Computer Science Journal of Moldova, vol.8, no.2(23), 2000

Computer Modelling of Dynamic Processes

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Abstract

Results of numerical modeling of dynamic problems are summed in the article up. These problems are characteristic for various areas of human activity, in particular for problem solving in ecology. The following problems are considered in the present work: computer modeling of dynamic effects on elastic-plastic bodies, calculation and determination of performances of gas streams in gas cleaning equipment, modeling of biogas formation processes.

The computers and communication systems are a corner stone of modern company. Not only scientists and engineers, but also the whole branches, such as banking, insurance, transport, public health services essentially depend on modern computers and communication systems. The success of computers is explained by their adaptation to the broadest circle of diverse problems. Growing possibilities of computer systems allow to satisfy increased needs of applications.

The main researches in the field of applied problems are carried out with the help of mathematical modeling of various processes, such as a mechanics of a liquid and gas, mechanics of a deformable solid bodies, structural analysis, modeling of electronic circuits etc. The mathematical modeling of those processes is based on the use of laws of mass, momentum and energy conservation, Navier-Stokes, Hooke, Maxwell, Newton equations.

To development new models in technique – machines, airplanes, rockets, etc., computer experiments are used wider and wider. Use of computer models has many advantages:

- computer modeling is cheaper and faster, than physical experiment;

- computer model is more flexible and permits to decide a broader circle of problems, than the experimental labware;
- computer experiments are limited only by speed and memory of the computer, while the physical experiments have many material restrictions.

The advantages of modeling can considerably speed up project development, and there is also a possibility to find more economical variant satisfying broader request spectrum. Nevertheless, restrictions of computer modeling are also widely known. The calculation complexity can grow with dimensionality of a problem faster, than possibility of computers. Possibilities to solve large number of various scientific problems were constantly increased during the computer era. E.g., it was not long ago that computing systems (CS) permitted to solve boundary problems for two-dimensional partial differential equations an a grid of ~ 1000 points. Now CS allow to solve systems of partial differential equations for three-dimensional problems, where a system of the algebraic equations with millions unknowns should be designed on each step. Such problems essentially depend both of the computer memory and its speed.

1 Computer modeling of hypervelocity collision

1.1 Modeling of the crater propagation

The investigation of the behavior of solid substance exposed to big pressure, shearing, temperatures etc. is of great interest both from practical and pure scientific points of view. To achieve the required accuracy, the calculations should be performed on an enough dense lattice taking into account the large number of various factors affecting the process of high-speed deformation of the substance. It considerably complicates the solving algorithms and leads to increased demand to the operative memory volume and estimated time.

Now there is a lot of debris on low near-earth orbits. This debris has appeared as a result of failures in launching spaceships, collision of space apparatuses fragments or of wrecked satellites, of last stages of rocket-carriers, etc. As the result of such collusions, further fragmentation occurs, and velocities and orbit inclination angles of fragments are re-distributed. The prolongation of the time of space apparatuses presence in space increases the probability of their construction elements being hit. In this connection, the creation of mathematical models that allow to investigate numerically processes occurring at mutual hitting is becoming a very important task.

We consider it interesting to evaluate the impact of the striker's shape on formation of loading and unloading waves, the distribution of the striker's initial impulse on the axial and radial components and their influence upon the deformation process of the target. The striker has the shape of a rotating body – a cylinder, a sphere, an ellipse. The interaction between the striker and the target is a complex, multi-stage process. At high interaction speed, the pressure in the striking zone leads to melting and vaporization of the striker's and target's substance. After lowering the loads, the durability properties of the substance are in the foreground. That is why the interaction process should provide a good description of substance's behavior both in the hydrodynamic and solid phases. The main equations, expressing the laws of preservation of mass, the quantity of motion and energy in case of two spatial variables with cylindrical symmetry will be written as follows:

$$\frac{\partial \rho}{\partial t} = \frac{\partial(\rho u)}{\partial r} - \frac{\partial(\rho v)}{\partial z} - \frac{\rho u}{r}, \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t} = -\frac{\partial P}{\partial r} + \frac{\partial s_{rr}}{\partial r} + \frac{\partial s_{rz}}{\partial z} + \\ + \frac{s_{rr} - s_{\theta\theta}}{r}, \tag{2}$$

$$\frac{\partial(\rho v)}{\partial t} = -\frac{\partial P}{\partial z} + \frac{\partial s_{zz}}{\partial z} + \frac{\partial s_{zr}}{\partial r} + \frac{s_{rz}}{r}, \tag{3}$$

$$\frac{\partial(\rho E)}{\partial t} = -\frac{P}{\rho}\frac{\partial \rho}{\partial t} + s_{rr}\frac{\partial u}{\partial r} + s_{zz}\frac{\partial v}{\partial z} + \tag{4}$$
$$+ s_{\theta\theta}\frac{u}{r} + s_{rz}(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}),$$

here ρ – is the density, $\{u, v\} = \vec{w}$ – are the velocity vector components, P – is the hydrostatic pressure, $\sigma_{i,j} = -P\delta_{i,j} + s_{i,j}$, $\sigma_{i,j}$ – the stress tensor, i, j – running by the values $r, z, \theta,$ – are the components of stresses tensor deviator, E – is the specific internal energy. The components of the stresses tensor deviator are connected with the components of the deformation velocities tensor $\varepsilon_{i,j}$ as follows [1]:

$$\dot{s}_{zz}^{\nabla} = 2\mu(\dot{\varepsilon}_{zz} + \frac{1}{3\rho}\frac{\partial \rho}{\partial t}),$$
$$\dot{s}_{rr}^{\nabla} = 2\mu(\dot{\varepsilon}_{rr} + \frac{1}{3\rho}\frac{\partial \rho}{\partial t}), \tag{5}$$
$$\dot{s}_{\theta\theta}^{\nabla} = 2\mu(\dot{\varepsilon}_{\theta\theta} + \frac{1}{3\rho}\frac{\partial \rho}{\partial t}), \qquad \dot{s}_{zz}^{\nabla} = \mu(\dot{\varepsilon}_{rz}).$$

Here ∇ is the derivative along the deformation way in the sense of Yauman:

$$\dot{s}_{ij}^{\nabla} = \dot{s}_{ij} - s_{ik}\omega_{jk} - s_{jk}\omega_{ik}, \tag{6}$$
$$\omega_{ij} = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}).$$

v_i are the velocity vector components, x_j are the Dekart co-ordinates. The components of deformation velocities tensor are determined as follows:

$$\dot{\varepsilon}_{zz} = \frac{\partial v}{\partial z}; \quad \dot{\varepsilon}_{rr} = \frac{\partial u}{\partial r}; \quad \dot{\varepsilon}_{\theta\theta} = \frac{v}{r}; \tag{7}$$
$$\dot{\varepsilon}_{rz} = \frac{1}{2}(\frac{\partial u}{\partial r} - \frac{\partial v}{\partial z}).$$

Let us assume that the medium satisfies Mizes's plasticity criterion:

$$s_{rr}^2 + s_{zz}^2 + s_{rr}s_{zz} + s_{rz}^2 \leq \frac{2}{3}Y_0^2. \tag{8}$$

In equations (5)-(7) μ - the shear modulus, Y_0^2 -is the dynamic limit of yield. The $P = P(\rho, E)$ characteristic equation determining the dependence of hydrostatic pressure upon density and energy will be taken in Grunairen's form. The influence of material's durability properties on the appearing flow will be determined according to the Shteinberg-Guinan model [1] – [3].

The algorithm, offered in [4] – [5] is used for numerical decision of the problem. The method of large particles with appropriate updating [2], [3], [7], is used in this algorithm. The offered updating permits to remove the noninvariant of the method of large particles relatively transformations of coordinates. Besides a special algorithm, is entered enabling to set and to move on Euler grid contact and free borders. The choice of such scheme is stipulated by the necessity of account of large deformations of substance. Application of known Lagrangian methods does not permit to perform accounts with large deformations because of strong distortions differences of the grid.

The estimated two-dimensional lattice is divided into rectangular domains according to the number of available nodes of the multiprocessor system. Such an approach, called the principle of geometrical parallelism "Domain Decomposition" [9], is an effective way for solving the tasks of mathematical physics. In this case, all equal estimated domain is divided into sub-domains with equal number of points and the solution of the task is done on a separate processor. Among processors there occurs an exchange of necessary information [6] – [8]. The computer system consists of N processors connected according to lattice type. All processors have their own local memory. After concluding the next time step, the processors exchange information about the parameters of thermo-elastic-plastic flow in contiguous domains.

Method of account used in the given work, permits to look after the process of formation of a crater, its change in due course, to study the gear of ejections of substance from face sheet of a target, to look after formation and development in due course of those zones, where

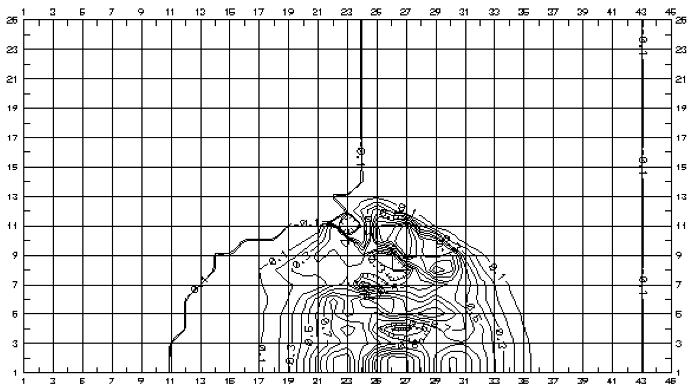

Figure 1.

the substance is in melted state. At impacts with speeds, large than 12 km/s, for strikers and targets, executed from aluminium, it is enough energy of impact for partial or complete evaporation of substance too.

In given work we shall study impact by a spherical striker on a target, for which the relation of a diameter of a striker to thickness of an obstruction is equal too $I(D/H = 1)$. The speeds of impact are accepted equal too 1, 3, 5, 7 km/s. Besides the case of impact with speed 7 km/s with attitude of a diameter of a striker to thickness of a target $(D/H = 0.5)$ is considered.

The indicated range of speeds is chosen on the following reasons. First, in this range the most probable speeds of collisionof separate fragments rotating on low Earth orbits among themselves lay. Secondly, in given range of speeds of collision the levels of pressure for initial material permit to investigate all stages of substance from hydrodynamic up to purely elastic. In third, for given range of speeds of

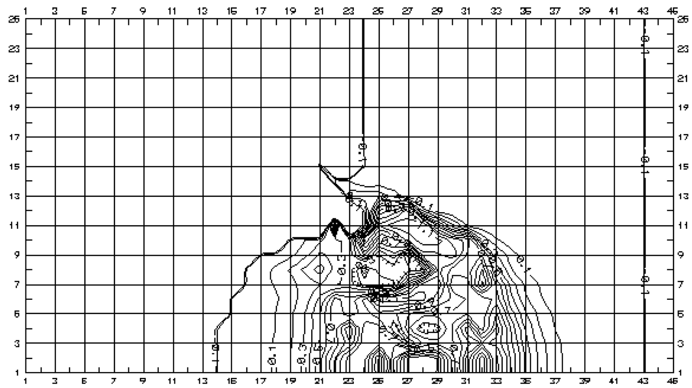

Figure 2.

collision there is the plenty of experimental researches.

In the course of time except this zone there arise one more area with large values of pressure at the place of contact of a spherical surface of the striker with the target. Up to the moment of an input in target of middle of a striker this zone is moved upwards - left to right. Thus between this area and the central zone will form area with smaller significances of pressure (fig. 1-4 : time = 10,14,16,20 mks respectively).

The interaction of these areas among themselves, the proceeding load from the side of the striker and the inertia of environment result in occurrence an ejection of substance from obverse surface of a target towards the striker and the formation of an edge of a crater. The formation of the edge of the crater begins with about 7 mks. At initial moments of time it has the curvature, coincided with the curvature of the striker. Between the striker and the edge of the crater exists an

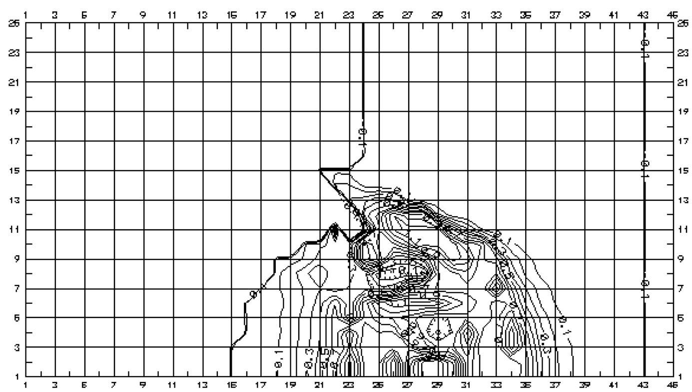

Figure 3.

interval After the moving striker fills in this interval, the curvature of an the edge of the crater changes its sign.

The analysis of indicated accounts on the forecasting of the final sizes of craters, arising at a hyper velocity impact, shows, that to that time, when the striker will be completely destroyed, the process of formation of a crater not finishs yet. Thus, it is possible to break a field of a flow, on area of the extending shock wave, which already does not render essential influence on the proceeding process of formation of the crater and on the unmoving area. The final crater sizes depend on the residual strength of material in the field of the formed crater and on the residual pulse of substance in its neighborhood.

The results of numerical investigations show that the shape of the indentor (sphere, ellipse, cylinder) exercises significant influence upon the distribution of the initial impulse of the striker on the axial and radial components in the target. Besides, the process of crater's forma-

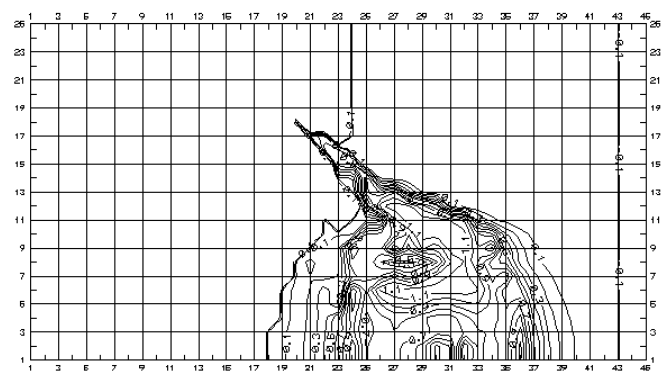

Figure 4.

tion and its shape depend considerably upon the striker's geometrical parameters.

1.2 Modelling underground objects

The influence of the impact load on various objects is of interest from the point of view of these objects stability and the preservation of their integrity. The dynamic load may be of varius nature: seismic waves, impact waves of explosions, ect. Experimental investigation of occurring non-stationary processes like the formation and spreading of impact processes, formation and speding of impact and discharge waves, as well as the formation of microdamages and splits is guite a complicated task. Application of mathematical modelling methods makes it possible to carry out numerical calculations for determining the optimal parameters of the object under construction.

An important task is the calculation of underground constructions which are under the influence of dynamic loads. In particular, of great interest is the task concerning the prediction of the holes, walls behavior when waves, from point or extended sources fall on them. The hole's walls are generally an "n" stratified construction made of different materials and placed in a hard rock or ground. When a rather intensive wave from a seismic or explosion source acts upon such a multilayered obstacle, damages or cracks may occur in the medium.

The adeguate mathematical modelling of the problem of the dynamic effect upon the complicated obstacle should take into account many factors. There are two main types of mathematical models for such problems: precise models in whose composition of basic eguations there enter the parameters which take into consideration the destruction processes; and the models in which the splitting durability and the dynamic limit of fluidity are the function of integral characteristics of the substance. Let us deal with the second approach. The eguations showing the movement of deformed solid state in Lagrange variables will be written as follows:

$$\frac{1}{V}\frac{\partial V}{\partial t} = \frac{1}{r^{\nu-1}}\frac{\partial (r^{\nu-1}U)}{\partial r}, \tag{1}$$

$$\frac{1}{V}\frac{\partial U}{\partial t} = \frac{\partial \sigma_1}{\partial r} + (\nu-1)\frac{(\sigma_1-\sigma_2)}{r}, \tag{2}$$

$$\frac{\partial E}{\partial t} = -p\frac{\partial V}{\partial t} + V[s_1\frac{\partial \varepsilon_1}{\partial t} + (\nu-1)s_2\frac{\partial \varepsilon_2}{\partial t}]. \tag{3}$$

Where: $V = \frac{1}{\rho}$ – is the specific volume, $V = \frac{1}{\rho}$ – density, U – velocity, E – specific (per mass unit) internal energy, $V = \frac{1}{\rho}$ – stress tensor components divided into two mutually orthogonal tensors: spheric – $\sigma_{kk}\delta_{ij}/3$ and deviator – s_{ij}: $\sigma_{ij} = \sigma\delta_{ij} + s_{ij}$; ε_{ij}, ε_{ij}^e, ε_{ij}^p – components of deformation tensors, elastic deformations and plastic deformations, respectively. We shall consider that $\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p$, i.e. the plastic flow is not contracted; $e_{ij} = \varepsilon_{ij} - 1/3\varepsilon_{kk}\delta_{ij}$ - components of the deformation tensor deviator, δ_{ij} Kroneker's symbol. Large deviations of the substance reguire the use of Jauman's derivate along the load trajectory:

$$s_{ij}^{\nabla} = \lambda \dot{s}_{ij} - s_{ik}\omega_{jk} - s_{jk}\omega_{ik}, \tag{4}$$

$$\omega_{ik} = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}),$$

where v_k – denote the velocity vector components, x_i – the Cartesian coordinates; the dot above the symbol denots the time derivate; x_i - denots the Jauman's time's derivate from tensor's components; λ is determined from Mizes's plasticity conditions: $s_{ij}s_{ij} \leq 2/3Y^2$

$$\lambda = 0$$

– in the elastic field;

$$\lambda = \frac{3\mu_0 \dot{s}_{ij}\dot{e}_{ij}}{Y^2} H(\dot{s}_{ij}\dot{e}_{ij})$$

– in the field of plastic flow,
$\dot{e}_{ij}$ – the components of the deviator of the deformation velocities tensor,
μ_0 – modulus of material's shift, Y – yield stress,
$H(x)$ – Heaviside's function.

The above system of equations describes the Prandtle - Race model of elastic-plastic yield stress with Mizes's plasticity criterion [10]. It is assumed that the yield stress limit Y and the modulus μ_0 depend on temperature, pressure and other parameters as follows (Shteinberg-Guinan's model [1]):

$$Y = Y_0(1+\beta\varepsilon_n^p)^n(1-b\sigma(\frac{\rho_0}{\rho})^{1/3} - h(T-T_0)),$$

$$Y_0(1+\beta\varepsilon_n^p)^n \leq Y_{max}, Y_0 = 0, T > T_m,$$

$$T_m = T_{m0}(\frac{\rho_0}{\rho})^{2/3} exp(2\gamma_0(1-\frac{\rho_0}{\rho})),$$

$$\mu_0 = \mu_{00}(1-b\sigma(\frac{\rho_0}{\rho})^{1/3} - h(T-T_0)),$$

here $\varepsilon_n^p = \sqrt{2\varepsilon_{ij}^p\varepsilon_{ij}^p/3}$ – denotes the intensivity of plastic deformations tensor; T_m – the temperature of material's melting; Y_0, Y_{max}, μ_{00}, T_{m0}, β, h, b, γ_0 – the constants of the material.

Equations (1)-(4) are supplemented by the state equation for the spheric component of tension tensor $\sigma = -p$ (p – the hydrostatic pressure).

$$p = p(\rho, E) \tag{5}$$

There is a great number of concrete state's equations of type (5). In [1] the state's equation of Grunazen type is given in the form:

$$p = k_1(1 - \frac{\rho_0}{\rho}) + k_2(1 - \frac{\rho_0}{\rho})^2 + k_3(1 - \frac{\rho_0}{\rho})^3 + \gamma_0\rho_0 E, \tag{6}$$

where k_1, k_2, k_3, are the constants of the material which are known for a wide class of materials. This equation describes the behavior of materials for a wide range of pressure and temperature.

Of wide use is the characteristic equation given as Teta's law:

$$p = \frac{K}{n}[(\frac{\rho_0}{\rho})^n - 1] \tag{7}$$

or the equation using the impact adiabat of the substance:

$$p = \frac{\rho_0 c^2 \chi}{(1 - A\chi)^2}, \qquad \chi = 1 - (\frac{\rho_0}{\rho}), \tag{8}$$

c – the material's sound velocity, A, k, n – constants.

One of the main task's of deformed solid state mechanics is the estimation of the strength of materials and of construction elements created from these materials. Its investigation is conected with great difficulties since the destruction is a complicated multistage process which depends on many factors. The study of this process is based on the determination of the characteristic types of defects and their increase depending upon the dynamics of intensively deformed state. Simplified approaches, which are based on the treatment of the destruction process as a threshold phenomenon, are applied in engineering practice. In such case, each criterion should correspond to that model of continuous medium and to those restrictions for whose description it is applied. In one-dimensional problems concerning the impulse deformation of stratified media the criterion for accumulation of damages [11],

[12] produces good results. In this model the parameter ω, which characterizes the number of microdamages and micropores in the material, was taken to serve as the measure of microdamages accumulation. The microcrackes appear after the tensile stress reaches some critical value σ_*. The growth and accumulation of microdamages is described by Tuler-Bucher's equation:

$$\frac{d\omega}{dt} = \begin{cases} B(\frac{\sigma'}{\sigma_*} - 1)^m, & \sigma' \geq \sigma_* \\ 0, & \sigma' < \sigma_* \end{cases}, \tag{9}$$

where $\sigma' = \frac{\sigma}{(1-\omega)}$, B, m – constants. With the increase and formation of new microcracks the stress, necessary for their growth, decreases when ω increases.

The value σ' increases too at the same values of current stress σ. The formation of the main crack and the full separation of the splitting plate occur simultaneously after the tensile stersses reach the $\sigma' \geq \Sigma_*$ value. Such a model is an acoustic one since parameter ω does not enter the defining equations. For concrete materials the numerical values of parameters m, σ_*, Σ_* are determined from the equation of the results of numerical and physical experiments of plates hitting one-another [12].

The most reliable and informative method for determining the characteristics of materials at impulse effects is the method of continuous registration of the velocity of plate-target free surface [13]. The performing of numerical experiments and the further comparison with experimental data allows to find the dynamical characteristics of investigated substances.

Let us consider the problem of dynamical effect on the stratified medium. The sourse of such effect is the wave which propagates as a result of an explosion or an earthquake. Basing upon the analysis of observations in the medium when several underground nuclear explosions occur the analytical approximation of the pressure profile form [14] is chosen

$$p(t) = (P_0 e^{-\alpha t} + P_{0c}) H(t), \tag{10}$$

here $P_0 + P_{0c} = P_{0s}$ is the peak impact pressure, P_{0c} – the established pressure, α – the constant attenuation. Equation (11) is true for distances that exceed the "elastic radius". The term "elastic radius" denotes the spherical surfase surrounding the point of explosion outside which the theory of infinitely small deformations is applicable. Let's mark the elastic radius by means of r_{l1}. It is necessary to determine the functional connection of r_{l_1}, α, P_{0c} and P_{0s} with the measured and experimental parameters of the explosion source such as the charge energy, its depth and the characteristics of the medium.

Parameter α is the value which is opposite to the characteristic time and is connected with the impact pressure upon the elastic radius. It is assumed that the similarity law for seismic sources is being realized, thus all characteristic times will be proportional to the cubic root of the charge magnitude. Then

$$\alpha = k\omega_0; \qquad \omega_0 = \frac{c}{r_{l1}}, \tag{11}$$

where c denotes the sound velocity, k – the proportionality coefficient which depends only on the properties of the medium. The values of coefficients for several media are given in Table 1:

The P_{0s} pressure is described by the following expression

$$P_{0s} = 1.5\rho g h, \tag{12}$$

where ρ is the density, g – the acceleration of the gravity force, h – the charge depth. The elasticity radius is determined from the following equation

$$\frac{r_{e1}}{W^{1/3}} = \frac{r_{e1c}}{(\rho h)^{1/n}}(\frac{A}{A_c})^{1/n}, \tag{13}$$

where W denotes the charge energy, n – the attenuation constant, r_{e1c} and A_c – the values for the calibration experiment when

$$W = 1\ kT, \qquad \rho h = 1.$$

The r_{e1} and n values are determined empirically on the basis of data for several explosions in similar media, proceeding from data [6] for varios media. Values A/A_c are given in Table 1.

The pressure established on the elastic radius P_{0c} and which is due the cavity expansion, is calculated by formula:

Table 1.

$Parameter$	Tuf	$Shale$	$Solt$
k	1.5	2.4	4.5
A/A_c	0.23	5.3	12
M	1.93	1.76	1.72

$$P_{0c} = \frac{4\mu}{3}(\frac{r_c}{r_{e1}})^3, \tag{14}$$

where r_c is the evaporation radius determined experimentally as a result of the cavity explosion. The empirical dependence of the cavity radius on the charge energy, its depth and properties of the medium has the following form:

$$r_c = 16.3W^{0.29}Mh^{-0.11}, \tag{15}$$

here

$$M = E^{0.62}\rho^{-0.24}\mu^{-0.67}, \tag{16}$$

dimensions of r_c and h values are given in meters; W – in kilotons; E and μ (Joung's and the shift's moduluses, respectivily) in megabars, ρ in g/cm^3. The values of M for several media are given in Table 1.

Thus, with given values of W, h, c formulas (12)-(17) fully determine the coefficients in equation (11) for the profile of the pressure upon the elastic radius. The choice of such a boundary condition allows to estimate the effect of the single pressure impulse upon the construction. The effect of the waves packet coming from the seismic sourse has a more complicated character.

We shall finally formulate the initial boundary problem: it is necessary to find the solution of the system of equations (1)-(4) with the condition equation of type (5), with account for the possibility of destruction (10) for the three-layered obstacle (salt-cement-steel). On the left boundary there acts the pressure which is given by equation (11), the right boundary is tensionless. The contact between the layers is considered ideal.

The calculations were carried out according to a modified differential scheme which realizes Wilkins's method [15]. The scheme has the second precision order (in those areas where this notion is meaningful) both as regards the time and space. To suppress the non-physical oscillations use was made of artificial viscosity as follows:

$$q = (k_1 \varepsilon + k_2 c) \varepsilon \rho,$$

where

$$\varepsilon = \frac{u_{i+1} - u_i}{r_{i+1} - r_i},$$

where c – sound velocity, ρ – density, k_1 and k_2 – the coefficients of quadratic and linear viscosity, respectively.

Initial values of parameters that determine the materials of layers are given in Table 2.

Table 2.

Material	ρ g/cm^3	C_0 cm/s	σ^* $Mbar$	I
Steel	7.85	0.4	$1.8 * 10^{-2}$	3.14
Cement	2.03	0.18	$5 * 10^{-5}$	0.37
Solt	2.16	0.44	$9.81 * 10^{-6}$	0.95

The process of propagation of waves in a multilayered medium is of a complex character. The acoustic impedances of investigated materials differ from one-another by about an order. It is the steel that has the greatest acoustic impedance. The impedance of cement is almost three times less than that of salt and by one order less than the impedance of steel. When the wave moves on the boundary dividing the layers arranged in ascending order of the acoustic impedance the wave's amplitude increases while the mass velocity decreases. When the layers are arranged in descending order of impedances the wave's amplitude decreases and its velocity increases.

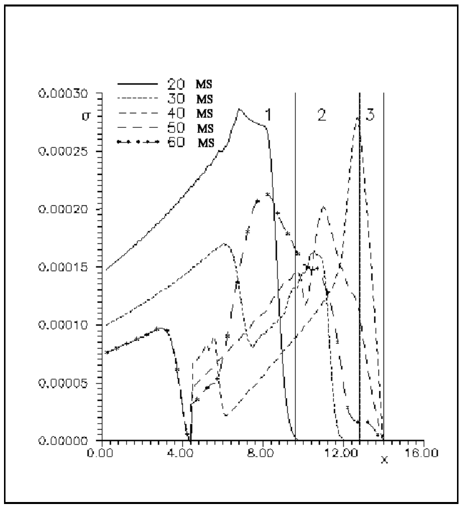

Figure 5.

The calculations, carried out according to the acoustic approximation, allow to determine the amplitude of the wave which is reflected from the boundary of layers division:

$$\sigma_{ot} = \frac{a_{01}\rho_{01}a_{02}\rho_{02}}{a_{01}\rho_{01} + a_{02}\rho_{02}} V_m = \frac{I_{01}I_{02}}{I_{01} + I_{02}} V_m. \tag{17}$$

Substituting the data from Table 2 we obtain that the amplitude of the wave reflected from the boundary of the division of salt-cement layers is equal to $1.42 * 10^4$ Mbars (the value of mass velocity behind the wave front $V_m = 5.32 * 10^4$. These data are in good argument with numeric calculations given in Figurs 5 and 6. Figure 5 shows the results of calculations of load's effect given by equation (11) with $\alpha = 0.04$, $P_0 = 3.2 * 10^{-4}$, $P_{0c} = 2.8 * 10^{-6}$. In Fig.2 – calculations with $\alpha = 0.008$, $P_0 = 3.2 * 10^{-4}$, $P_{0c} = 2.8 * 10^{-6}$. Given also are dependences $\sigma = f(x)$ in time moments t = 20,30,40,50,60 mks. The figures indicate the layers: of salt – 1, of cement – 2, of steel – 3.

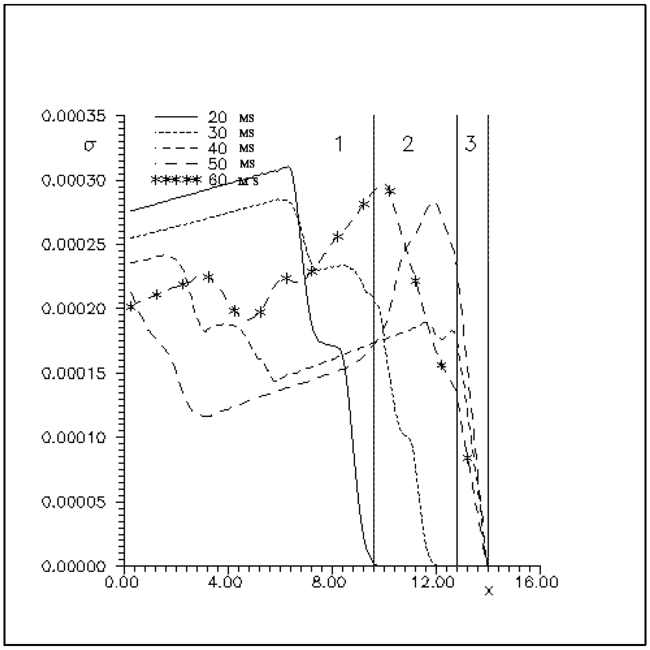

Figure 6.

In the time moment ~ 22 mks the wave front falls on the boundary of the salt-cement division and disintegrates into two waves: one is reflected from the boundary as a discharge wave which spreads in the salt, while the second one passes into the cement. The amplitude of the wave decreases up to $\sim 1.4 * 10^{-4}$ Mbars. When the wave moves on the boudary of cement-steel division its amplitude increases. A similar picture is observed in case of other values of α and P_0 too.

It should be noted that in case of a greater steepness of the front of the incident wave (5) a split occurs in the salt layer. This split is observed in the time moment $t = 41.5$ mcs, its coordinate is equal to $x = 4.2$ cm. When the wave is more gentle, $\alpha = 0.008$ and no split is observed (5).

The carried out investigations make it possible to draw the following conclusions: when the wave falls on the wall of an underground gas storage hole located in the salt stratum the disturbing of medium's continuity occurs, this depending on the parameters of the incident wave. Split phenomena occur not in the walls of the hole but at some distance from it. This phenomenon is connected with a dreat difference of acoustic impedances of salt and cement layers. To avoid the destruction of holes in the areas of increased seismicity, it is necessary to increase the value of the acoustic impedance of cement.

2 Computing modelling in the gazdynamics

The solution of the number of important applied problems in the field of ecology requires to use the methods of mathematical experiment, and subsequently to resort to the numerical experiment using computers. Research of currents in multiphase dispersion flows of gas containing solid particles is a very urgent problem of mechanics of continuous media. Especially acute it is at gas cleaning installations for thermal power stations, at cement and ferro-concrete factories etc. Increase of degree of purification of gas flow results in essential improvement of ecological conditions. Let us consider statement of a problem, main equations and method of solution. The flow of polydispersion gas through an installation of complex shape is simulated.

For the simulation usual assumptions of mechanics of continuous media are used: each phase is considered to be ideal, viscosity and heat conductivity of phases are taken into account only for inter-phase friction and heat and mass transfer processes; collisions of solid particles, their splitting or coagulation shall be neglected [22]. Let as also limit to the situations where one can assume the particles and the distances between them very small in comparison with characteristic size scale of the flow.

We shall represent the part of volume of the suspension occupied by phase i by the value of its volumetric contents $\alpha_i \quad (i = 1, 2)$. To each point the relative phase densities ρ_i, characterizing their masses in volume unit of the suspension, and the real densities, characterizing the densities of substances they consist ρ_i^u, will be put in correspondence.

$$\rho_1 = \alpha_1 \rho_1^u, \qquad \rho_2 = \alpha_2 \rho_2^u.$$

The volumetric content of the suspended phase of particles will be determined by the volume of an individual particle θ and the number of particles in the volume unit of the suspension, which can be called the numerical density of the suspension $\alpha_2 = \theta n$. We shall study the movement of the suspension media using the two-phase example (with two speed values and two temperature values), i.e. let us assume that all particles in the solid phase are identical.

Note that in this case the number of differential equations to be solved is two times more than in the case of usual one phase gas. Therefore the questions of an optimal construction of the program implementing the parallel algorithm of their solution become especially important.

Proceeding from aforesaid, the system of differential equational describing unsteady spatial currents of gas suspension could be written down in the following form:

$$\frac{\partial \rho_1}{\partial t} + \nabla \rho_1 \mathbf{W}_1 = 0, \quad \frac{\partial \rho_2}{\partial t} + \nabla \rho_2 \mathbf{W}_2 = 0,$$

$$\rho_1 \frac{d_1 \mathbf{W}_1}{dt} = -\alpha_1 \nabla p - n\mathbf{f}, \quad \rho_2 \frac{d_2 \mathbf{W}_2}{dt} = n\mathbf{f} - \alpha_2 \nabla p,$$

$$\frac{\partial}{\partial t}(\rho_1 E_1 + \rho_2 E_2) + \nabla(\rho_1 E_1 \mathbf{W}_1 + \rho_2 E_2 \mathbf{W}_2) + \nabla p(\alpha_1 \mathbf{W}_1 + \alpha_2 \mathbf{W}_2) = 0,$$

$$\frac{\partial \rho_2 J_2}{\partial t} + \nabla(\rho_2 J_2 \mathbf{W}_2) = ng, \tag{18}$$

$$(E_i = J_i + \mathbf{W}_i^2/2).$$

Here $\mathbf{W}_i, J_i, E_i$ are the speed, the specific internal and the specific full energy of phase i, p is the pressure in the suspension, $\mathbf{f}$ is the total force effecting upon a separate particle from of the gas, g is the intensity of heat inflow to the surface of a separate particle. Through ∇ and $\frac{d_i}{dt}$ the symbolical operators are designated:

$$\nabla \equiv \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k},$$

$$\frac{d_i}{dt} \equiv \frac{\partial}{\partial t} + (\mathbf{W}\nabla).$$

To close the system of differential equations (18) one should exactly specify the laws of phase interaction and formulate the equations of

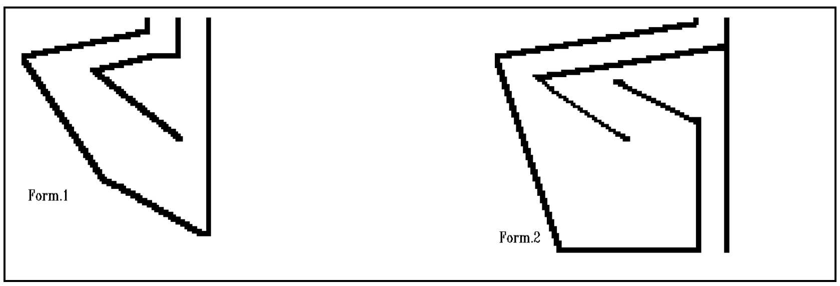

Figure 7.

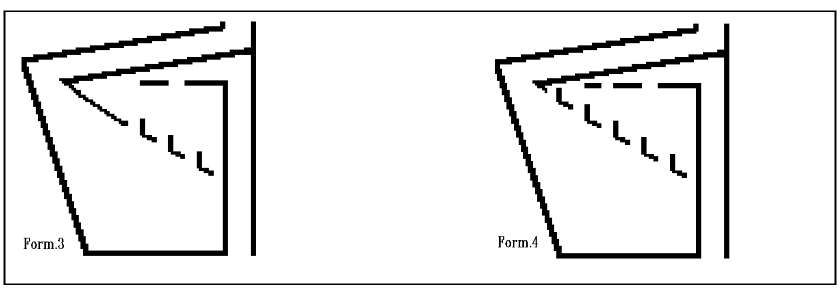

Figure 8.

state.

$$P = P(\rho_1^u, T_1), \quad J_1 = J_1(\rho_1^u, T_1), \quad \rho_2^u = const, \quad J_2 = J_2(T_2). \quad (19)$$

Here $T_i \quad (i = 1, 2)$ - is the temperature of phase i - of the suspension.

To choose the physical model we shall follow works [22], [21]. Specifying the laws of phase interaction we shall presume that suspended particles are spheres of diameter δ.

Assume, besides that, the intensities of thermal and force interaction of a particle with the gas do not depend on presence of another particles at its nearest vicinity. Then to determine the intensity of heat exchange one can use usual formulae:

$$\begin{gathered} g = \pi\delta^2\beta(T_1 - T_2) = \pi\delta\lambda_1 Nu((T_1 - T_2), \\ Nu = Nu(Re, Pr, M), \qquad Re = \rho_1^u\delta|\mathbf{W}_1 - \mathbf{W}_2|/\mu_1, \\ Pr = c_{p1}\mu_1/\lambda_1, \qquad M = |\mathbf{W}_1 - \mathbf{W}_2|/C_2. \end{gathered} \quad (20)$$

Here β is the heat transfer factor, Nu and Pr are Nusselt and Prandtl numbers. Re and M are Reynolds and Mach numbers for the relative flow about the particle. Trough λ_1 and μ_1 the heat conductivity and the dynamic viscosity of the gas are designated, c_{p1} is the heat capacity of the gas at constant pressure C_1 is the local sound speed in the gas.

We shall determine the resulting force of interaction between phases $\mathbf{f}$ in view of non-stationary effects in the flow of the gas about the particle in its relative movement. For this purpose, we shall approximately represent it as the sum:

$$\mathbf{f} = \mathbf{f}_\mu + \mathbf{f_A} + \mathbf{f_m}. \quad (21)$$

Here $\mathbf{f}_\mu$ is the force of viscous friction, $\mathbf{f_A}$ is the Archimedes force, $\mathbf{f_m}$ is the force of the appended mass:

$$\mathbf{f}_\mu = \frac{1}{8}\pi\delta^2\rho_1^u C_\delta|\mathbf{W}_1 - \mathbf{W}_2|(\mathbf{W}_1 - \mathbf{W}_2), \quad C_\delta = C_\delta(Re, M),$$

$$\mathbf{f}_A = \theta \rho_1^u \frac{d\mathbf{W}_1}{dt}, \quad \mathbf{f}_m = \frac{1}{2}\theta \rho_1^u (\frac{d\mathbf{W}_1}{dt} - \frac{d\mathbf{W}_2}{dt}), \tag{22}$$

here C_δ is the aerodynamic drag of the particle.

Note, that because of essential distinction of the sizes of suspended particles and the sizes of suspended particles and the body flown about, the Reynolds number of an external flow is in many orders more than the Reynolds number for interaction between the gas and particles. Therefore, for enough large class of problems one may take into account the viscosity and the heat conductivity only for phase interaction, counting the gas in the gas dynamical sense as non-viscous and non-heat-transferable.

If we take into account the forces of phase interaction (21) and (22) from phase acceleration, then it is reasonable to solve the equations for the system moment relative to the derivatives: $\frac{d\mathbf{W}_1}{dt}$, $\frac{d\mathbf{W}_2}{dt}$. When we fulfill this procedure concerning the gas suspensions with enough small pressures $\rho_1^u/\rho_2^u \ll 1$ and concentrations of the suspended phase $\alpha_2 \ll 1$, neglecting the terms with the order $O(\alpha_2, \rho_1^u/\rho_2^u)$ it gives us:

$$\rho_1 \frac{d\mathbf{W}_1}{dt} = -(1 - \frac{3}{2}\alpha_1)\nabla p - \chi n \mathbf{f}_\mu,$$

$$\rho_2 \frac{d\mathbf{W}_2}{dt} = -\frac{3}{2}\alpha_2 \nabla p + \chi n \mathbf{f}_\mu, \tag{23}$$

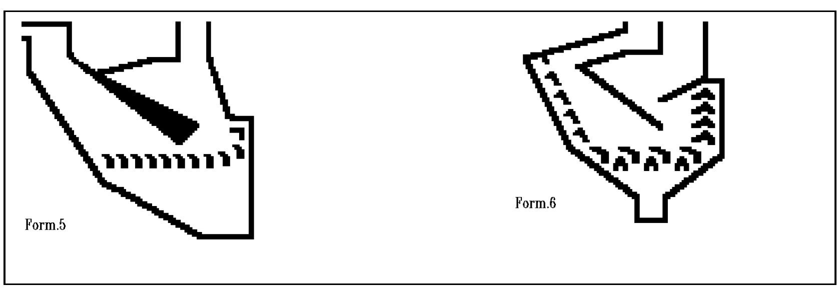

Figure 9.

$$\chi = (1 - \frac{3}{2}\alpha_2 - \rho_1^u/2\rho_2^u).$$

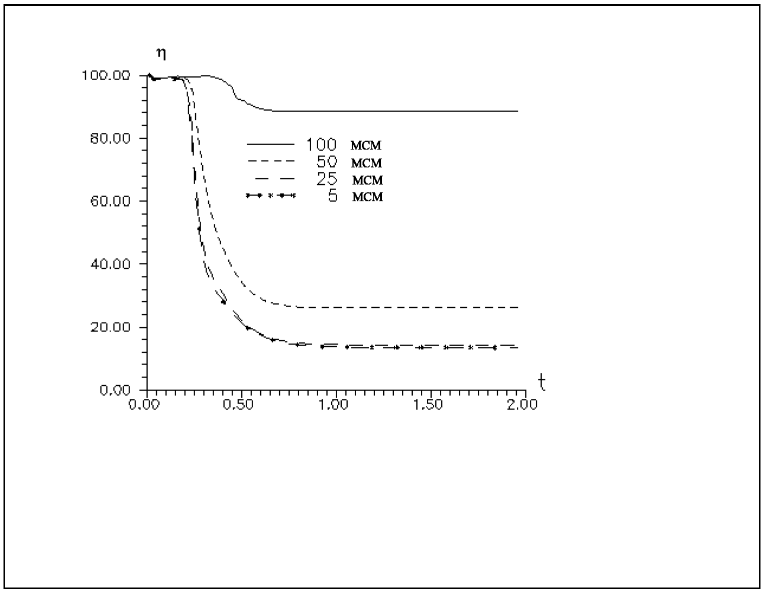

Figure 10.

Equations (23) could be conveniently rewritten in the quasidivergent form:

$$\frac{\partial \rho_1 \mathbf{W_1}}{\partial t} + \nabla \rho_1 \mathbf{W_1}(\mathbf{W_1}, \mathbf{l}) + (1 - \frac{3}{2}\alpha_2)\nabla p = -\chi n \mathbf{f}_\mu,$$

$$\frac{\partial \rho_2 \mathbf{W_2}}{\partial t} + \nabla \rho_2 \mathbf{W_2}(\mathbf{W_2}, \mathbf{l}) + \frac{3}{2}\alpha_2 \nabla p = \chi n \mathbf{f}_\mu. \tag{24}$$

Here $\rho_i \mathbf{W_i}(\mathbf{W_i}, \mathbf{l})$ is the momentum vector fluxes for phase i through surface, which is perpendicular to the unit vector $\mathbf{l}$, $(\mathbf{W_i}, \mathbf{l})$ is the scalar product of vectors $\mathbf{W_i}$ and $\mathbf{l}$.

The effects in (23) and (24) have the order of value of volumetric gas content α_2. In a number of practically important cases of an external aerodynamical flow of dusted gas about the body the value of volumetric contents $\alpha_2 < 1\%$ of the suspend phase is low, although the mass contents of the particles can even exceed the mass content of the gas because of its low pressure. Therefore we shall further neglect the terms of order α_2. Correspondingly we shall limit ourselves to the cases of moderate pressures, when $\rho_1^u/\rho_2^u \ll 1$.

Resistance factor for the particle C_δ is found using the empirical dependence.

$$C_\delta = \begin{cases} \frac{24}{Re} + \frac{4}{Re^{0.33}}, & \text{if} \quad 0 < Re \leq 700, \\ 4.3(lgRe)^{-2}, & \text{if} \quad 700 < Re \leq 2000. \end{cases}$$

Nusselt number Nu, Prandtl number Pr and Reynolds number Re are interconnected as follows:

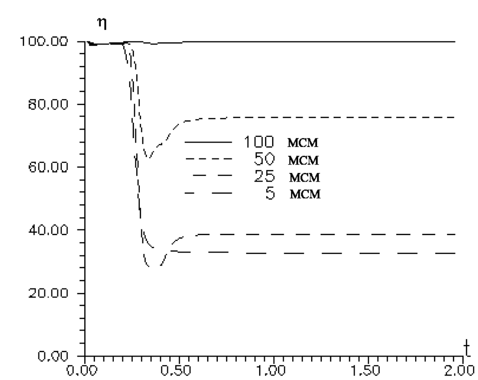

Figure 11.

$$Nu = 2.0 + 0.6 Re^{1/2} Pr^{1/3}.$$

The differential scheme calculating the flows of heterogeneous media is the developed one calculating the movement of the single-phase gas [5] [23].

At Eulerian stage of the traditional formulae are used:

$$\begin{aligned}
\frac{\tilde{u}^n_{i,j} - u^n_{i,j}}{\triangle t} &= -\frac{p^n_{i+1,j} - p^n_{i-1,j}}{2\triangle x}\frac{\alpha^n_{i,j}}{\rho^n_{i,j}}, \\
\frac{\tilde{v}^n_{i,j} - v^n_{i,j}}{\triangle t} &= -\frac{p^n_{i,j+1} - p^n_{i,j-1}}{2\triangle y}\frac{\alpha^n_{i,j}}{\rho^n_{i,j}}, \\
\frac{\tilde{E}^n_{i,j} - E^n_{i,j}}{\triangle t} &= -\frac{\alpha^n_{i,j}}{\rho^n_{i,j}}\left\{\frac{p^n_{i+1,j}u^n_{i+1,j} - p^n_{i-1,j}u_{i-1,j}}{2\triangle x} + \right. \\
&+ \left.\frac{p^n_{i,j+1}v^n_{i,j+1} - p^n_{i,j-1}v^n_{i,j-1}}{2\triangle y}\right\}.
\end{aligned} \tag{25}$$

Values $\tilde{u}^n_{i,j}$ and $\tilde{v}^n_{i,j}$, $\tilde{E}^n_{i,j}$ are determined using equations (25). All values concern to the gas phase.

At Lagrangian stage the mass transfer through cell borders for each phase is calculated. The fluxes of values for the gas phase are calculated using the formula:

$$\triangle(M_1\phi_1)^n_{i+1/2,j} =$$

$$\begin{cases} \rho^n_{(1)i,j}\phi^n_{(1)i,j}\frac{\tilde{u}^n_{i,j}+u^n_{i,j}}{2}\triangle y\triangle t, & \text{if } \tilde{u}^n_{i,j} + u^n_{i,j} > 0, \\ \rho^n_{(1)i+1,j}\phi^n_{(1)i+1,j}\frac{\tilde{u}^n_{i,j}+u^n_{i,j}}{2}\triangle y\triangle t, & \text{if } \tilde{u}^n_{i,j} + u^n_{i,j} < 0, \end{cases} \tag{26}$$

$$\phi_1 = (1, u, v, E).$$

The fluxes of solid phase parameters are determined similarly (26). At a final stage, on the basis of laws of conservation, the values of parameters for both phases at a new temporal level are found ρ^{n+1}_ω, E^{n+1}_ω, u^{n+1}_ω, v^{n+1}_ω, $\omega = 1, 2$.

Doing this, the phase interaction $\mathbf{f}$ and heat flow q are taken into account. The differential formulae for the final stage have the following form:

$$\begin{aligned}\rho^{n+1}_{(1)i,j} &= \rho^{n}_{(1)i,j} + \Big\{ \triangle(M_1)^n_{i,j-1/2} + \triangle(M_1)^n_{i-1/2,j} - \\ &- \triangle(M_1)^n_{i,j+1/2} - \triangle(M_1)^n_{i,j-1/2} \Big\} \\ &= \rho^{n+1}_{(1)i,j} + \sum \triangle(M_1)/(\triangle x \triangle y),\end{aligned}$$

$$u^{n+1}_{(1)i,j} = (\rho^{n}_{(1)i,j}/\rho^{n+1}_{(1)i,j})\tilde{u}^{n}_{(1)i,j} + \sum \triangle(M_1)\tilde{u}_{(1)}/(\triangle x \triangle y \rho^{n+1}_{(1)i,j}) -$$
$$-\triangle t f_x/\rho^{n+1}_{(1)i,j}.$$

The formulae determining the parameters of a suspended solid phase are similar to ones adduced above.

For realization of test accounts the experimental data indicated in the patent of USA N 287889 from 06.07.1979 [19] were selected. The

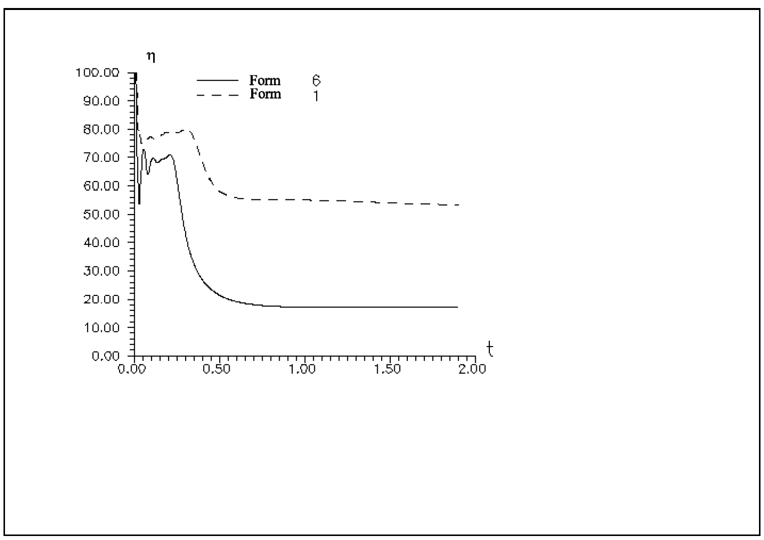

Figure 12.

multiphase stream, to blow in gas cleaning installation, consists of an air and aluminum dust. The volumetric denseness aluminum dust is equal 1121,3 kgs / m ^{3}, the size of particles makes 100 microns. A velocity of receipt of an air 2,407 m ^{3}/ of mines, aluminum dust 23,58 kgs / m, denseness of an air of $0,96 kgs/m^3$. the source diameter of installation makes 5,06 cm, target - 4,5 see. Efficiency of the gas cleaning installation according to experimental data makes $\eta_{\text{exp}} = 89,1\%$. the Numerical accounts conducted on the program, give significance of efficiency, equal $\eta_{\text{calc}} = 92,3\%$. The accounts conducted on a grid with reduced twice sizes of cells as on x, and on y, have given significance $\eta = 92,1\%$. A comparison of zones of concentration of a rigid faction obtained in account and in experiment, show practically full concurrence.

Series of accounts for gas cleaning of installations of various geometry (fig. 7 - fig. 9 Form.1 - Form.6) and for various parameters of volumetric concentration of a rigid faction and for a various diameter

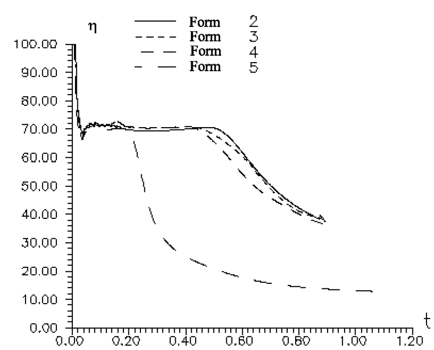

Figure 13.

of particles were conducted. In a fig. 10 the schedules of significances of efficiency as functions of time for particles of a diameter $d = 5$ micron - curve 1, $d = 25$ micron - curve 2 are indicated, $D = 50$ micron - curve 3 and $d = 100$ micron - curve 4 for geometry indicated in the fig. 7 Form.1.

In a fig. 11 the significances of efficiency for geometry gas cleaning installation represented in the fig. 8 Form.4 are indicated. To curve 1 there corresponds a diameter of particles $d = 5$ micron, curve 2 - $d = 25$ micron, 3 - $d = 50$ micron and 4 - $d = 100$ micron. As follows from the indicated schedule, the geometry of clearing installation considerably influences factor gas cleaning.

The influence of geometry gas cleaning of meanses is reflected in a fig. 12, 13. In a fig.12 the accounts for particles of a rigid faction equal $d = 20$ microns for geometry, indicated on a fig.7 (Form.1) and represented on a fig.9 (Form.6) are indicated. In a fig.13 the outcomes of accounts of factors gas cleaning for geometry of meanses indicated in the fig.8 - fig.9 (accordingly Form.2 - Form.5) are represented. The diameter of particles of a rigid faction makes $d = 10$ micron.

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Received April 24, 2000