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Small Universal Circular Post Machines

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Abstract

We consider a new kind of machines with a circular tape and moving in one direction only, so-called Circular Post machines. Using 2-tag systems we construct some small universal machines of this kind.

1 Introduction

In 1956 Shannon [11] introduced the problem of constructing very small universal (deterministic) Turing machines. The underlying model of Turing machines is defined by instructions in form of quintuples $(\mathbf{p}, x, y, m, \mathbf{q})$ with the meaning that the machine is in state $\mathbf{p}$, reads symbol $x \in \Sigma$, overwrites it by y, moves by $m \in \{-1, 0, 1\}$, and goes into state $\mathbf{q}$. Another equivalent model is defined by quadruples $(\mathbf{p}, x, \alpha, \mathbf{q})$ where $\alpha \in \Sigma \cup \{-1, 0, 1\}$. This model is also equivalent to so called *Post* machines [6]. Whereas the quintuple model allows to construct equivalent machines with 2 states this is impossible for the quadruple model [1].

We introduce (deterministic) Circular Post machines (**CPM**). These are similar to those presented in [2], with the difference that the head can move only in one direction on the circular tape. It is also possible to erase a cell or to insert a new one. We consider 5 variants of such machines, distinguished by the way a new cell is inserted. It is shown that all variants are equivalent to each other, and also to Turing machines. We also show that for all variants there exist equivalent Circular Post machines with 2 symbols, and for 3 variants with 2 states.

To construct small universal Circular Post machines we use a method first presented in [4] (see also [9, 10]). This method uses tag

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systems [3] which are special cases of monogenic Post Normal systems [8], namely of the form $s_i vu \to u\alpha_i$ with $v \in \Sigma^{k-1}$ and $k > 1$ a constant. In [4] it is also shown that 2-tag systems (i. e. $k = 2$) suffice to simulate all Turing machines, with halting only when encountering a special symbol s_H. Since Circular Post machines are also monogenic Post Normal systems we expect to get a more natural simulation of tag systems and perhaps smaller universal machines.

We show that a still unsolved problem of Post from 1921 [7, 5] can be simulated by a CPM0(6,2), i.e. a machine of variant 0 with 6 states and 2 symbols. We present a CPM0(5,3) simulating the also unsolved (?) Collatz ($3n+1$) problem. Finally, we construct universal machines UCPM0(13,4), UCPM0(11,5), UCPM0(8,6), and UCPM0(7,7).

2 Definitions and Basic Results

Here we introduce some variants of circular Post machines.

Definition 1: (Circular Post machine (**CPM0**))

A Circular Post machine is a quintuple $(\Sigma, Q, \mathbf{q}_0, \mathbf{q}_f, P)$ *with a finite alphabet* Σ *where* 0 *is a blank, a finite set of states* Q*, an initial state* $\mathbf{q}_0 \in Q$*, a terminal state* $\mathbf{q}_f \in Q$*, and a finite set of instructions of the forms*

$\mathbf{p}x \to \mathbf{q}$ *(erasing of the symbol read)*

$\mathbf{p}x \to y\mathbf{q}$ *(overwriting and moving to the right)*

$\mathbf{p}0 \to y\mathbf{q}0$ *(overwriting and creation of a blank)*

The storage of such a machine is a circular tape, the read and write head moving only in one direction (to the right), and with the possibility to cut off a cell or to create and insert a new cell with a blank.

This version is called variant 0. Note that by erasing symbols the circular tape might become empty. This can be interpreted that the machine, still in some state, stops. However, in the universal machines constructed later, this case will not occur.

In this article it will assumed that all machines are deterministic.

□

There are variants equivalent to such machines.

Definition 2: (Variant **CPM1**)
The instructions are of the form
$\mathbf{p}x \to \mathbf{q}$ $\quad\mathbf{p}x \to y\mathbf{q}$ $\quad\mathbf{p}x \to x\mathbf{q}0$ *(0 blank)*

□

Definition 3: (Variant **CPM2**)
The instructions are of the form
$\mathbf{p}x \to \mathbf{q}$ $\quad\mathbf{p}x \to y\mathbf{q}$ $\quad\mathbf{p}x \to y\mathbf{q}0$ *(0 blank)*

□

Definition 4: (Variant **CPM3**)
The instructions are of the form
$\mathbf{p}x \to \mathbf{q}$ $\quad\mathbf{p}x \to y\mathbf{q}$ $\quad\mathbf{p}x \to yz\mathbf{q}$.

□

Definition 5: (Variant **CPM4**)
The instructions are of the form
$\mathbf{p}x \to \mathbf{q}$ $\quad\mathbf{p}x \to y\mathbf{q}$ $\quad\mathbf{p}x \to yx\mathbf{q}$

□

Lemma 1: *All variants of Circular Post machines are equivalent.*
Proof : Variant 0 simulates variants 1, 2, 3, 4 by
$\mathbf{p}xu \to \bar{0}\mathbf{q}_1 u \to \mathbf{q}_1\bar{0}\bar{u} \to 0\mathbf{q}_2\bar{u} \to \mathbf{q}_2 0u \to x\mathbf{q}0u$
using the instructions ($\bar{u}$ denoting u with 0 replaced by $\bar{0}$)

$\mathbf{p}x \to \bar{0}\mathbf{q}_1$		
$\mathbf{q}_1 s \to s\mathbf{q}_1$ ($s \neq 0$, $s \neq \bar{0}$)	$\mathbf{q}_1 0 \to \bar{0}\mathbf{q}_1$	$\mathbf{q}_1\bar{0} \to 0\mathbf{q}_2$
$\mathbf{q}_2 s \to s\mathbf{q}_2$ ($s \neq 0$, $s \neq \bar{0}$)	$\mathbf{q}_2 0 \to x\mathbf{q}0$	$\mathbf{q}_2\bar{0} \to 0\mathbf{q}_2$

$\mathbf{p}xu \to \bar{0}\mathbf{q}_1 u \to \mathbf{q}_1\bar{0}\bar{u} \to 0\mathbf{q}_2\bar{u} \to \mathbf{q}_2 0u \to y\mathbf{q}0u$
using the instructions

$\mathbf{p}x \to \bar{0}\mathbf{q}_1$		
$\mathbf{q}_1 s \to s\mathbf{q}_1$ ($s \neq 0$, $s \neq \bar{0}$)	$\mathbf{q}_1 0 \to \bar{0}\mathbf{q}_1$	$\mathbf{q}_1\bar{0} \to 0\mathbf{q}_2$
$\mathbf{q}_2 s \to s\mathbf{q}_2$ ($s \neq 0$, $s \neq \bar{0}$)	$\mathbf{q}_2 0 \to y\mathbf{q}0$	$\mathbf{q}_2\bar{0} \to 0\mathbf{q}_2$

$\mathbf{p}xu \to \bar{0}\mathbf{q}_1 u \to \mathbf{q}_1\bar{0}\bar{u} \to 0\mathbf{q}_2\bar{u} \to \mathbf{q}_2 0u \to y\mathbf{q}_3 0u \to yz\mathbf{q}u$
using the instructions

$\mathbf{p}x \to \bar{0}\mathbf{q}_1$ $\mathbf{q}_1 s \to s\mathbf{q}_1$ $(s \neq 0,\ s \neq \bar{0})$ $\mathbf{q}_2 s \to s\mathbf{q}_2$ $(s \neq 0,\ s \neq \bar{0})$	$\mathbf{q}_1 0 \to \bar{0}\mathbf{q}_1$ $\mathbf{q}_2 0 \to y\mathbf{q}_3 0$ $\mathbf{q}_3 0 \to z\mathbf{q}$	$\mathbf{q}_1\bar{0} \to 0\mathbf{q}_2$ $\mathbf{q}_2\bar{0} \to 0\mathbf{q}_2$

where the states $\mathbf{q}_i$ are understood to carry the information yz as $\mathbf{q}_1(yz)$, $\mathbf{q}_2(yz)$ and $\mathbf{q}_3(z)$.

CPM4 is a special case of **CPM3**.

Variant 1 simulates variant 0 by
$\mathbf{p}xu \to \bar{y}\mathbf{q}_1 u \to \mathbf{q}_1\bar{y}u \to \bar{y}\mathbf{q}_2 0u \to \mathbf{q}_2\bar{y}0u \to y\mathbf{q}0u$
using instructions

$\mathbf{p}x \to \bar{y}\mathbf{q}_1$ $\mathbf{q}_1 s \to s\mathbf{q}_1$ $\mathbf{q}_2 s \to s\mathbf{q}_2$	$\mathbf{q}_1\bar{s} \to \bar{s}\mathbf{q}_2 0$ $\mathbf{q}_2\bar{s} \to s\mathbf{q}$

Variant 2 includes variant 1 as a special case.

Variant 3 simulates variant 0 by
$\mathbf{p}xu \to \bar{x}0\mathbf{q}_1 u \to \mathbf{q}_1\bar{x}0u \to x\mathbf{q}0u$
using instructions

$\mathbf{p}x \to \bar{x}0\mathbf{q}_1$ $\mathbf{q}_1 s \to s\mathbf{q}_1$	$\mathbf{q}_1\bar{s} \to s\mathbf{q}$

Variant 4 simulates variant 0 by
$\mathbf{p}xu \to \tilde{0}\mathbf{q}_1 u \to \mathbf{q}_1\tilde{0}u \to \bar{y}\tilde{0}\mathbf{q}_2 \to \mathbf{q}_2\bar{y}\tilde{0}u \to \bar{y}\mathbf{q}_3\tilde{0}u \to \bar{y}0\mathbf{q}_3 u \to \mathbf{q}_3\bar{y}0u \to y\mathbf{q}0u$
using instructions

$\mathbf{p}x \to \tilde{0}\mathbf{q}_1$ $\mathbf{q}_1 s \to s\mathbf{q}_1$ $(s \neq \tilde{0})$ $\mathbf{q}_2 s \to s\mathbf{q}_2$ $\mathbf{q}_3 s \to s\mathbf{q}_3$	$\mathbf{q}_2\bar{s} \to \bar{s}\mathbf{q}_3$ $\mathbf{q}_3\bar{s} \to s\mathbf{q}$	$\mathbf{q}_1\tilde{0} \to \bar{y}\tilde{0}\mathbf{q}_2$ $\mathbf{q}_3\tilde{0} \to 0\mathbf{q}_3$

□

Proposition 1: *Any Turing machine (in quintuple version) can be simulated by a Turing machine with the following restrictions.*

1 : The configuration is represented by $\$u\mathbf{p}xv\$$ *where \$ is a marker for the left and right end of the tape inscription (the rest of the tape contains only blanks* 0*)*

2 : The inscription is enlarged on the left end by

$0\mathbf{p}\$u \rightarrow \mathbf{q}_1 00u \rightarrow \$\mathbf{q}_2 0u \rightarrow \mathbf{q}\$0u$

and on the right end by

$u\mathbf{p}\$0 \rightarrow u0\mathbf{q}_1 0 \rightarrow u\mathbf{q}_2 0\$ \rightarrow u0\mathbf{q}\$$

3 : The inscription is shortened on the left end by

$\mathbf{p}\$0xu \rightarrow 0\mathbf{q}_1 0xu \rightarrow 0\$\mathbf{q}_2 xu \rightarrow 0\mathbf{q}\$xu$

and on the right end by

$ux0\mathbf{p}\$ \rightarrow ux\mathbf{q}_1 00 \rightarrow u\mathbf{q}_2 x\$0 \rightarrow ux\mathbf{q}\$0$

4 : A left move is given by

$uz\mathbf{p}xv \rightarrow u\mathbf{q}zyv$

and a right move by

$u\mathbf{p}xzv \rightarrow uy\mathbf{q}zv.$

□

Theorem 1: *Any Turing machine can be simulated by a Circular Post machine of variant 0.*

Proof : A configuration $\$v\mathbf{p}u\$$ of a Turing machine is represented by $\mathbf{p}u\$v$ on a circular tape.

Adding a 0 at the left is simulated by ($\bar{u}$ denoting u with 0 replaced by $\bar{0}$)

$\mathbf{p}\$u \rightarrow \bar{\$}\mathbf{q}_1 u \rightarrow \mathbf{q}_1\bar{\$}\bar{u} \rightarrow 0\mathbf{q}_2\bar{u} \rightarrow \mathbf{q}_2 0\bar{u} \rightarrow 0\mathbf{q}_3 0\bar{u} \rightarrow 0\$\mathbf{q}_4 0\bar{u} \rightarrow 0\$0\mathbf{q}_5\bar{u} \rightarrow \mathbf{q}_5 0\$0u \rightarrow \mathbf{q}\$0u$

using the instructions

$\mathbf{p}\$ \rightarrow \bar{\$}\mathbf{q}_1$ $\mathbf{q}_1 x \rightarrow x\mathbf{q}_1$ ($x \neq 0$, $x \neq \bar{\$}$) $\mathbf{q}_2 x \rightarrow x\mathbf{q}_2$ ($x \neq 0$) $\mathbf{q}_3 0 \rightarrow \$\mathbf{q}_4 0$ $\mathbf{q}_4 0 \rightarrow 0\mathbf{q}_5$ $\mathbf{q}_5 x \rightarrow x\mathbf{q}_5$ ($x \neq 0$, $x \neq \bar{0}$)	 $\mathbf{q}_1 0 \rightarrow \bar{0}\mathbf{q}_1$ $\mathbf{q}_2 0 \rightarrow 0\mathbf{q}_3 0$ $\mathbf{q}_5 0 \rightarrow \mathbf{q}$	 $\mathbf{q}_1 \bar{\$} \rightarrow 0\mathbf{q}_2$ $\mathbf{q}_5 \bar{0} \rightarrow 0\mathbf{q}_5$

Adding a 0 at the right by ($\bar{u}$ denoting u with 0 replaced by $\bar{0}$)

$\mathbf{p}\$u \rightarrow \bar{0}\mathbf{q}_1 u \rightarrow \mathbf{q}_1\bar{0}\bar{u} \rightarrow 0\mathbf{q}_2\bar{u} \rightarrow \mathbf{q}_2 0\bar{u} \rightarrow 0\mathbf{q}_3 0\bar{u} \rightarrow 0\$\mathbf{q}_4\bar{u} \rightarrow \mathbf{q}_4 0\$u \rightarrow 0\mathbf{q}\$u$

using the instructions

$\mathbf{p}\$ \rightarrow \bar{0}\mathbf{q}_1$ $\mathbf{q}_1 x \rightarrow x\mathbf{q}_1$ ($x \neq 0$, $x \neq \bar{0}$) $\mathbf{q}_2 x \rightarrow x\mathbf{q}_2$ ($x \neq 0$) $\mathbf{q}_3 0 \rightarrow \$\mathbf{q}_4$ $\mathbf{q}_4 x \rightarrow \mathbf{q}_4$ ($x \neq 0$, $x \neq \bar{0}$)	 $\mathbf{q}_1 0 \rightarrow \bar{0}\mathbf{q}_1$ $\mathbf{q}_2 0 \rightarrow 0\mathbf{q}_3 0$ $\mathbf{q}_4 \bar{0} \rightarrow 0\mathbf{q}_4$	 $\mathbf{q}_1 \bar{0} \rightarrow 0\mathbf{q}_2$ $\mathbf{q}_4 0 \rightarrow 0\mathbf{q}$

For erasing 0 at the left or right end it may be assumed that 0 has been written there just before.

Erasing 0 at the left is simulated by

$\mathbf{p}\$0u \rightarrow \bar{0}\mathbf{q}_1 0u \rightarrow \bar{0}\$\mathbf{q}_2 u \rightarrow \mathbf{q}_2\bar{0}\$u \rightarrow \mathbf{q}\$u$

using instructions

$\mathbf{p}\$ \rightarrow \bar{0}\mathbf{q}_1$ $\mathbf{q}_1 0 \rightarrow \$\mathbf{q}_2$ $\mathbf{q}_2 x \rightarrow x\mathbf{q}_2$ ($x \neq \bar{0}$)	 $\mathbf{q}_2\bar{0} \rightarrow \mathbf{q}$

Erasing 0 at the right by

$\mathbf{p}\$u0 \rightarrow \$\mathbf{q}_1 v0xw0 \rightarrow \$v\mathbf{q}_1 0xw0 \rightarrow \$v\bar{0}\mathbf{q}_2 xw0 \rightarrow \$v\bar{0}x\mathbf{q}_3 w0 \rightarrow \$v\mathbf{q}_3\bar{0}xw0 \rightarrow \$v0\mathbf{q}_1 xw0 \rightarrow \cdots \rightarrow \$u\mathbf{q}_1 0 \rightarrow \mathbf{q}_2\$u\bar{0} \rightarrow \$\mathbf{q}_4 u\bar{0} \rightarrow \$u\mathbf{q}_4\bar{0} \rightarrow \mathbf{q}\$u$

using instructions

$\mathbf{q}\$ \to \$\mathbf{q}_1$	
$\mathbf{q}_1 x \to x\mathbf{q}_1 \ (x \neq 0)$	$\mathbf{q}_1 0 \to \bar{0}\mathbf{q}_2$
$\mathbf{q}_2 x \to x\mathbf{q}_3 \ (x \neq \$)$	$\mathbf{q}_2 \$ \to \$\mathbf{q}_4$
$\mathbf{q}_3 x \to x\mathbf{q}_3 \ (x \neq \bar{0})$	$\mathbf{q}_3 \bar{0} \to 0\mathbf{q}_1$
$\mathbf{q}_4 x \to x\mathbf{q}_4 \ (x \neq \bar{0})$	$\mathbf{q}_4 \bar{0} \to \mathbf{q}$

Moving right is simulated by
$\mathbf{p}xu \to y\mathbf{q}u$
using instruction

$\mathbf{p}x \to y\mathbf{q}$

Moving left by

1. length 1:
$\mathbf{p}x \to \tilde{x}\mathbf{q}_1 \to \mathbf{q}x$
2. length 2:
$\mathbf{p}xz \to \tilde{y}\mathbf{q}_1 z \to \mathbf{q}_2\tilde{y}\bar{z} \to \tilde{y}\mathbf{q}_3\bar{z} \to \mathbf{q}_4\tilde{y}z \to y\mathbf{q}z$
3. length 3:
$\mathbf{p}xsz \to \tilde{y}\mathbf{q}_1 sz \to \tilde{y}\bar{s}\mathbf{q}_2 z \to \mathbf{q}_3\tilde{y}\bar{s}z \to y\mathbf{q}_4\bar{s}z \to ys\mathbf{q}z$
4. length 4:
$\mathbf{p}xstz \to \tilde{y}\mathbf{q}_1 stz \to \tilde{y}\bar{s}\mathbf{q}_2 tz \to \tilde{y}\bar{s}t\mathbf{q}_3 z \to \mathbf{q}_5\tilde{y}\bar{s}tz \to \tilde{y}\mathbf{q}_5\bar{s}tz \to \tilde{y}s\mathbf{q}_1 tz \to \tilde{y}s\bar{t}\mathbf{q}_2 z \to \mathbf{q}_3\tilde{y}s\bar{t}z \to y\mathbf{q}_4 s\bar{t}z \to yst\mathbf{q}z$
5. length ≥ 4:
$\mathbf{p}xsturz \to \tilde{y}\mathbf{q}_1 sturz \to \tilde{y}\bar{s}\mathbf{q}_2 turz \to \tilde{y}\bar{s}t\mathbf{q}_3 urz \to \cdots \to \tilde{y}\bar{s}tru\mathbf{q}_5 z \to \mathbf{q}_5\tilde{y}\bar{s}turz \to \tilde{y}\mathbf{q}_5\bar{s}turz \to \tilde{y}s\mathbf{q}_1 turz \to \cdots \to \tilde{y}s\bar{t}ur\mathbf{q}_5 z \to \cdots \to \tilde{y}stu\mathbf{q}_1 rz \to \tilde{y}stu\bar{r}\mathbf{q}_2 z \to \mathbf{q}_3\tilde{y}stu\bar{r}z \to y\mathbf{q}_4 stu\bar{r}z \to ystu\mathbf{q}_4\bar{r}z \to ystu\mathbf{q}z$

using instructions

$\mathbf{p}x \to \tilde{y}\mathbf{q}_1$		
$\mathbf{q}_1 x \to \bar{x}\mathbf{q}_2$		$\mathbf{q}_1\tilde{x} \to x\mathbf{q}$
$\mathbf{q}_2 x \to x\mathbf{q}_3$		$\mathbf{q}_2\tilde{x} \to \tilde{x}\mathbf{q}_3$
$\mathbf{q}_3 x \to x\mathbf{q}_5$	$\mathbf{q}_3\bar{x} \to x\mathbf{q}_4$	$\mathbf{q}_3\tilde{x} \to x\mathbf{q}_4$
$\mathbf{q}_4 x \to x\mathbf{q}_4$	$\mathbf{q}_4\bar{x} \to x\mathbf{q}$	$\mathbf{q}_4\tilde{x} \to x\mathbf{q}$
$\mathbf{q}_5 x \to x\mathbf{q}_5$	$\mathbf{q}_5\bar{x} \to x\mathbf{q}_1$	$\mathbf{q}_5\tilde{x} \to \tilde{x}\mathbf{q}_5$

□

Theorem 2: *For any* **CPMi** *(i = 2, 3, 4) there exists an equivalent* **CPMi** *with 2 states (excluding the halting state).*

Proof : We use a method as in [11, 2]. Let the states of the simulated CPM0 be $\{\mathbf{1}, \cdots, \mathbf{n}\}$.

An instruction $\mathbf{p}x \to \mathbf{q}$ is simulated by

length 1 :

$\mathbf{1}\binom{x}{p} \to \binom{\hat{x}}{q}\mathbf{2} \to \mathbf{1}$

length ≥ 2 :

$\mathbf{1}\binom{x}{p}su \to \binom{\hat{x}}{q}\mathbf{2}su \to \binom{\hat{x}}{q}\binom{s}{0}\mathbf{1}u \to \cdots \to \mathbf{1}\binom{\hat{x}}{q}\binom{s}{0}u \to \binom{\hat{x}}{q-1}\mathbf{2}\binom{s}{0}u \to \binom{\hat{x}}{q-1}\binom{s}{1}\mathbf{1}u \to \cdots \to \mathbf{1}\binom{\hat{x}}{0}\binom{s}{q}u \to \mathbf{1}\binom{s}{q}u$

using instructions

$\mathbf{1}\binom{x}{p} \to \binom{\hat{x}}{q}\mathbf{2}$	$\mathbf{2}\binom{\hat{x}}{q} \to \mathbf{1}$
$\mathbf{1}s \to s\mathbf{1}$	$\mathbf{2}s \to \binom{s}{0}\mathbf{1}$
$\mathbf{1}\binom{\hat{x}}{i} \to \binom{\hat{x}}{i-1}\mathbf{2} \quad (1 \leq i \leq q)$	$\mathbf{2}\binom{s}{i} \to \binom{s}{i+1}\mathbf{1} \quad (0 \leq i < q)$
$\mathbf{1}\binom{\hat{x}}{0} \to \mathbf{1}$	

An instruction $\mathbf{p}x \to y\mathbf{q}$ is simulated by

length 1 :

$\mathbf{1}\binom{x}{p} \to \binom{\bar{y}}{q}\mathbf{2} \to \mathbf{1}\binom{y}{q}$

length ≥ 2 :

$\mathbf{1}\binom{x}{p}su \to \binom{\bar{y}}{q}\mathbf{2}su \to \binom{\bar{y}}{q}\binom{s}{0}\mathbf{1}u \to \cdots \to \mathbf{1}\binom{\bar{y}}{q}\binom{s}{0}u \to \binom{\bar{y}}{q-1}\mathbf{2}\binom{s}{0}u \to \binom{\bar{y}}{q-1}\binom{s}{1}\mathbf{1}u \to \cdots \to \mathbf{1}\binom{\bar{y}}{0}\binom{s}{q}u \to y\mathbf{1}\binom{s}{q}u$

using instructions

$\mathbf{1}\binom{x}{p} \to \binom{\bar{y}}{q}\mathbf{2}$	$\mathbf{2}\binom{\bar{y}}{q} \to \binom{y}{q}\mathbf{1}$
$\mathbf{1}s \to s\mathbf{1}$	$\mathbf{2}s \to \binom{s}{0}\mathbf{1}$
$\mathbf{1}\binom{\bar{y}}{i} \to \binom{\bar{y}}{i-1}\mathbf{2} \quad (1 \leq i \leq q)$	$\mathbf{2}\binom{s}{i} \to \binom{s}{i+1}\mathbf{1} \quad (0 \leq i < q)$
$\mathbf{1}\binom{\bar{y}}{0} \to y\mathbf{1}$	

An instruction $\mathbf{p}x \to y\mathbf{q}0$ is simulated by
$\mathbf{1}\binom{x}{p}u \to \binom{\tilde{y}}{q}\mathbf{1}\bar{0}u \to \binom{\bar{y}}{q}\binom{0}{0}\mathbf{1}u \to \mathbf{1}\binom{\bar{x}}{q}\binom{0}{0}u \to \binom{\bar{y}}{q-1}\mathbf{2}\binom{0}{0}u \to \binom{\bar{y}}{q-1}\binom{0}{1}\mathbf{1}u \to \cdots \to \mathbf{1}\binom{\bar{y}}{0}\binom{0}{q}u \to y\mathbf{1}\binom{0}{q}u$
using instructions

$\mathbf{1}\binom{x}{p} \to \binom{\bar{y}}{q}\mathbf{1}\bar{0}$ $\mathbf{1}\bar{0} \to \binom{0}{0}\mathbf{1}$	and then as above ($\bar{0}$ is the new blank).

An instruction $\mathbf{p}x \to yz\mathbf{q}$ is simulated by

length 1 :
$\mathbf{1}\binom{x}{p} \to y\binom{\bar{z}}{q}\mathbf{2} \to \binom{y}{0}\mathbf{1}\binom{\bar{z}}{q} \to \cdots \to \binom{y}{q}\mathbf{1}\binom{\bar{z}}{0} \to \mathbf{1}\binom{y}{q}z$

length ≥ 2 :
$\mathbf{1}\binom{x}{p}su \to y\binom{\bar{z}}{q}\mathbf{2}su \to y\binom{\bar{z}}{q}\binom{s}{0}\mathbf{1}u \to y\mathbf{1}\binom{\bar{z}}{q}\binom{s}{0}u \to y\binom{\bar{z}}{q-1}\mathbf{2}\binom{s}{0}u \to y\binom{\bar{z}}{q-1}\binom{s}{1}\mathbf{1}u \to \cdots \to y\mathbf{1}\binom{\bar{z}}{0}\binom{s}{q}u \to yz\mathbf{1}\binom{s}{q}u$
using instruction

$\mathbf{1}\binom{x}{p} \to y\binom{\bar{z}}{q}\mathbf{2}$	and then as above.

An instruction $\mathbf{p}x \to zx\mathbf{q}$ is simulated similarly by

length 1 :
$\mathbf{1}\binom{x}{p} \to \mathbf{1}\binom{zx}{q} \to \mathbf{1}\binom{\tilde{z}}{q}\binom{zx}{q} \to z\mathbf{2}\binom{zx}{q} \to \mathbf{2}z\binom{\bar{x}}{q} \to \binom{z}{0}\mathbf{1}\binom{\bar{x}}{q} \to \cdots \to x\mathbf{1}\binom{z}{q}$

length ≥ 2 :
$\mathbf{1}\binom{x}{p}su \to \binom{zx}{q}\mathbf{1}su \to \cdots \to \mathbf{1}\binom{zx}{q}su \to \binom{\tilde{z}}{q}\binom{zx}{q}\mathbf{1}su \to \mathbf{1}\binom{\tilde{z}}{q}\binom{zx}{q}su \to z\mathbf{2}\binom{zx}{q}su \to z\binom{\bar{x}}{q}\mathbf{2}su \to z\binom{\bar{x}}{q}\binom{s}{0}\mathbf{1}u \to \cdots \to zx\mathbf{1}\binom{s}{q}u$

using instructions

$\mathbf{1}\binom{x}{p} \to \binom{zx}{q}\mathbf{1}$ $\mathbf{1}\binom{zx}{q} \to \binom{\tilde{z}}{q}\binom{zx}{q}\mathbf{1}$ $\mathbf{1}\binom{\tilde{z}}{q} \to z\mathbf{2}$	$\mathbf{2}\binom{zx}{q} \to \binom{\bar{x}}{q}\mathbf{2}$	and then as above.

□

Theorem 3: *For every* **CPMi** *(i = 0, 1, 2, 3, 4) there exists an equivalent* **CPMi** *with 2 symbols.*
Proof : Let the alphabet be $\{s_1 \cdots, s_m\}$. Let $k = \lfloor log_2(m) \rfloor + 2$

The symbols s_i are encoded by $1bin_{k-1}(i)$ where $bin_{k-1}(i)$ is the binary encoding of i in $k-1$ digits. Note that 0 is encoded by 10^{k-1}, and that 0^k will be used for special information. Let x be encoded by $x_1 \cdots x_k$. Then

$$\mathbf{p}x_1 \cdots x_k u \to 0\hat{\mathbf{p}}(x_1)x_2 \cdots x_k u \to \cdots$$
$$\cdots \to 0^{k-1}\hat{\mathbf{p}}(x_1 \cdots x_{k-1})x_k u \to 0^k \bar{\mathbf{p}}_1(x)u$$

using instructions

$\mathbf{p}s_1 \to 0\hat{\mathbf{p}}(s_1)$	
$\hat{\mathbf{p}}(s_1 \cdots s_i)s_{i+1} \to 0\hat{\mathbf{p}}(s_1 \cdots s_{i+1})$	$(2 \leq i < k)$
$\hat{\mathbf{p}}(s_1 \cdots s_{k-1})s_k \to 0\bar{\mathbf{p}}_1(s_1 \cdots s_k)$	

Instructions

$\bar{\mathbf{p}}_1(x)s \to s\bar{\mathbf{p}}_2(x)$	$(s \neq 0)$
$\bar{\mathbf{p}}_i(x)t \to t\bar{\mathbf{p}}_{i+1}(x)$	$(2 \leq i < k)$
$\bar{\mathbf{p}}_k(x)t \to t\bar{\mathbf{p}}_1(x)$	

give $\quad 0^k\bar{\mathbf{p}}_1(x)u \to \cdots \to \bar{\mathbf{p}}_1(x)0^k u.$

An instruction $\mathbf{p}x \to \mathbf{q}$ is simulated by

$\bar{\mathbf{p}}_1(x)0^k u \to \tilde{\mathbf{p}}_2 0^{k-1}u \to \cdots \to \tilde{\mathbf{p}}_k 0u \to \mathbf{q}$

using instructions

$\bar{\mathbf{p}}_1(x)0 \to \tilde{\mathbf{p}}_2$	
$\tilde{\mathbf{p}}_i 0 \to \tilde{\mathbf{p}}_{i+1}$	$(2 \leq i < k)$
$\tilde{\mathbf{p}}_k 0 \to \mathbf{q}$	

An instruction $\mathbf{p}x \to y\mathbf{q}$ is simulated by

$\bar{\mathbf{p}}_1(x)0^k u \to y_1\tilde{\mathbf{p}}_2(x)0^{k-1}u \to \cdots \to y_1 \cdots y_{k-1}\tilde{\mathbf{p}}_k(x)0u \to y_1 \cdots y_k \mathbf{q}u$

using instructions

$$\begin{array}{|ll|}\hline \bar{\mathbf{p}}_1(x)0 \rightarrow y_1\tilde{\mathbf{p}}_2(x) & \\ \tilde{\mathbf{p}}_i(x)0 \rightarrow y_i\tilde{\mathbf{p}}_{i+1}(x) & (2 \leq i < k) \\ \tilde{\mathbf{p}}_k(x)0 \rightarrow y_k\mathbf{q} & \\ \hline \end{array}$$

To simulate instructions with insertion we use instructions

$$\begin{array}{|l|}\hline \bar{\mathbf{p}}_1(x)0 \rightarrow 0\hat{\mathbf{p}}_1(x)0 \\ \hat{\mathbf{p}}_1(x)0 \rightarrow 0\dot{\mathbf{p}}_2(x) \\ \dot{\mathbf{p}}_i(x)0 \rightarrow 0\hat{\mathbf{p}}_i(x)0 \\ \hat{\mathbf{p}}_i(x)0 \rightarrow 0\dot{\mathbf{p}}_{i+1}(x) \\ \dot{\mathbf{p}}_k(x)0 \rightarrow 0\hat{\mathbf{p}}_k(x)0 \\ \hat{\mathbf{p}}_k(x)0 \rightarrow 0\tilde{\mathbf{p}}_1(x) \\ \hline \end{array}$$

or

$$\begin{array}{|l|}\hline \bar{\mathbf{p}}_1(x)0 \rightarrow 00\hat{\mathbf{p}}_2(x) \\ \hat{\mathbf{p}}_i(x)0 \rightarrow 00\hat{\mathbf{p}}_{i+1}(x) \\ \hat{\mathbf{p}}_k(x)0 \rightarrow 00\tilde{\mathbf{p}}_1(x) \\ \hline \end{array}$$

together with

$$\begin{array}{|ll|}\hline \tilde{\mathbf{p}}_1(x)s \rightarrow s\tilde{\mathbf{p}}_2(x) & (s \neq 0) \\ \tilde{\mathbf{p}}_i(x)t \rightarrow t\tilde{\mathbf{p}}_{i+1}(x) & (2 \leq i < k) \\ \tilde{\mathbf{p}}_k(x)t \rightarrow t\tilde{\mathbf{p}}_1(x) & \\ \hline \end{array}$$

giving $\quad \bar{\mathbf{p}}_1(x)0^k u \rightarrow 0^k 0^k \tilde{\mathbf{p}}_1(x)u \rightarrow \tilde{\mathbf{p}}_1(x)0^k 0^k u$.

Instructions $\mathbf{p}x \rightarrow yz\mathbf{q}$, $\mathbf{p}x \rightarrow yx\mathbf{q}$ are simulated by using instructions (replacing z by x for the second case)

$$\begin{array}{|l|}\hline \tilde{\mathbf{p}}_1(x)0 \rightarrow y_1\check{\mathbf{p}}_2(x) \\ \check{\mathbf{p}}_i(x)0 \rightarrow y_i\check{\mathbf{p}}_{i+1}(x) \\ \check{\mathbf{p}}_k(x)0 \rightarrow y_k\breve{\mathbf{p}}_1(x) \\ \breve{\mathbf{p}}_i(x)0 \rightarrow z_i\breve{\mathbf{p}}_{i+1}(x) \\ \breve{\mathbf{p}}_k(x)0 \rightarrow z_k\mathbf{q} \\ \hline \end{array}$$

giving $\quad \tilde{\mathbf{p}}_1(x)0^k 0^k u \rightarrow y_1 \cdots y_k z_1 \cdots z_k \mathbf{q} u$.

Instructions $\mathbf{p}0 \rightarrow y\mathbf{q}0$, $\mathbf{p}x \rightarrow x\mathbf{q}0$, $\mathbf{p}x \rightarrow y\mathbf{q}0$ are simulated by using instructions (only given for the last case)

$\tilde{\mathbf{p}}_1(x)0 \to 0\ddot{\mathbf{p}}_1(x)$	$\breve{\mathbf{p}}_i(x)0 \to 0\breve{\mathbf{p}}_{i+1}(x) \quad (2 \le i < k)$
$\ddot{\mathbf{p}}_i(x)0 \to 0\ddot{\mathbf{p}}_{i+1}(x) \quad (1 \le i < k)$	$\breve{\mathbf{p}}_k(x)0 \to 0\breve{\mathbf{p}}_1(x)$
$\ddot{\mathbf{p}}_k(x)0 \to 0\check{\mathbf{p}}_1(x)$	$\breve{\mathbf{p}}_1(x)s \to s\breve{\mathbf{p}}_2(x) \quad (s \neq 0)$
$\check{\mathbf{p}}_1(x)0 \to 1\breve{\mathbf{p}}_2(x)$	$\breve{\mathbf{p}}_i(x)t \to t\breve{\mathbf{p}}_{i+1}(x) \quad (2 \le i < k)$
	$\breve{\mathbf{p}}_k(x)t \to t\breve{\mathbf{p}}_1(x)$

This gives $\tilde{\mathbf{p}}_1(x)0^k0^ku \to 0^k\check{\mathbf{p}}_1(x)0^ku \to 0^k10^{k-1}\breve{\mathbf{p}}_1(x)u \to \breve{\mathbf{p}}_1(x)0^k10^{k-1}u$.

Finally, using instructions

$\breve{\mathbf{p}}_1(x)0 \to y_1\acute{\mathbf{p}}_2(x)$
$\acute{\mathbf{p}}_i(x)0 \to y_i\acute{\mathbf{p}}_{i+1}(x) \quad (2 \le i < k)$
$\acute{\mathbf{p}}_k(x)0 \to y_k\mathbf{q}$

gives $\breve{\mathbf{p}}_1(x)0^k10^{k-1}u \to y_1 \cdots y_k\mathbf{q}10^{k-1}u.$

□

The following CPM0(6,2) simulates the still unsolved problem stated by Emil Post in 1921 [7, 5] to decide whether the iteration of the 3-tag system $0 \to 00$, $I \to II0I$ on a word $w \in \{0, I\}^+$ either ends on one of $\{0, I, 00, 0I, I0, II\}$, enters a loop, or diverges.

	0	I
1	**2**	I**4**I
2	0**3**	0**3**
3	0**1**	0**1**
4		I**5**
5	0**6**	0**6**
6	I**1**	I**1**

blank : I

□

The next Machine CPM0(5,3) simulates another unsolved (?) problem, namely the *Collatz* or $(3n+1)$ problem. This is to decide whether the procedure

$n \to \frac{n}{2}$ if $n = 2m$, $n \to 3n+1$ if $n = 2m+1$

always enters the loop $(1, 4, 2, 1)$ (this is the conjecture), enters another loop or diverges.

	0	1	c
$\mathbf{p}_0$	$\mathbf{p}_0$	$0\mathbf{p}_{11}$	
$\mathbf{p}_{00}$	$0\mathbf{p}_{00}$	$1\mathbf{p}_{10}$	$c\mathbf{p}_0$
$\mathbf{p}_{01}$	$1\mathbf{p}_{00}$	$0\mathbf{p}_{11}$	$1\mathbf{p}_{00}c$
$\mathbf{p}_{10}$	$1\mathbf{p}_{00}$	$0\mathbf{p}_{11}$	$1\mathbf{p}_{00}c$
$\mathbf{p}_{11}$	$0\mathbf{p}_{01}$	$1\mathbf{p}_{11}$	$0\mathbf{p}_{01}c$

blank : c

An integer n is encoded in binary by $n_1 \cdots n_k c$ with $n_k \neq 0$, and the initial configuration is $\mathbf{p}_0 n_1 \cdots n_k c$. In case $n = 2m+1$ the machine simulates the addition $2n + n + 1$ with the index i of $\mathbf{p}_{ij}$ denoting the symbol last read, and j the carrier.

□

3 Universal Circular Post Machines

In this part we present some small universal machines of variant 0, with the halting state not included but represented by **H** in the program.

The machines are constructed by simulation of tag systems. From [4] it is known that 2-tag systems suffice, and that halting occurs only if a special symbol s_H is encountered. Let the alphabet be $\Sigma = \{s_1, \cdots, s_{n+1}\}$ with $s_H = s_{n+1}$. A symbol s_i is encoded in unary form by some number N_i, together with a separator. In a 2-tag instruction $s_i \to \alpha_i$ with $\alpha_i = a_{i1} \cdots a_{im(i)}$ the symbols a_{ij} are encoded in the same way, with other separators.

In the tables an entry y stands for an instruction $\mathbf{p}x \to y\mathbf{p}$, and an entry $\mathbf{q}$ for an instruction $\mathbf{p}x \to \mathbf{q}$.

UPM0(13,4)

	I	c	b	a
1	**2**	**6**	**H**	
2	I	c	a**3**	a**4**
3	I**5**	c**1**	b	
4	c	c	a**3**	a
5	I	c**6**	b	a**3**
6	**6**	**7**	b**1**	
7	I	c		a**8**
8	c**9**	c	a**0**	a
9	I	c	b	I**8**a
0	I**A**	c	b**B**	
A	I	c	b	c**8**a
B	I	c	b	c**C**a
C		I	b**5**	b

$N_1 = 2$, $N_{k+1} = N_k + m_k + 1$ $(1 \le k < n)$
blank : a
Encoding of symbol s_i: $I^{N_i}c$
Encoding of α_i :
$I^{N_{i1}}b \cdots bI^{N_{im(i)}}bb$
Separators : b, bcb

The initial configuration is

$$bbI^{N_{11}}b \cdots bI^{N_{1m(1)}}bb \cdots bbI^{N_{n1}}b \cdots bI^{N_{nm(n)}}bbbcb\mathbf{1}I^{N_r}cI^{N_s}c \cdots cI^{N_w}c.$$

In the first stage I^{N_r} is read, N_r b's are changed into a's, the I's in the middle - into c's, and $I^{N_r}cI^{N_s}c$ is erased, giving

$$aac^{N_{11}}a \cdots ac^{N_{1m(1)}}aa \cdots aaI^{N_{r1}}b \cdots bI^{N_{rm(r)}}bb \cdots$$
$$\cdots bI^{N_{nm(n)}}bbbcb\mathbf{8}I^{N_t}c \cdots cI^{N_w}c.$$

In the second stage, starting with **8**, the part $I^{N_{r1}}b \cdots I^{N_{rm(r)}}bb$ is copied to the end of $I^{N_t}c \cdots I^{N_w}c$ as $I^{N_{r1}}c \cdots cI^{N_{rm(r)}}c$.

In the third stage, starting with **B**, the instruction part of the tape is restored, and a new cycle may start.

The machine stops if in the first stage **4** encounters bcb.

□

UPM0(11,5)

	I	c	a	b	d
1	**2**	**3**	a**7**	b**9**	d**4**
2	I	c	I	c**3**	d
3	**3**	**4**	a	b	d**1**
4	I	c	I**5**	c**1**	d**5**
5	**H**	c**4**	a	b	d**6**
6	I	c			I**1**d
7			a	b	d**8**
8	I	c			c**4**d
9			a	b	d**0**
0	I	c		b**3**	c**A**d
A	a	b		b**3**	d**A**

$N_1 = 1$, $N_{k+1} = N_k + m_k + 1$
$(1 \leq k < n)$
blank : d
Encoding of symbol s_i: $I^{N_i}c$
Encoding of α_i :
$a^{N_{i1}}b \cdots ba^{N_{im(i)}}bb$
Separator : d

The initial configuration is

$$dba^{N_{11}}b \cdots ba^{N_{1m(1)}}bb \cdots bba^{N_{n1}}b \cdots ba^{N_{nm(n)}}bbd\mathbf{1}I^{N_r}cI^{N_s}c \cdots cI^{N_w}c.$$

In the first stage I^{N_r} is read, N_r b's are changed into c's, the a's in the middle - into I's, and $I^{N_r}cI^{N_s}c$ is erased, giving

$$dcI^{N_{11}}c \cdots cI^{N_{1m(1)}}cc \cdots cca^{N_{r1}}b \cdots ba^{N_{rm(r)}}bb \cdots$$

$$\cdots ba^{N_{nm(n)}}bbd\mathbf{4}I^{N_t}c \cdots cI^{N_w}c.$$

In the second stage, starting with **4**, the part $a^{N_{r1}}b \cdots a^{N_{rm(r)}}bb$ is copied to the end of $I^{N_t}c \cdots I^{N_w}c$ as $I^{N_{r1}}c \cdots cI^{N_{rm(r)}}c$.

In the third stage, starting with **A**, the instruction part of the tape is restored, and a new cycle may start.

The machine stops if in the first stage **4** encounters d.

□

UPM0(8,6)

	I	c	a	b	d	e
1	**2**	**3**	a**6**	b**7**		
2	I	c	I	c**3**		e
3	**3**	**4**	a	b	d**1**	
4	I	c	I**5**	c**1**	**H**	e
5	I	c	a	b	d	I**4**e
6	I	c	a	b	d	c**4**e
7	I	c	a	b	d	c**8**e
8	a	b	a	b	d**1**	e

$N_1 = 1$,
$N_{k+1} = N_k + m_k + 1$
$(1 \leq k < n)$
blank : e
Encoding of symbol s_i:$I^{N_i}c$
Encoding of α_i:
$a^{N_{i1}}b \cdots ba^{N_{im(i)}}bb$
Separators : e, d

The initial configuration is

$$eba^{N_{11}}b \cdots ba^{N_{1m(1)}}bb \cdots bba^{N_{n1}}b \cdots ba^{N_{nm(n)}}bbd\mathbf{1}I^{N_r}cI^{N_s}c \cdots cI^{N_w}c.$$

In the first stage I^{N_r} is read, N_r b's are changed into c's, the a's in the middle - into I's, and $I^{N_r}cI^{N_s}c$ is erased, giving

$$ecI^{N_{11}}c \cdots cI^{N_{1m(1)}}cc \cdots cca^{N_{r1}}b \cdots ba^{N_{rm(r)}}bb \cdots$$

$$\cdots ba^{N_{nm(n)}}bbd\mathbf{4}I^{N_t}c \cdots cI^{N_w}c.$$

In the second stage, starting with **4**, the part $a^{N_{r1}}b \cdots a^{N_{rm(r)}}bb$ is copied to the end of $I^{N_t}c \cdots I^{N_w}c$ as $I^{N_{r1}}c \cdots cI^{N_{rm(r)}}c$.

In the third stage, starting with **8**, the instruction part of the tape is restored, and a new cycle may start.

The machine stops if in the first stage **4** encounters d.

□

UPM0(7,7)

	I	c	a	b	d	h	f
1	**2**	**3**	a**6**	b**7**		h**3**	
2	I	c	f	h**3**		h	f
3	**3**	**4**	a	b	d**1**	b	a
4	I	c	f**5**	h**1**	**H**	h	f
5	I	c	a	b	d	I**4**h	
6	I	c	a	b	d	c**4**h	
7	I	c	a	b	d	c**1**h	

$N_1 = 1$,
$N_{k+1} = N_k + m_k + 1$
$(1 \leq k < n)$
blank : h
Encoding of symbol s_i:
$I^{N_i}c$
Encoding of α_i:
$a^{N_{i1}}b \cdots ba^{N_{im(i)}}bb$
Separators : h, d

The initial configuration is

$$hba^{N_{11}}b \cdots ba^{N_{1m(1)}}bb \cdots bba^{N_{n1}}b \cdots ba^{N_{nm(n)}}bbd\mathbf{1}I^{N_r}cI^{N_s}c \cdots cI^{N_w}c.$$

In the first stage I^{N_r} is read, N_r b's are changed into h's, the a's in the middle - into f's, and $I^{N_r}cI^{N_s}c$ is erased, giving

$$hhf^{N_{11}}h \cdots hf^{N_{1m(1)}}hh \cdots hha^{N_{r1}}b \cdots ba^{N_{rm(r)}}bb \cdots$$

$$\cdots ba^{N_{nm(n)}}bbd\mathbf{4}I^{N_t}c \cdots cI^{N_w}c.$$

In the second stage, starting with **4**, the part $a^{N_{r1}}b \cdots a^{N_{rm(r)}}bb$ is copied to the end of $I^{N_t}c \cdots I^{N_w}c$ as $I^{N_{r1}}c \cdots cI^{N_{rm(r)}}c$.

In the third stage, starting with **1**, the instruction part of the tape is restored, and a new cycle may start.

The machine stops if in the first stage **4** encounters d.

□

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