set of actions, allowed for execution with respect to elements of V and K. When the problem is solved, environment fragments (i.e. situations) are operated with. The situation is a pair $< V_s, K_s >$, where V_s and K_s are subsets of, respectively V and K. Assume that an initial situation consists of a finite number of objects. Their features and states, as well as relations between them are described by a set of **initial** variables. Variables, describing a goal situation, are **goal** variables. A goal-oriented system may make use of certain actions in order to transform the initial situation into the goal situation. Variables, describing these actions, are **transforming** variables. Consider the problem of attainment of some goal situation, described by the certain goal variable value set $(y_1, \ldots, y_Q)$. Denote initial situation variables by vector $(x_1, \ldots, x_N)$. Transforming influences are described by variables $(u_1, \ldots, u_K)$. Predicted variables result from approximation of goal variables performed by means of situation and transforming variables. Main variables are understood as goal, initial and transforming ones. Let values of every main variable be observed at time moments $t = 1, \ldots, n$.

2 Types of Confluent Intelligent Solving Systems

The intelligent solving systems, aimed at functional modelling and considered in [3], construct a functional model of an environment. It is supposed in this case, that previously unknown functional relations between goal, initial and transforming variables are present. If these variables are random in accordance with their nature, then structural modelling systems are used to construct a model. In this case, an environment model is described by structural relations and not by functional relations between variables. And such a model is called structural model. Note, that initial and transforming variables are considered as exactly known in the solving systems of functional modelling. Although these variables are random in the structural modelling systems, their values are measured without errors. The present paper goes on discussing the functional modelling systems, but it is supposed now, that, when initial and transforming variables are measured, errors are present. If randomly emerging initial and transforming variable measurement errors are treated in a model, then it is possible to construct generalized models for functional and structural ones. Such generalized models are described by structural relations between observed variables, resulting from confluence of main variables and random errors. Therefore, they are, respectively, **confluent-functional** and **confluent-structural** models. The confluent-functional intelligent solving systems (CFISs) and confluent structural intelligent solving systems are used to construct unknown confluent-functional and confluent-structural models of an environment, respectively, on the basis of the statistical analysis of an obtained input information. Also, the above-mentioned systems are designed in order to solve different problems, when the found environment model is applied. Note, that, when functional environment models are dealt with, then it is possible to determine goal variable values according to initial and transforming variable ones, but when structural and confluent models are taken into consideration, then one may only say about a probability, with which goal variable values are found in stated confidence intervals under specified initial and transforming variable values. Only the shortest intervals are interesting in practice.

The paper considers two types of the CFISs: a) the open systems, characterized by their operation with observed main variables, when values of initial and transforming variables, onto which random disturbances exert their influence during problem solution, are known; and b) the closed systems, characterized by their operation with observed main variables, when values of initial and transforming variables, onto which random disturbances exert their influence during problem solution, are not known. Depending on the CFIS type, the environment models, built up by them, are divided, respectively, into the open and closed confluent-functional ones.

Finding of a confluent environment model is based on the statistical analysis of observed main variable values, arrived at a goal-oriented system input at time moments $t = 1, \dots, n$. When models are constructed by means of the CFISs, it is supposed that unknown functional relations between variables, which describe a confluent-functional environment model, are expressed by known functions with unknown parameters Θ_0. The paper considers the case, when values of initial and transforming variables $x_1(t), \dots, x_N(t), \quad u_1(t), \dots, u_K(t)$, required for problem solution, cannot be found exactly enough, and goal variables $y_i(t), \quad i = 1, \dots, Q$ are observed with some errors. Therefore, an **open confluent model of an environment** may be represented as the set of the following structural relations:

$$\begin{aligned} y'_i(t) &= f_i(x'_1(t), \dots, x'_N(t), u'_1(t), \dots, u'_K(t); \Theta_0) + \varepsilon_i(t), \\ & \quad i = 1, \dots, Q; \end{aligned} \tag{1}$$

$$\begin{aligned} x'_j(t) &= x_j(t) + \varepsilon_{jx}(t), \quad j = 1, \dots, N; \\ u'_k(t) &= u_k(t) + \varepsilon_{ku}(t), \quad k = 1, \dots, K; \end{aligned} \tag{2}$$

where $x_1(t), \dots, x_N(t), u_1(t), \dots, u_K(t)$ are known initial and transforming variable values, specified for problem solution and under which goal variable $y_i(t)$ must be observed. The values of $y'_i(t), x'_j(t), u'_k(t)$ are the results, obtained in observation of main variables $y_i(t), x_j(t) u_k(t)$, respectively, and real value of $y_i(t)$ is not known. $\varepsilon_i(t)$ is an error of observation of goal variable, $\varepsilon_{jx}(t), \varepsilon_{ku}(t)$ are random errors in fixation of initial and transforming variables, $j = 1, \dots, N$; $k = 1, \dots, K$; $i = 1, \dots, Q$.

A **closed confluent model of an environment** may be represented as the set of the following structural relations:

$$\begin{aligned} y'_i(t) &= f_i(x_1(t), \dots, x_N(t), u_1(t), \dots, u_K(t); \Theta_0) + \varepsilon_i(t), \\ & \quad i = 1, \dots, Q; \end{aligned} \tag{3}$$

$$\begin{aligned} y'_i(t) &= y_i(t)+\varepsilon_i(t),\ x'_j(t)=x_j(t)+\varepsilon_{jx}(t), \quad j=1,\dots,N; \\ u'_k(t) &= u_k(t)+\varepsilon_{ku}(t), \quad k=1,\dots,K; \end{aligned} \tag{4}$$

where $\varepsilon_{jx}(t)$, $\varepsilon_{ku}(t)$, $\varepsilon_i(t)$, $j=1,\dots,N$, $k=1,\dots,K$, $i=1,\dots,Q$ are uncontrollable random errors. $y'_i(t)$, $i=1,\dots,Q$; $x'_j(t)$, $j=1,\dots,N$; $u'_k(t)$, $k=1,\dots,K$ can be measured. The values of $y_i(t)$, $x_j(t)$, $u_k(t)$, $j=1,\dots,N$; $k=1,\dots,K$; $i=1,\dots,Q$ are unknown.

Introduce not very strict assumptions, which, from the computational point of view, allow to reduce construction of confluent environment models to construction of some auxiliary functional models. The procedure of construction of the latter is considered in [3].

Let the following conditions be met:

A) functions $f_i(x_1(t),\dots,x_N(t),u_1(t),\dots,u_K(t);\Theta_0)$ have mixed derivatives as for main variables up to the third-order derivative inclusive, and these derivatives are uniformly restricted on the set of admissible initial and transforming variable values;

B) random errors, included into main observed variables $\varepsilon_{jx}(t)$, $\varepsilon_{ku}(t)$, $\varepsilon_i(t)$, $j=1,\dots,N$; $k=1,\dots,K$; $i=1,\dots,Q$ are independent in their totality; and the mathematical expectations are

$$\begin{aligned} E\varepsilon_{jx}(t) &= 0,\ E\varepsilon_{ku}(t)=0,\ E\varepsilon_i(t)=0,\ E\varepsilon_i^2(t)=\sigma^2, \\ \varepsilon_{jx}(t) &= \gamma\nu_{jx}(t),\ \varepsilon_{ku}(t)=\gamma\nu_{ku}(t), \quad E(\nu(\mathbf{t})\nu^T(\mathbf{t}))=\mathbf{d}. \end{aligned}$$

$\nu(\mathbf{t})=(\nu_{1x}(t),\dots,\nu_{Nx}(t),\nu_{1u}(t),\dots,\nu_{Ku}(t))$ is a vector of standardized random values (for instance, with a unique dispersion) with the restricted third-order mixed moment:

$$E(|\nu_{j.}(t)\nu_{k.}(t)\nu_{s.}(t)|)=c, \quad j,k,s=1,\dots,N+P;$$

c is some constant. The dimensionality of $\nu(\mathbf{t})$ is $p=N+K$. $\mathbf{d}$ is a diagonal pxp-dimensional matrix.

3 Constructing Open Models by CFISs under Known Noise Parameters.

Introduce the denotations

$$\begin{aligned} X'(t) &= (x_1(t)+\varepsilon_{1x}(t),\ldots,x_N(t)+\varepsilon_{1x}(t);\\ &\quad u_1(t)+\varepsilon_{1u}(t),\ldots,u_K(t)+\varepsilon_{1u}(t);) \end{aligned}$$

and

$$X_0(t)=(x_1(t),\ldots,x_N(t);u_1(t),\ldots,u_K(t);).$$

It is easy to check that the following relation takes place for the mathematical expectation and dispersion of goal variable:

$$My_i'(t)=M[f_i(X'(t);\Theta_0)+\varepsilon_i(t)]=f_i^*(X_0(t);\Theta_0)+o(\gamma^3); \tag{5}$$

$$Dy_i'(t)=\lambda^{-1}(X_0(t);\Theta_0)+o(\gamma^3); \tag{6}$$

where

$$\begin{aligned} f_i^*(X_0(t);\Theta_0) &= f_i(X_0(t);\Theta_0)+\\ &\quad +0.5v^2 Sp\left(\mathbf{d}\bullet\left.\frac{\partial^2 f(X(t);\Theta_0)}{\partial X\partial X^T}\right|_{X=X_0}\right); \end{aligned}\tag{7}$$

$$\begin{aligned} \lambda^{-1}(X_0(t);\Theta_0) &= \sigma^2+v^2\cdot\frac{\partial f(X(t);\Theta_0)}{\partial X^T}\cdot\\ &\quad\cdot\left(\mathbf{d}\cdot\left.\frac{\partial f(X(t);\Theta_0)}{\partial X}\right|_{X=X_0}\right); \end{aligned}\tag{8}$$

T is the transposition operation.

Therefore, construction of an open confluent-functional environment model is reduced with accuracy within $o(\gamma^3)$ to construction of the following auxiliary nonconfluent functional environment model:

$$\begin{aligned} y_i'(t) &= f_i^*(x_1(t),\ldots,x_N(t),u_1(t),\ldots,u_K(t);\Theta_0)+\mu_i(t),\\ &\quad i=1,\ldots,Q; \end{aligned}\tag{9}$$

here $\mu_i(t)$ is a transformed error,

$$E\mu_i(t) = 0, \quad E\mu_i^2(t) = \lambda^{-1}(X_0(t);\Theta_0). \tag{10}$$

The essential difference between the present functional environment model and the one in [3] consists in the fact that the dispersion of the transformed error μ_i(t) depends on the unknown parameters Θ_0, σ^2, **d**.

If the goal variable error dispersion σ^2 and the covariant error matrix **d** for initial and transforming variables are known, then, to find unknown parameters, determining the functional environment model, the procedure [4]

$$\Theta_n^* = \lim_{s\to\infty} \Theta_s, \tag{11}$$

where

$$\Theta_s = \arg\min_{\Theta} n^{-1}\sum_{t=1}^{n}\lambda(X_0,\Theta_{s-1})[y_i'(t) - f_i^*(X_0,\Theta)]^2,$$

or the modification of Newton-Rafson method

$$\Theta_s = \Theta_{s-1} + \alpha_s Z_n^{-1}(\Theta_{s-1})W_n(\Theta_{s-1}), \tag{12}$$

where

$$Z_n(\Theta) = n^{-1}\sum_{t=1}^{n}\lambda(X_0(t),\Theta)\ \dot{F_i}\ (X_0(t),\Theta)\ \dot{F}_i^T\ (X_0(t),\Theta), \tag{13}$$

$$W_n(\Theta) = n^{-1}\sum_{t=1}^{n}\lambda(X_0(t),\Theta)[y_i'(t) - f_i^*(X_0(t),\Theta)]\ \dot{F_i}\ (X_0(t),\Theta),$$

$$\dot{F_i}\ (X_0(t),\Theta) = \left.\frac{\partial f_i^*(X(t);\Theta)}{\partial\Theta}\right|_{X=X_0}$$

are used.

The factor α_s is chosen in the same way as in the usual Newton-Rafson procedure.

It is possible to show [4], that under some additional conditions, except conditions A) and B), the considered iterative procedures converge with the probability 1, and the obtained estimates are consistent and normal.

These conditions are as follows.

C) The sequence

$$w_n^2(\Theta) = n^{-1}\sum_{t=1}^{n}\lambda(X_0(t),\Theta_0)[f_i^*(X_i(t),\Theta) - f_i^*(X_0(t),\Theta_0)]^2$$

uniformly converges with respect to $\Theta \in \Omega$, and $\lim w_n^2(\Theta) = w^2(\Theta)$ takes here place, and the function $w^2(\Theta)$ has the only one minimum under $\Theta = \Theta_0$.

D) Under all $\Theta \in \Omega$, there exist Θ-continuous derivatives

$$\frac{\partial f_i^*(X_0(t);\Theta)}{\partial\Theta}, \quad \frac{\partial^2 f_i^*(X_0(t);\Theta)}{\partial\Theta\partial\Theta^T}$$

and the sequences

$$\{n^{-1}\sum_{t=1}^{n}\lambda(X_0(t),\Theta_0)\varphi(X_0(t),\Theta)\psi(X_0(t),\Theta)\},$$

where the functions $\varphi(X_0(t),\Theta)$, $\psi(X_0(t),\Theta)$ may coincide with any above-mentioned derivatives, and the sequences uniformly converge with respect to $\Theta \in \Omega$.

E) The matrix

$$Z(\Theta_0) = \lim_{n\to\infty} n^{-1}\sum_{t=1}^{n}\lambda(X_0(t),\Theta_0)\ \dot{F}_i\ (X_0(t),\Theta_0)\ \dot{F}_i^T\ (X_0(t),\Theta_0)$$

is not unusual.

When conditions A)–E) are met, then:

1. $\lim\limits_{n\to\infty} P_n = 1$, where P_n is the probability that iteration procedure (11) converges under selection of a volume n;

2. the estimate Θ_n^*, determined by procedure (11), is strongly consistent, and if there are some solutions under X_0, than any of them are taken as Θ_n^*;

3. the estimation Θ_n^* is asymptotically normal, i.e.

$$\lim_{n\to\infty} P\{\sqrt{n}(\Theta_n^* - \Theta_0) < s\} = \Phi(s;\ 0,\ Z^{-1}(\Theta_0)),$$

and, in this case, $Z_n(\Theta_n^*)$ is the strongly asymptotical estimation of the matrix $Z(\Theta_n)$.

4 Constructing Open Models by CFISs under Unknown Noise Parameters.

If a goal variable error dispersion and a covariant initial and transforming variable error matrix are unknown, then, to find unknown parameters which determine an auxiliary functional environment model, the more complicated iteration procedure is applied. This procedure allows to additionally state a goal variable error dispersion and a covariant main variable error matrix.

To find the estimates Θ^* for Θ_0 under unknown σ^2 and $\mathbf{d}$, introduce the following new unknown parameter vector:

$$\Xi_0 = \begin{pmatrix} \Theta_0 \\ \sigma^2 \\ \mathbf{d} \end{pmatrix},$$

where $\mathbf{d}^T{=}(d_{11}d_{12}\ldots d_{jl}\ldots d_{pp})$

By analogy, as it is done in the case with expressions (9) and (10), consider the auxiliary regression problem:

$$y_i'(t) = f_i^*(X_0(t);\Xi_0) + \mu_i(t), \quad i = 1,\ldots,Q; \tag{14}$$

$$E\mu_i(t) = 0, \quad E\mu_i^2(t) = \lambda^{-1}(X_0(t);\Xi_0). \tag{15}$$

Note, that the functions $f_i^*(X_0(t);\Xi_0)$ and $\lambda^{-1}(X_0(t);\Xi_0)$ depend on different groups of the parameters in Ξ_0. Assume that the random values $\mu_i(t)$ are distributed normally. To meet this condition, the errors contained in main variables should be supposed to be normal in the initial problem.

The estimates Ξ^* for Ξ_0 are found according to the following iteration procedure:

$$\Xi^* = \lim_{s\to\infty} \Xi_s^* \tag{16}$$

$$\begin{aligned}\Xi_s^* &= \arg\min_{\Theta^*} n^{-1}\sum_{t=1}^{n}\{\lambda(X_0(t),\Xi_{s-1}^*)[y_i'(t) - f_i^*(X_0(t),\Xi_{s-1}^*)]^2 + \\ &+\frac{1}{2}\lambda^2(X_0(t),\Xi_{s-1}^*)[\lambda^{-1}(X_0(t),\Xi_{s-1}^*) - \\ &-(y_i'(t) - f_i^*(X_0(t),\Xi_{s-1}^*))^2]^2\}.\end{aligned}$$

Consider the convergence conditions for this procedure and the features of obtained estimates. Let the functions $f_i^*(X_0(t);\Xi_0)$ and $\lambda^{-1}(X_0(t);\Xi_0)$ satisfy condition D). Besides this, the function

$$\begin{aligned}v_n^2(\Xi^*) &= n^{-1}\sum_{t=1}^{n}\{\lambda(X_0(t),\Xi^*)[y_i'(t) - f_i^*(X_0(t),\Xi^*)]^2 + \\ &+\frac{1}{2}\lambda^2(X_0(t),\Xi_1^*)[\lambda^{-1}(X_0(t),\Xi^*) - \\ &-(y_i'(t) - f_i^*(X_0(t),\Xi^*))^2]^2\}\end{aligned}$$

satisfies condition C).

Introduce the matrix

$$\begin{aligned}G_n(\Xi^*) &= n^{-1}\sum_{t=1}^{n}[\lambda(X_0(t);\Xi^*)p(X_0(t);\Xi^*)p'(X_0(t);\Xi^*) + \\ &+\frac{1}{2}\lambda^2(X_0(t);\Xi^*)q(X_0(t);\Xi^*)q'(X_0(t);\Xi^*)],\end{aligned}$$

where

$$p(X_0; \Xi^*) = \frac{\partial f_i^*(X_0; \Xi^*)}{\partial \Xi^*}, \quad q(X_0; \Xi^*) = \frac{\partial \lambda^{-1}(X_0; \Xi^*)}{\partial \Xi^*}.$$

Instead condition E), the matrix

$$G(\Xi_0^*) = \lim_{n \to \infty} G_n(\Xi_0^*)$$

is required, and it is necessary for this matrix to be nondegenerated.

If these five conditions are met, the result is as follows [5]:

1. $\lim_{n \to \infty} P_n = 1$, where P_n is the probability that iteration procedure (16) converges;

2. the estimate Ξ_n^*, determined by procedure (16), is strongly consistent, and if there are some solutions under n, then any of them is taken as Ξ_n^*;

3. the estimation Ξ_n^* is asymptotically normal, i.e.

$$\lim_{n \to \infty} P\{\sqrt{n}(\Xi_n^* - \Xi_0) < s\} = \Phi(s;\ 0,\ G^{-1}(\Xi_0)),$$

and, in this case, $G_n(\Xi_n^*)$ is the strongly consistent estimation of the matrix $G(\Xi_0)$.

5 Constructing Closed Models by CFISs

If a closed confluent-functional environment model is constructed, this point is much more difficult, than the open model case. To construct a closed model, the CFISs use the least-distances method instead of the least-squares one. Consider some $1 + N + K$-dimensional Euclidean space for those points, coordinates $(y_i, x_1, \ldots, x_N, u_1, \ldots, u_K)$ of which correspond to main (goal, initial and transforming) variables. For each coordinate of this Euclidean space, the respective mean-square variable error deviation is used as a length unit. A variable error here corresponds to some given coordinate. It is σ^2 for the first coordinate and it is v^2 for the rest of them. Such points are dealt with in the present space, coordinates of which are equal to observed values of main variables for each observation time moment, i.e. to $(y_i'(t), x_1'(t), \dots, x_N'(t), u_1'(t), \dots, u_K'(t))$. Number of points is equal to the number of observation time moments. Let these points be called observable. Parameter values are taken as an estimate of unknown parameters of the given functional relation, included into a closed confluent-functional model. And those parameter values are taken as the estimate, under which the sum of distances from every observed point in this metrics to the surface

$$y_i(t) = f_i(x_1(t), \dots, x_N(t), u_1(t), \dots, u_K(t); \Theta_0)$$

is minimum. The surface

$$y_i(t) = f_i(x_1(t), \dots, x_N(t), u_1(t), \dots, u_K(t); \Theta_0)$$

is determined by the functional relation

$$\Theta^* = \arg\min_{\Theta_0} \sum_{t=1}^{n} l_i^2(\Theta_0), \tag{17}$$

where

$$\begin{aligned} l_i^2(\Theta_0) \quad = \quad & \min_{X_0(t)} \left[\left(\frac{y_i' - f(X_0(t); \Theta_0)}{\sigma} \right)^2 + \right. \\ & \left. + \gamma^2 \left(X_0(t) - X'(t) \right)^T d^{-1} \left(X_0(t) - X'(t) \right) \right], \end{aligned} \tag{18}$$

$$\begin{aligned} X_0(t) &= (x_1(t), \dots, x_N(t); u_1(t), \dots, u_K(t);), \\ X'(t) &= (x_1'(t), \dots, x_N'(t); u_1'(t), \dots, u_K'(t);), \end{aligned}$$

$$\begin{aligned} y_i'(t) &= y_i(t) + \varepsilon_i(t), \quad x_j'(t) = x_j(t) + \varepsilon_{jx}(t), \quad j = 1, \dots, N; \\ u_k'(t) &= u_k(t) + \varepsilon_{ku}(t), \quad k = 1, \dots, K; \end{aligned}$$

Estimates (17) are the estimates for the least-distances method [5].

In the linear case, i.e. under

$$y_i = f_i(X_0(t); \Theta_0) = \Theta_0^T X_0(t),$$

formula (18) is

$$l_i^2(\Theta_0) = \frac{(y_i' - \Theta_0^T X'(t))^2}{\sigma^2 + \nu^2 \Theta^T d\Theta}.$$

In the case with the arbitrary function $y_i = f_i(X_0(t); \Theta_0)$, and when conditions A) and B) are met, the approximate formula

$$l_i^2(\Theta_0) = (y_i' - f_i^{**}(X'(t); \Theta_0))^2 \lambda(X'(t); \Theta_0) + o(\nu^3)$$

takes place, where

$$\begin{aligned} f_i^{**}(X'(t); \Theta_0) &= f_i(X'(t); \Theta_0) - \\ &\quad -0.5 v^2 Sp \left(\mathbf{d} \bullet \left. \frac{\partial^2 f(X(t); \Theta_0)}{\partial X \partial X^T} \right|_{X=X'} \right); \end{aligned} \tag{19}$$

$$\lambda^{-1}(X'(t); \Theta_0) = \sigma^2 + \nu^2 \cdot \left. \frac{\partial f(X(t); \Theta_0)}{\partial X^T} \cdot d \cdot \frac{\partial f(X(t); \Theta_0)}{\partial X} \right|_{X=X'}; \tag{20}$$

Therefore, the estimates of the least-squares methods are as follows:

$$\Theta_n^* = \arg\min_{\Theta_0} \sum_{t=1}^{n} (y_i - f_i^{**}(X'(t); \Theta_0))^2 \lambda(X'(t); \Theta_0). \tag{21}$$

Consider the conditions, when the obtained estimates are strongly consistent and asymptotically normal.

Assume

$$\overset{\circ}{\Psi}(X(t); \Theta) = \frac{\partial f_i^{**}(X(t); \Theta)}{\partial \Theta},$$

$$d^*(X; \Theta) = \frac{\partial \overset{\circ}{\Psi}(X(t); \Theta)}{\partial X^T} \cdot d \cdot \frac{\partial \overset{\circ}{\Psi}(X(t); \Theta)}{\partial X}$$

and let the conditions, similar to conditions A)-E), be met, when $f_i^*(X_0(t), \Theta_0)$ is replaced by $f_i^{**}(X'(t); \Theta_0)$ and $\dot{F}_i\ (X_0(t), \Theta)$ is replaced by $\overset{\circ}{\Psi}\ (X'(t); \Theta)$. Besides this, there is the limit

$$z(\Theta_0) = \lim_{n\to\infty} n^{-1}\sum_{t=1}^{n}\lambda(X_0(t), \Theta_0)d^*(X_0(t), \Theta_0).$$

Then:

1) estimates (21) are strongly consistent and asymptotically normal [4], i.e.

$$\lim_{n\to\infty} P\{\sqrt{n}(\Theta_n^* - \Theta_0) < s\} = \Phi(s;\ 0,\ \Delta(\Theta_0)),$$

where

$$\Delta(\Theta_0) = Z^{-1}(\Theta_0)[Z(\Theta_0) + \gamma^2 z(\Theta_0)]Z^{-1}(\Theta_0);$$

2) the matrix

$$n^{-1}\sum_{t=1}^{n}\lambda(X_0(t); \Theta_n^*)[\overset{\circ}{\Psi}\ (X_0(t);\Theta_n^*)\overset{\circ}{\Psi}{}^T\ (X_0(t);\Theta_n^*) - \gamma^2 d^*(X_0(t); \Theta_n^*)$$

is the strongly consistent estimate for the matrix $Z(\Theta_0)$.

3) the matrix

$$n^{-1}\sum_{t=1}^{n}\lambda(X_0(t); \Theta_n^*)d^*(X_0(t); \Theta_n^*)$$

is the strongly consistent estimate for the matrix $z(\Theta_0)$.

Note, that, in contrary to the open model case, since the information that goal variable error dispersion and covariant initial and transforming variable error matrix are absent, it is impossible for CFISs to construct strongly consistent and asymptotically normal estimates for closed model.

6 Example.

The task for one of the main methods, used to control mechanical features of products made of steels and other materials is to measure hardness. Measurement of hardness is carried out in the majority of cases in order to check up correctness of modes, in which products made of structural steels pass their heat treatment. Hardness is measured at different devices depending on product strength levels, dimensions and forms, on an applied technology and on many other factors. Let HB be Brinel hardness, and Rockwell hardness is abbreviated as HRC. Consider the problem, concerned with measurement of ultimate strength σ_b , when proceeding from the values of HB or HRC. The tables of conversion of HB into σ_b , of HRC into σ_b , of HB into HRC and vice versa were made up as far back as in 1920s on the basis of the experiments. However, when these conversion tables were applied, it was noticed that practical values of mechanical characteristics were considerably different from the values in the conversion tables. According to the table data, the hardness value and the ultimate strength are interrelated by the linear functional dependence. To correct the present functional model, it is necessary to construct a confluent-functional model instead of it. The dependences between the following pairs of variables are now of interest: HB and σ_b, HRC and σ_b, HB and HRC. Each variable here may be treated as a goal variable, i.e. six interrelations between these variables are dealt with. Let y be the goal variable, and the initial variable is denoted by x. The following factors exert their influence onto x, y and interrelations between them: chemical composition ω_1 ; heat treatment ω_2 ; features of a specified specimen (local chemical composition and heat treatment, dimensions of a grain in a print zone, etc.) ω_3 and measurement errors ω_4 . The dependences of x and y on these factors are determined by the following expressions:

$$x = x(\omega_1, \omega_2, \omega_3, \omega_4) \tag{22}$$

$$y = y(\omega_1, \omega_2, \omega_3, \omega_4) \tag{23}$$

If many products are manufactured for one melting (fixed ω_1) and if it is necessary to make them pass heat treatment in one mode (fixed

ω_2), then it is possible to estimate mean values for studied mechanical characteristics as for the specified ω_1 and ω_2 with sufficient accuracy. Denote these mean values by x_{12} and y_{12} . Introduce the denotations for the balances:

$$\begin{aligned} x_{3,4} &= x(\omega_3, \omega_4/\omega_1, \omega_2) = x - x_{12}, \\ y_{3,4} &= y(\omega_3, \omega_4/\omega_1, \omega_2) = y - y_{12}. \end{aligned}$$

It follows from these formulas that, under fixed ω_1 , $\omega_{2,}$ the conventional mean values are:

$$M_{12}x_{3,4} = 0 \text{ and } M_{12}y_{3,4} = 0. \quad (24)$$

The collected experimental data, on the basis of which the tables are made up, provide the assumption that there exists the fundamental relation between mean values of x and y, and this relation may depend on chemical composition ω_1:

$$y_{12} = f(x_{12}, \omega_1). \quad (25)$$

According to the data a priori (table), this relation is linear within sufficiently broad intervals of values of x:

$$y_{12} = a(\omega_1) + b(\omega_1)x_{12}. \quad (26)$$

Therefore, the observed variables are described by the following closed confluent-functional model:

$$x = x_{12} + x_{3,4}, \quad (27)$$

$$y = a(\omega_1) + b(\omega_1)x_{12} + y_{3,4}. \quad (28)$$

When ω_1 , ω_2 are fixed, conditions (24) are met as well as the following assumptions:

1) the pairs of random values $(x_{3,4}, y_{3,4})$ for different specimens do not depend on each other.

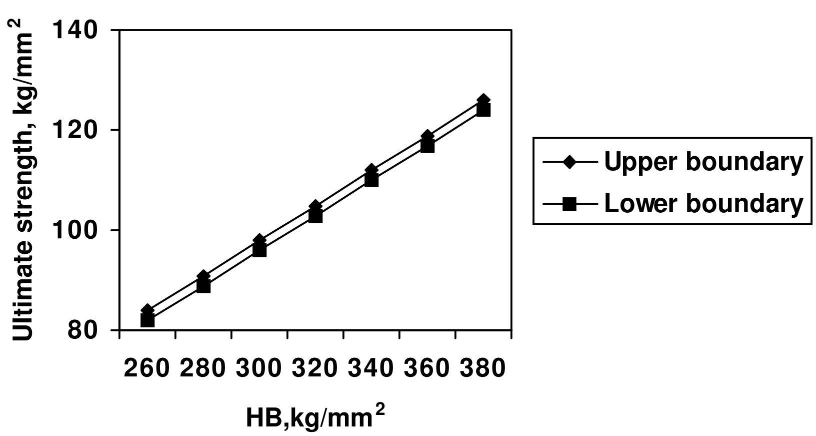
2) there exist the dispersions for $x_{3,4}, y_{3,4} : D_{12}x_{3,4}, D_{12}y_{3,4}$.

The model is constructed on the basis of the test results, concerned with the groups of the products of one melting for each steel quality. The products pass here heat treatment as for different strength [6]. There are only 15 values of HB and σ_b for different heat treatment $\omega_2(j), \quad j = 1, \dots, 5$:

Specimen No.	$\omega_2(1)$ HB	$\omega_2(1)$ σ_b	$\omega_2(2)$ HB	$\omega_2(2)$ σ_b	$\omega_2(3)$ HB	$\omega_2(3)$ σ_b	$\omega_2(4)$ HB	$\omega_2(4)$ σ_b	$\omega_2(5)$ HB	$\omega_2(5)$ σ_b
1	263	88.5	277	95.5	331	109.0	363	120.0	383	126.0
2	262	90.0	275	95.5	335	111.5	356	117.0	383	127.0
3	262	90.5	278	94.0	331	109.5	350	118.5	385	128.0
4	262	87.5	278	93.5	331	109.5	352	117.5	390	129.0
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…
14	265	89.5	275	92.5	341	110.5	352	120.0	383	129.0
15	263	90.0	277	93.5	327	109.5	354	119.0	384	128.5

The test results provide the dependencees between HB and σ_b , HRC and σ_b , HB and HRC, which are linear within the interval $\sigma_b = 90 - 130\ kg/mm^2$. For instance, the points in the above table are between the lines, depicted in Figure 1.

Figure 1.



When x is converted into y according to the table and if steel quality is taken into account, the spreading of actual values of $y(\omega_1, \omega_2, \omega_3, \omega_4)$

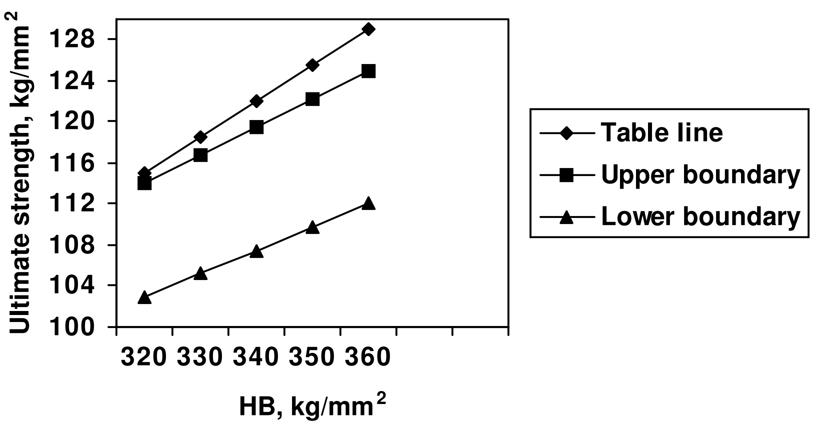
near the table value is the sum of spreading inside melting (i.e. of deviations of $y(\omega_1, \omega_2, \omega_3, \omega_4)$ from a line $a(\omega_1) + b(\omega_1)x_{12}$) plus difference between this line and the table one. A contribution, made by the first term, may be characterized by an in-melting dispersion averaged with respect to different meltings. It is easy to characterize a contribution of the second term by its mean square of a distance between a table line and $a(\omega_1) + b(\omega_1)x_{12}$, when this mean square is averaged with respect to the studied interval of values for x. In the latter case, the totality of meltings and chemical composition are taken into consideration. Here arises the question: which value of any one characteristic corresponds to the measured value x of another characteristic? It follows from the above considerations, that this question cannot be answered exactly, i.e. it is impossible to give one number. It is possible to speak only about the probability, with which the value of y is found within the certain interval (y_1, y_2) under the specified value of x. If this probability is assumed to be equal to 0.95, then, when conventional distribution of y under specified x is supposed to be normal, it is possible to obtain upper and lower interval boundaries for the 30 HGSA steel quality (Figures 2–4). Figures 2–4 show the line, yielded on the basis of the table data.

The upper and lower confidence boundaries in Figures 2–4 allow to correctly foresee values of the considered goal variables with the probability of 0.95.

7 Conclusion.

Any experimental variable is always measured under influences of some noises, which never can be removed completely. Measurements of these and similar variables are faced with in quantum-mechanical and biological investigations, in some chemical kinetic problems and in a number of other scientific and technical branches. Many processes run under influences, exerted by the factors, which not always can be fully taken into consideration. Many production processes are just those ones, under which features of manufactured products depend, for instance, on different properties of raw materials and on some technological pro-

Figure 2.

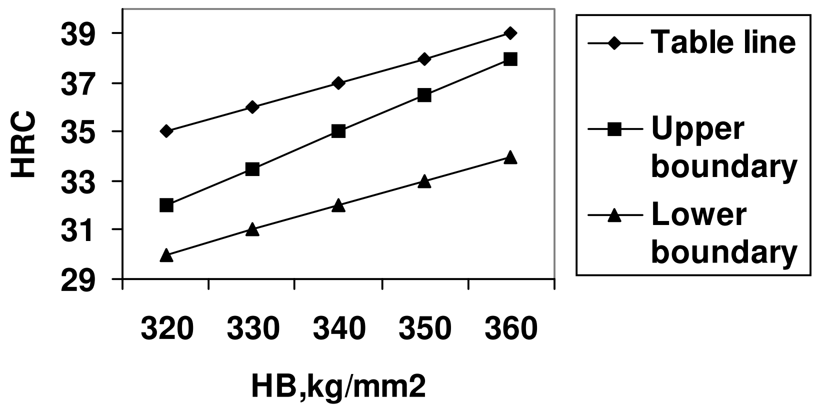


cess parameters, which cannot undergo continuous and sufficiently full examination. These properties and parameters include, for example, admixtures in raw materials, changes in temperature and humidity, accidentally emerging abnormalities in machine operation, etc.

Confluent-functional solving systems allow to study dependences between variables, which describe environment on the basis of the statistical analysis of input information. They can find unknown structural relations between environment variables, distorted by random noises. They help to solve a whole number of urgent problems: to establish a dependence between features of a product and factors, characterizing technological product manufacturing process, to examine relations between some characteristics and parameters, stating product operation conditions. In addition, they are able not only to state unknown regularities, existing in the environment, but also to foresee further development of situations.

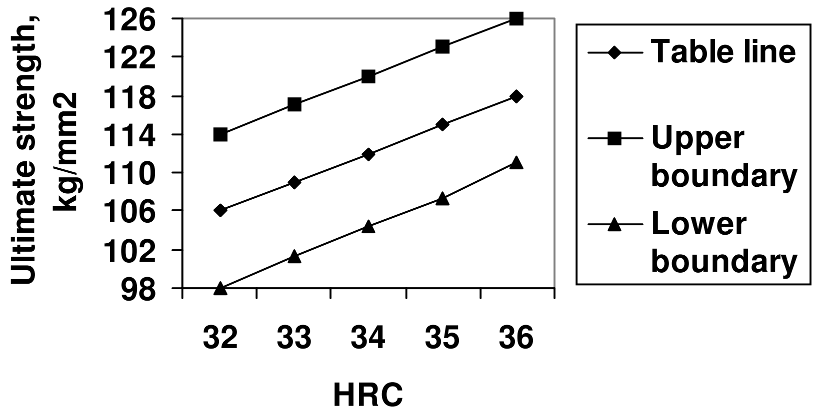
Regularities, existing in the environment, are revealed and situations are made foreseen, and, after this, the goal-oriented solving systems use them in order to solve a concrete problem, concerned with transformation of an initial situation into a goal situation. Due to this

Figure 3.



circumstance, it turns out that it is quite necessary to apply the methods, which would provide not only data processing way, but also allow to optimally arrange problem solution. The problem of acquisition of as many knowledge about studied processes under limited costs as possible is rather urgent at present. Note, that problem solution planning is expedient only when a CFIS has stated a final goal of environment model construction. The statistical problem solution planning methods are the tool, by means of which it becomes easier to achieve the stated goal. For instance, when new chemical-technological processes are developed, the optimality criterion consists in required maximum amount of reaction products. Planning in this case consists in finding of such values of temperature, pressure, composition percentage, etc., for reagents, under which the stated task can be fulfilled. To solve this problem, it becomes necessary to reveal the dependence of a reaction product yield on temperature, pressure, etc., for reagents, i.e. to find a function, able to state a correspondence between this yield and values, exerting their influence onto the reaction process. In other words, a model of this process is needed. The confluent-functional models, derived as the CFIS operation result, can serve as the efficient math-

Figure 4.



ematical apparatus, when highly-dimensional systems are designed for functioning in different branches of economy, in scientific investigations, in military art and in other spheres.

References

[1] V.N. Koval, Yu.V. Kuk. *Goal-Oriented Solution-Planning Systems. KDS-99: Knowledge-Dialog-Solution*, Proc. VII International Conference on Artificial Intelligence, September 13–18, 1999, Katsively, Crimea, Ukraine, Donetsk, 1999, pp. 158–165. (In Russian).

[2] V.P. Gladun. *Planning of Solutions.* Kiev, Naukova Dumka, 1987. 168 p. (In Russian).

[3] V.N. Koval, Yu.V. Kuk. *Finding Unknown Rules of an Environment by Intelligent Goal-Oriented Systems International Journal.* "Information Theories & Applications", Vol.7, N3, pp. 127–138, Sofia, FOA-COMMRS, 2001

[4] V.V. Fyodorov. *Regression Analysis under Errors in Determination of a Predictor*. Voprosy Kibernetiki, 1978, vol. 47, pp.69–75. (In Russian).

[5] C.A. Aivazian. *Applied Statistics: Examination of Dependences.Moscow, Finansy I Statistika*, 1985, 487 p. (In Russian).

[6] L.D. Meshalkin, B.M. Strunin. *A Methodology of Conversion of Hardness Indices*. Zavodskaya Laboratoriya, 1967, vol. 33, N 11, pp. 1408–1417.

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