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Regional Computer Networks: Research Aspects, Solutions

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Abstract

Here are described the problems and are proposed models and algorithms for the macrosynthesis and for an efficient resource utilization of the computer nodes, of the data trunks, of some network fragments and of the regional computer networks as a whole

Introduction

The most advanced form of exploration of informatic means are the computer networks. The obvious advantages and the implementation of some efficient technologies have resulted in a great development, especially in the 90s years. During the last years many computer networks have been created in Moldova too.

The creation and exploration of computer networks imply considerable expenditures. In such a way, the minimization and an efficient usage of the disposable resources have a significant importance. For this aim, there are elaborated models and algorithms, and more solutions are obtained. But because of the nonlinearity and great dimensions it is difficult to obtain exact solutions to these problems. Experts use simplifications which reduce the efficiency of the elaborated models and algorithms. The improvement of the existing results can lead to considerable economic and social effects.

Depending on the area of extension, we can distinguish local, metropolitan and wide area networks. Wide area networks can be subdivided into regional and global area networks. Regional networks are oriented toward the surface of some middle-scale geographical regions and they include only some basic public computer nodes (PCN). For such networks they can use models and algorithms within detailed description of characteristics. Mainly this category of problems in the field is of a greater interest for the Republic of Moldova.

In this work actual aspects are emphasized, ways are determined and solutions are proposed for bettering the models and algorithms of macrosynthesis and resources' capitalization of the regional computer networks (RCN). This represents a summary of the obtained results.

1 The general problem

On the base of the performed researches, the essential factors for the improvement of the already known models and algorithms of RCN examination are consideredto be the following: the differentiation of the users' tasks depending on the operativity, laboriousity and on the complexity of processing and, also, on the mode of users interaction with the computer system; the modeling of public computer nodes, inclusively of two level hierarchy structure, taking account of the differentiation of the tasks and their fulfillment in an order of priorities; the experimental research of the features of the data channels (DC); the modeling of trunks from two types of data channels with priority in the transfer of the packages for larger capacity channels; the introduction of the local public computer nodes (LPCN) in the structure of the network; the motivated choice of the places where the basic PCNs (BPCN) could be initially installed; the attachment of users' stations with PCNs basing on minimal total expenditures regarding the transfer, as well as the data processing; the determination of PCN performances consequently with the whole distribution of tasks held by users within the network; the efficient coordination with the works traffic in the network and the arrangement of works accomplishment at the network's components.

The methodology of experimental features evaluation of the task flows held by users is elaborated basing on the data analysis of the inquiry of some economical entities in the field of informatics. For the accomplishment of summary calculus are sufficient such tasks characteristics as: the regime of users' interaction with the computer system (dialog, inquiry-answer and batch), the processing laboriousity u_i, the volume of initial data v_i, the average period of answer T_{di} and the volume of output information w_i, that are specific to a task of $i = \overline{1, I}$ category.

For some regional computer networks the topological-functional structure with four levels is recommended. It includes such functional elements as: user stations or client nodes (UN); data channels or trunks (DT); switching nodes (SN); local, basic and main (MPCN) public computer nodes. For MPCN they choose one of basic PCN from the network. Basic PCNs are interconnected according to the opened radial-ring scheme with a center – MPCN. The switching nodes are placed together with some PCN. Users' nodes are connected through DT (as a rule DC) with one and just a single PCN of the network. They can be nonhomogeneous and gifted with diverse informatics means depending on functional performances, from ordinary terminals and personal computers to computer nodes or LAN.

Taking account of the upper-denoted factors, the macrosynthesis problem of the regional computer networks can be formulated, in general, as follows. Suppose we know: the set $X = \{x\}$ of the clients, with the intensity α_1 of dialog tasks generation and the rates γ_{xi} of the task flows with the categories $i = \overline{1, I}$; the users' tasks parameters v_i, w_i, u_i, T_{di} $(i = \overline{1, I})$; the set $Y_o \subseteq X$ of potential locations of BPCN placing; the set $Z \subseteq X$ of potential locations of LPCN placing; the maximal accepted productivity $U_{\max}$ of BPCN; the functions of expenditures with PCN $C_z(\cdot)$ and DT $\tilde{C}_{ij}(\cdot)$ $(i \in X,\ j \in X)$ between the i and j locations, by the features of the task flows.

We are to determine: the number $m^*(m^*_{Zy})$ and the set $Y^* \subseteq Y_o$ $(Z^*_y \subseteq X,\ y \in Y^*)$ of placing locations, the productivity $U^*_y \left(\breve{U}_z\right)$ of basic (local) PCN $y \in Y^*$ $(z \in Z^*_y,\ y \in Y^*)$; the productivity U^*_x of UN of the clients $x \in X$; the location $y^*_o \in Y^*$ of placing of MPCN; the rates λ_{xi}, $\hat{\lambda}_{xzi}$, λ_{xyi} $(i = \overline{1, I})$of the x client's task flows, distributed respectively at the x client's UN, LPCN z to witch the x client is attached and BPCN y, for $x \in X^*_{zy}$, $z \in Z^*_y$, $y \in Y^*$; the capacities

Q^*_{xz}, Q^*_{zy}, $Q^*_{yy_o}$ of DT between, respectively, the x client's UN and LPCN z, LPCN z and BPCN y, BPCN y and MPCN y_o for $x \in X^*_{zy}$, $z \in Z^*_y$, $y \in Y^*$, that ensures the minimum of the expenditures C for the elaboration and exploration of the computer network

$$\begin{aligned} C \;&=\; \tilde{C}_{Y\backslash y_o} + \sum_{y\in Y} \Biggl\{ C_y + \tilde{C}_{yy_o} + \sum_{z\in Z_y} \Bigl(\breve{C}_z + \tilde{C}_{zy} + \\ &\quad + \sum_{x\in X_{yz}} \Bigl(\hat{C}_x + \tilde{C}_{xz} \Bigr) \Bigr) \Biggr\} \to \min \end{aligned} \tag{1}$$

at the restrictions:

1) each client is attached directly to a single PCN and each LPCN is attached at a single BPCN

$$X_{yz} \cap X_{yz'} = \emptyset, \; \Biggl(\bigcup_{z\in Z_y} X_{yz} \Biggr) \cap \Biggl(\bigcup_{z\in Z'_y} X_{y'z} \Biggr) = \emptyset, \; \bigcup_{y\in Y} \bigcup_{z\in Z_y} X_{yz} = X,$$
$$z \neq z', \; z \in Z_y, \; z' \in Z_y, \quad y \neq y', \; y \in Y, \; y' \in Y; \tag{2}$$

$$Z_y \cap Z_{y'} = \emptyset, \qquad \bigcup_{y\in Y} Z_y = Z, \qquad y \neq y', \; y \in Y, \; y' \in Y; \tag{3}$$

2) all the clients' tasks are proceeded entirely in the network

$$\lambda_{xi} \geq 0, \; \hat{\lambda}_{xzi} \geq 0, \; \lambda_{xyi} \geq 0, \; \lambda_{xi} + \hat{\lambda}_{xzi} + \lambda_{xyi} \geq \gamma_{xi}, \tag{4}$$
$$i = \overline{1, I}, \; x \in X_{yz}, \; z \in Z_y, \; y \in Y;$$

$$\begin{aligned} \lambda_{zi} \;&\geq\; \sum_{x\in X_{yz}} \lambda_{xzi}, \; z \in Z_y, \; y \in Y, \; i = \overline{1, I}; \\ \lambda_{yi} \;&\geq\; \sum_{z\in Z_y} \sum_{x\in X_{yz}} \lambda_{xyi} + \begin{cases} 0, & y \in Y\backslash y_o \\ \lambda_{di}, & y = y_o \end{cases}, \; i = \overline{1, I}; \end{aligned} \tag{5}$$

3) the effective capacity Q_{kj} of DT between the locations $k \in X_{yj}$ and $j \in Z_y$ or $k \in Z_j$ and $j \in Y$ is sufficient for transferring volume V_{kj} of data that correspond to the task flows between these locations

$$Q_{kj} \geq V_{kj},\ k \in X_{yj} \text{ and } j \in Z_y \text{ or } k \in Z_j \text{ and } j \in Y; \qquad (6)$$

4) the mean average T_{xi} of serving the users' $x \in X$ tasks of i category doesn't exceed the maximal accepted value T_{di} for $i = \overline{1, I}$

$$T_{xi} \leq T_{di}, \quad i = \overline{1, I}, \quad x \in X, \qquad (7)$$

where: λ_{di} – the flow data rate of derived tasks of i category that is processed at MPCN; $\hat{C}_x$, $\breve{C}_z$, C_y – the expenditures with, respectively, UN x, LPCN z and BPCN y; $\tilde{C}_{xz}$, $\tilde{C}_{zy}$, $\tilde{C}_{yy_o}$ – the expenditures with DT between, respectively, the client x and LPCN z, LPCN z and BPCN y, BPCN y and MPCN y_o; $\tilde{C}_{Y \setminus y_o}$ – the total expenditures with DT between BPCN, except MPCN.

The aspects concerning the capitalization of the informatic resources of networks are also important, like: the efficient control of the network functioning; the creation of a wide variety of services oriented toward potential users, at one taking account of peculiarities and of the necessary priorities of serving; the rational combining of the usage of own resources and of the other networks' resources; the time phasing of the computer networks creation and development, taking account of priorities etc.

In order to solve the general problem they need to know: the DT configuration depending of the data traffic and of the requirements of their serving; the configuration and determining the operating regime of the computer nodes, needed for the users' task flows processing; the elaboration of the algorithms for rational distribution of task flows between the computer nodes and of network macrosynthesis as a whole; the efficient utilization of the networks' resources basing on the rational routing of the packages, on the ordering the works' execution etc. These aspects of the general problem are examined further.

2 Data transfer

The data system is composed of channels or data trunks and switching nodes. The data channels are created, mainly, on the basing of switched channels of telephone networks. Because at the implementation of telephone networks they didn't take account of data transfer specific requirements, the given analysis of data channels based on the switched channels is welcome.

The single channel trunks are studied carefully. But there are also many cases when it is rational to use multichannel trunks. Samples: the insurance of a high reliability of the functioning, the lack of the channels with the needed capacity; the capacity of a channel lakes values from a discreet set of values at respective costs. The possibility of multichannel trunks' usage is foreseen in the respective ISO standards, for example, in the protocol MLP (ISO TC 97/6 nr. 1951).

For the nonhomogenous trunks, the order of packages distribution between different channels influences the transmission features [1]. In some cases this influence can be of a great importance. That is why the problem of optimal packages distribution through the different channels of DT presents interest. In case of lack of the information regarding the current DT state and the inadmissibility of the package transmission's interruption, the solution is known.

The choice of a definite discipline is determined by the package transfer features, and by the complexity of its performance. In DT are used, as usual, disciplines without package transmission interruption. A relative discipline is implemented, for example, in the DATAPAC network [2]. Although [3] for DT between the switches nodes can be efficient the absolute disciplines. We are to mention that the DT performances can be improved, in the framework of the relative disciplines, by using a supplementary queue for the larger capacity channels [1].

So, regarding the data trunks we may state that the research of such aspects that follow are actual: the experimental evaluation of the data channels features; the determination of the characteristics of the data trunks from two types of channels with priority for larger capacity channels in the package transfer.

2.1 The experimental evaluation of the DC features

In this aim there is elaborated a methodology and a set of special tools [4]. For the experiment are used as follows: a remote station (terminal, computer), the data transfer channel that is examined and a computer (the central computer – CC). A control text is transferred through DC from the remote station to CC by repeating during the stated time period. The information, intercepted by CC, is analyzed in the aim of determining the basic characteristics of the selection. Basing on this, they compute three groups of DC characteristics: on errors and packages of errors; on data blocks; of reliability and capacity.

Using the mentioned methodology and tools, the characteristics of the 10 DC type, created in the basis of the Public Telephone Network of the Republic of Moldova, are evaluated. The DC are differentiating among them by the type of connection (dedicated, switched) and length; the switched channels, in turn, are differentiating by the type and quality of the telephone stations and of the entering message nodes that take part at the connection formation, and, also the direction of connection formation.

The results of the experimental researches [4] show that the value of the error coefficient depends greatly on the channels' type. The switched DC, that are based on the comunication channels of Chisinau PTN, as well as those dedicated throughout Moldova, are satisfying, according the error coefficient, the norms. With regard to the interurban switched channels in Moldova, the part of channels, for which the error coefficient value exceeds the normative, accepted value, can be essential.

2.2 Trunks of nonhomogenous data channels

There are studied DT of channels of two types- dedicated channels of different capacity or dedicated channels and switched channels, at the priority disciplines of serving the packages: absolute with a continuous serving; relative, inclusively with supplementary queue, and also generalized. In the modeling, the trunks are represented through queuing systems.

The DT flow of input packages is considered poissonian with the rate λ, and the length of the packages have an exponential distribution. If all the channels are occupied, the new-came packages are placed in the common queue with the number r of places. They are taken from the queue for transfer in the order of arrival. There are considered only the characteristics of the stationary state of the trunks.

In case of the absolute discipline with continuous serving, the system is described through a markovian process of birth and death. Using the general solution for such processes, we can obtain the expressions for the basic characteristics of the system's stationery state.

When using the relative priority of a larger capacity channels, the specific of the respectiv Chapman-Kolmogorov equations system allows to obtain the solution using a recurrent method. This method foresees the decrease of the problem dimension, basing on the representation of the system state probabilities through the probabilities of the least possible number of given states and then we can solve the reduced system of equations. In some cases we can obtain analytic solutions in such a way.

Let's denote by: $n(\breve{n})$ – the number and $\mu(\breve{\mu})$ – the intensity of package serving by the priority channels (non-priority); P_{ij} – the probability that in the system there are $i+j$ demands, from which i are served by the priority units, at the non-priority units are served j demands at $j \leq \breve{n}$ and $\breve{n}$ demands at $j > \breve{n}$; $\rho = \lambda \,/\, \mu$; $\breve{\rho} = \lambda \,/\, \breve{\mu}$; $\alpha = \mu \,/\, \breve{\mu}$; $x = \rho + \breve{n}\,\alpha$; $x_j = x + j - 1$. We present P_{ij} in the form: $P_{ij} = s_{ij} P_{0\,\breve{n}}$ at $i = \overline{0, n}$, $j = \overline{0, \breve{n}}$ and $i = n$, $j = \overline{\breve{n}+1, \breve{n}+r}$; $P_{ij} = a_{ij} P_{0\,\breve{n}} + b_{ij} P_{0,\breve{n}-1}$, $i = \overline{0, n}$, $j = \overline{0, \breve{n}-1}$; $P_{ij} = e_{ij} P_{0\,\breve{n}} + f_{ij} P_{0,\breve{n}-1} + g_{ij} P_{0j}$, $i = \overline{0, n}$, $j = \overline{0, \breve{n}-2}$.

For the coefficients $s_{i\,\breve{n}}$ $(i = \overline{0, n})$ are obtained the following explicit expressions:

$$s_{0\,\breve{n}} = 1, \quad s_{1\,\breve{n}} = \rho + \breve{n}\,\alpha, \quad s_{2\,\breve{n}} = \frac{1}{2}\left[\rho x + \breve{n}\,\alpha(x+1)\right],$$

$$
\begin{aligned}
s_{i\breve{n}} = {} & \frac{x\prod\limits_{j=2}^{i} x_j}{i!}\left(1+\sum_{j=1}^{u}(-\rho)^i\sum_{k_1=2j}^{i-1}\frac{k_1}{x_{k_1}(x_{k_1}+1)}\cdot\right.\\
& \left.\cdot\sum_{k_2=2j-1}^{k_1-2}\frac{k_2}{x_{k_2}(x_{k_2}+1)}\cdots\sum_{k_j=2}^{k_{j-1}-2}\frac{k_j}{x_{k_j}(x_{k_j}+1)}\right)-\\
& -\frac{\rho\prod\limits_{j=3}^{i} x_j}{i!}\left(1+\sum_{j=1}^{v}(-\rho)^i\sum_{k_1=2j+1}^{i-1}\frac{k_1}{x_{k_1}(x_{k_1}+1)}\cdot\right.\\
& \left.\cdot\sum_{k_2=2j-1}^{k_1-2}\frac{k_2}{x_{k_2}(x_{k_2}+1)}\cdots\sum_{k_j=3}^{k_{j-1}-2}\frac{k_j}{x_{k_j}(x_{k_j}+1)}\right), i=\overline{3,n},
\end{aligned}
$$

where for an even i correspond $k_0 = i-1$, $u = v = (i-1)/2$, and for an odd i, $u = (i-1)/2$, $v = u-1$. There are also obtained recurrent relations for the coefficients s_{ij}, a_{ij}, b_{ij}, e_{ij}, f_{ij} and g_{ij}, and basing of them we determine the probabilities P_{ij}, and after that the basic characteristics of the stationary system's state.

We are to mention the fact that the number of arithmetical operations u_{op}, needed for solving the given system of equations for the stationary system' state, using the described recurrent method, is approximately proportional to the total number of unknown variables. At once, at $n \geq 2$, $\breve{n} \geq 3$ is true the relation $20n\breve{n}+3r \leq u_{op} \leq 30n\breve{n}+3r$, while for getting its solutions using the Gauss method $u_{op} \sim [(n+1)(\breve{n}+1)+r]^3$, and using the modern numerical methods – $u_{op} \sim [(n+1)(\breve{n}+1)+r]^2$.

For $\breve{n} = 1$ the expressions for the trunk's stationary state features are obtained in explicit form, inclusively for the average period Θ of retain of a package in the system

$$\Theta = \frac{1}{\breve{\mu}} + \left\{ \frac{\breve{\mu} - \mu}{\lambda \breve{\mu}(1-\beta^r P_{n1}} \left(\frac{n\beta(1-\beta^r)}{1-\beta} + \right. \right.$$
$$+ n! \left(1 + \beta^{-1} - \frac{s_{n-1,1}}{s_{n1}} \right) \cdot$$
$$\left. \left. \cdot \sum_{i=1}^{n} \frac{\rho^{i-n}}{(i-1)!} \right) + \frac{1-\beta^r(1+r-r\beta)}{(n\mu+\breve{\mu})(1-\beta)^2} \right\} \cdot$$
$$\cdot \left(\frac{\beta(1-\beta^r)}{1-\beta} + n! \left(1 + \beta^{-1} - \frac{s_{n-1,1}}{s_{n1}} \right) \sum_{i=0}^{n} \frac{\rho^{i-n}}{i!} \right)^{-1} . \quad (8)$$

In case of relative priority, trunk characteristics can be improved, introducing an additional queue that is common to all the priority channels. The basic characteristics of such a trunk are determined using the upper mentioned method. Although the computation in a recurrent way is more comfortable, for the coefficients $s_{i\,\breve{n}}$ $(i = \overline{o, n+l})$ are obtained explicit expressions:

$$s_{0\,\breve{n}} = 0, \quad s_{1\,\breve{n}} = 1, \quad s_{2\,\breve{n}} = \tfrac{1}{2}(x_1 x_2 + \sigma),$$
$$s_{3\,\breve{n}} = \tfrac{1}{3}\left(\tfrac{x_1 x_2 x_3}{2} \sigma \left(x_1 + \tfrac{x_3}{2}\right)\right);$$

$$s_{i\,\breve{n}} = \frac{\prod_{j=3}^{i} x_j}{i!} \left(x_1 x_2 \left(1 + \sum_{j=1}^{h_i} \sigma^j \sum_{k_1=2j}^{i-1} y_{k_1} \sum_{k_2=2j-2}^{k_1-2} y_{k_2} \cdots \sum_{k_j=2}^{k_{j-1}-2} y_{k_j} \right) + \right.$$
$$\left. + \sigma \left(1 + \sum_{j=1}^{g_i} \sigma^j \sum_{k_1=2j+1}^{i-1} y_{k_1} \sum_{k_2=2j-1}^{k_1-2} y_{k_2} \cdots \sum_{k_j=3}^{k_{j-1}-2} y_{k_j} \right) \right),$$
$$i = \overline{4, n},$$

where $k_0 = i + 1$ and for i being even $h_i = g_i = \frac{i}{2} - 1$, but for an odd i, $h_i = g_i + 1 = \frac{i-1}{2}$;

$$s_{n+1,\breve{n}} = \left(1 + \tfrac{x}{n}\right) s_{n\breve{n}} + \tfrac{\sigma}{n} s_{n-1,\breve{n}};$$

$$s_{n+2,\breve{n}} = \left(\left(1 + \tfrac{x}{n}\right)^2 + \tfrac{\sigma}{n}\right) s_{n\breve{n}} + + \tfrac{\sigma}{n}\left(1 + \tfrac{x}{n}\right) s_{n-1,\breve{n}};$$

$$s_{i\breve{n}} = \tfrac{\sigma}{n} s_{n-1,\breve{n}} + s_{n\breve{n}} \left\{ \left(1 + \tfrac{x}{n}\right) \sum_{j=1}^{h_{i-n+2}} \binom{i-j-n}{j-1} \left(1 + \tfrac{x}{n}\right)^{2(h_{i-n+2}-j)} \cdot \right.$$

$$\left. \cdot \left(\tfrac{\sigma}{n}\right)^{j-1} + \sum_{j=1}^{h_{i-n+1}} \binom{i-j-n-1}{j-1} \left(1 + \tfrac{x}{n}\right)^{2(h_{i-n+1}-j)} \left(\tfrac{\sigma}{n}\right)^j \right\},$$

$$i = \overline{n+3, n+l}.$$

For the characteristics of DT from n priority units and one non-priority unit are obtained explicit analytic expressions, inclusively for the average period Θ of retain of a package in the system:

$$\Theta = P_{n+l,1} \left\{ -\tfrac{1}{\mu} + \tfrac{1-\beta^r(1+r-r\beta)}{(n\mu+\breve{\mu})(1-\beta)^2} + \tfrac{1}{n\mu} + \left(1 - \tfrac{\rho}{n}\right)^{-2} \cdot \right.$$

$$\cdot \left(1 + \tfrac{1}{\beta} - \tfrac{s_{n+l-1,1}}{s_{n+l,1}}\right) \left(\left(\tfrac{n}{\rho}\right)^l + 1 + l - \tfrac{l\rho}{n}\right) + \tfrac{n+l+1}{n\mu+\breve{\mu}} \cdot \tfrac{1-\beta^r}{1-\beta} +$$

$$+ \tfrac{n!n^l}{\mu\rho^{n+l}} \left(1 + \tfrac{1}{\beta} - \tfrac{s_{n+l-1,1}}{s_{n+l,1}}\right) \cdot$$

$$\left. \cdot \left(\sum_{i=0}^{n-1} \tfrac{\rho^i}{i!} + \tfrac{\rho^n}{i!} \left(\tfrac{\mu}{\breve{\mu}} \left(\tfrac{\rho}{n}\right)^l + \tfrac{1-\left(\frac{\rho}{n}\right)^l}{1-\frac{\rho}{n}} \right) \right) \right\},$$

where

$$P_{n+l,1} = \left[n! \tfrac{n^l}{\rho^l} \left(1 + \tfrac{1}{\beta} - \tfrac{s_{n+l-1,1}}{s_{n+l,1}}\right) \cdot \right.$$

$$\left. \cdot \left(\sum_{i=0}^{n} \tfrac{\rho^{i-n}}{i!} + \tfrac{\rho\left(1-\left(\frac{\rho}{n}\right)^l\right)}{n!\left(1-\frac{\rho}{n}\right)} + \tfrac{\beta(1-\beta^r)}{1-\beta} \right) \right]^{-1}.$$

The comparative analysis of DT, through calculus for a wide variety of initial data, has confirmed the importance of taking into consideration the DT state in the process of package distribution between unites with different capacities. The increase of loading DT, is accompanied by a growth in the superiority of disciplines which use the information about the current system's state: at the level of loading about 90 %, the average period of retain of packages in the system for such disciplines is almost 4 times less. In the consicered examples, the introduction of the relative priority permits the decrease of the average period of package retain in the system to almost 40 % when comparing to the priority-less discipline. Also the introduction of an additional queue to DT with a relative discipline permits, in some cases, the decrease of the average period of package retain in the system till 10–17 %. At the same time, during very low or very big loading, the effect of introduction of an additional queue is not important.

Basing on the obtained results, we have elaborated a methodology of DT configuration depending on the data traffic.

3 The configuration of the computer nodes

Computer nodes are considered as computer systems that combine the time-sharing mode, for serving the dialog and inquiry-answer tasks, and a mode without time-sharing mode – for batch tasks processing. The tasks are processed depending on priorities at the absolute discipline with a continuous serving.

There are elaborated analitical models for mono- and multiprocessor CN. In the models, CN are represented through queuing systems or networks. In case of a multidimensional tasks flow, CN are detailed till the level of computers, and in case of single-dimensional flow – till the level of processors, of input-output channels and of the external memory. The duration of task processing has an exponential distribution. They also, suppose, that inside each category from the tasks that are processed in the time-sharing mode is used the cyclical discipline, and for others – “first input-first output”. The highest priority of tasks are considered, conventionally, the categories with the less value of index i ($i = \overline{1, I}$).

3.1 Monoprocessor computer nodes

Noting with N the number of active users, which interact with the computer system in a dialog regime, the examined system can be considered as a monounity queuing system with one finite source and $I-1$ infinite sources of tasks. For $I = 2$ the expressions for T_1 and T_2 through the moments of first level ($E[e_1]$, $E[e_2]$) and those of the 2^{nd} level ($E[e_1^2]$, $E[e_2^2]$) which are referred to the duration of the state period, which deal with the processing of the prior task, and the duration of the cycle of the processing of an nonpriority task are knew [5]. In particular for the examined queuing system the following formula are obtained:

$$
\begin{aligned}
E[e_1] &= \tfrac{1}{\mu_1}\left(1 + (N-1)! \sum_{i=1}^{N-1} \tfrac{\sigma_1^i}{(N-i-1)!}\right), \\
E[c_2^2] &= \tfrac{1}{\mu_2}\left(\tfrac{2}{\mu_2}(1 + \alpha_1 N E[e_1])^2 + \alpha_1 N E[e_1^2]\right), \\
E[e_1^2] &= \tfrac{2(N-1)!}{\mu_1^2} \sum_{i=1}^{N} \tfrac{\sigma_1^{i-2}}{(N-j)!}\left(\tfrac{1}{j} - \sum_{1}^{j} \tfrac{1}{k} + \sum_{k=1}^{N} \tfrac{N!\sigma_1^k}{k(N-k)!}\right), \\
E[c_2] &= \tfrac{1}{\mu_2}(1 + \alpha_1 N E[e_1]).
\end{aligned}
$$

If $I > 2$ and the nondialog tasks have the same laboriousity of processing, i.e. $\mu_i = \mu_2$ ($i = \overline{3, I}$), for T_1 and T_2 the relations of the system with a two-dimensional flow are valid, and for T_i ($i = \overline{3, I}$) the following recurrent formula is obtained:

$$T_i = \frac{1}{\lambda_i}\left(\sum_{2}^{i}\lambda_j\left(E[c_2]+\frac{\sum\limits_{2}^{i}\lambda_j E[c_2^2]}{2(1-\lambda(i)E[c_2])}+\frac{\alpha_1 NE[e_1^2]}{2(1+\alpha_1 NE[e_1]}\right)-\right.$$
$$\left.-\sum_{2}^{i-1}\lambda_j T_j\right), \qquad i=\overline{3,I}. \tag{9}$$

In case when $I > 2$ and some of the μ_i, $i = \overline{2,I}$ values are not equal, it is demonstrated that the formula (3) can be replaced, on the base of the conservation law of L.Kleinrock [7], with an unessential error with the formula

$$T_i = \frac{1}{\lambda_i u_i}\left(\frac{\sum\limits_{j=1}^{i}\lambda_j u_j^2}{\mu_1 u_1 - \sum\limits_{j=1}^{i}\lambda_j u_j} - \sum_{j=1}^{i-1}\lambda_j T_j u_j\right),\ i=\overline{3,I}. \tag{10}$$

The relations between the CN productivity U and the u_i, λ_i, μ_i ($i = \overline{1,I}$) parameters are obtained, thus being finalized the analytic model of CN.

3.2 Dual computer nodes

The performant computer nodes are built as multicomputer or multiprocessor systems. Dual computer nodes (with an hierarchical two-level structure) are examined from one or more basic B computers and one or more auxiliary A computers at the suppositions: the flows of tasks of the categories $i > 1$ in the B system have a sufficiently close character to the poisson one; for the tasks of the first category the duration of the restraint in the source and at the A stage as well as the duration of the restraint in the B stage and in the source have a repartition, close to the exponential one with the parameters α_1^B and α_1^A respectively. Taking into account these suppositions, the dual system investigation is reduced to the examination of two A and B queuing systems of the above studied types.

There are formulated 12 problems of analysis and synthesis of the computer nodes using the elaborated models, inclusive for: the determination of the CN performances, which satisfy the demands of the tasks processing; the determination of the CN possibilities for the processing of the users tasks etc. There are elaborated algorithms for the solution of these problems. On the basis of the calculation results, using the respective programs, for many variants of initial data, it is demonstrated that:

a) the combination of the processing of the dialog tasks and of the other types of tasks on a computer (computer node) is rational in many practical cases; this is particularly convenient for the small amounts of the average allowed by the answer duration to the dialog tasks, of the intensity of the preparation of the dialog tasks and of the number of users;

b) at small rates of the incoming flows, at CN the dialog tasks prevail, and at big rates – nonoperative tasks;

c) for the computer nodes of high productivity, the dual structure is more efficient, due to the amount of the expressed expenses, that the monoprocessor's one.

3.3 Computer nodes with monodimentional flows

Let's consider a monolevel multiprocessor computer node (MPCN) with extern units of two types, which serves N users in a dialog regime. The duration of the dialog tasks' preparation has an exponential repartition with the rate α – equal for all the users. The tasks' processing is performed on time quantum of certain duration, by the expiration of which the tasks are placed at the end of the queue to the processor. Let's consider that the τ_{ir} duration of a stage of serving of the tasks by an unity of the system i ($i = \overline{1,3}$) at the current rank of multiprogramming r ($r = \overline{1,S}$) has an exponential repartition with the average $1/\mu_{ir}$ ($i = \overline{1,3}$; $r = \overline{1,S}$). The interruption of the processing of the tasks by the processor may be caused by the termination of the task's serving (with the probability π_{10r}), by the expiration of the time quantum granted to the task (with the probability π_{11r}), by the necessity of the information' changing between the operative memory and the first type memory units (with the probability π_{12r}) and by the necessity of the changing of information between the operative memory and the second type memory units (with the probability π_{13r}).

According to the W.Brandwain's [7] principle of almost complete divisibility, the initial system is replaced by an equivalent one for which the following relations take place:

$$\lambda_{kr} = (N-k-r)\alpha, \qquad r=\overline{0,S},\ k=N-r; \tag{11}$$

$$\hat{\mu}_{kr} = \alpha_r \pi_{10r} \mu_{1r}, \qquad r=\overline{1,S},\ k=N-r, \tag{12}$$

where: $\alpha_r(\lambda_{kr})$ is the probability of the busy state of the system (of the processor), and $\hat{\mu}_{kr}$ – the equivalent intensity of serving of the tasks by the quasiequivalent system at the current stage r of the multiprogramming and k tasks in the additional queue.

The probabilities α_r ($r=\overline{1,S},\ k=N-r$) are unknown in the formula (12). For these using the general solution for closed exponential queuing nets, proposed by Gordon and Newell [6], the following relations are obtained:

$$\alpha_r = 1 - \frac{t_3^r}{C_{r3}} \left(\frac{n^{n-r}}{n!} \sum_{r_2=0}^{i} \frac{t_{23n}^{r_2}}{r_2!} + \frac{m^m}{m!} \sum_{r_2=j+1}^{r} \frac{t_{23m}^{r_2}}{(r-r_2)!} + \right.$$
$$\left. + \begin{cases} \sum\limits_{r_2=i+1}^{j} \frac{t_{23}^{r_2}}{r_2!(r-r_2)!},\ i=r-n-1,\ j=m,\ i<j; \\ \frac{m^m n^n}{m!n!} \sum\limits_{r_2=i+1}^{j} \frac{t_{23m}^{r_2}}{n^{r_2}},\ i=m,\ j=r-n-1,\ i\le j \end{cases} \right), r=\overline{1,S},$$

where

$$C_{r3} = t_1^r \left\{ \sum_{i=0}^{n} \frac{t_{31}^i}{i!} + \left(\sum_{j=0}^{m} \frac{t_{21}^j}{j!} + \frac{m^m}{m!} \cdot \frac{t_{21m}^{m+1} - t_{21m}^{r-k+1}}{1 - t_{21m}} \right) + \right.$$

$$+ \frac{n^n}{n!} \cdot \frac{t_{31n}^{n+1} - t_{31n}^{r+1}}{1 - t_{31n}} \sum_{j=0}^{m} \frac{t_{21}^j}{j!} +$$

$$\left. + \frac{m^m n^n t_{21m}^{m+1}}{m! n! (1 - t_{21m})} \left(\frac{t_{31n}^{n+1} - t_{31n}^{r+1}}{1 - t_{31n}} - t_{21m}^{r-m} \cdot \frac{t_{32nm}^{n+1} - t_{32nm}^{r+1}}{1 - t_{32nm}} \right) \right\},$$

$$t_{ij} = \frac{t_i}{t_j}, \quad t_{ijs} = \frac{t_{ij}}{S}, \quad t_{ijmn} = \frac{t_{ijm}}{n}.$$

Knowing the probabilities α_r ($k = \overline{N-1, N-S}$, $r = N-k$), are determined the basic characteristics of the stationary state of the system, inclusively the average T duration of response to the task:

$$T = \frac{\sum_{j=1}^{S} \frac{jH_j}{(N-j)!} + H_S \sum_{i=1}^{N-S} \frac{(i+S)d^i}{(N-S-i)!}}{\alpha_1 \left(\frac{1}{(N-1)!} + \sum_{j=1}^{S} \frac{H_j}{(N-j-1)!} + H_S \sum_{i=1}^{N-S-1} \frac{d^i}{(N-S-i-1)!} \right)}, \qquad (13)$$

where $H_j = \alpha^i \,/\, \prod_{i=1}^{j} \alpha_i \mu_{1i} \pi_{10i}$; $d = \frac{\alpha}{\alpha_S \mu_{1S} \pi_{0S}}$.

The relation (13) is approximate. Its exactness is determined by the degree of satisfaction of the almost complete divisibility's principle. An orientated estimate of this may be given on the basis of the T calculation without the supposition regarding the respecting of this principle. In this purpose is examined a monoprogramming computer system – a particular case of the precedent case, which is obtained at $m = 1$, $n = 0$, $S = 1$, $\pi_{101} = \varphi_1$, $\pi_{111} = 0$, $\pi_{121} = 1 + \varphi_1 = \hat{\varphi}_1$, $\pi_{131} = 0$. We will consider that the system besides the dialog demands ($i = 1$) is processing also, $I - 1$ flows of demands of rates λ_i ($i = \overline{2, I}$). At the same time the demands of i category have absolute priority with the continuation of the serving regarding the demands of $i+1$ ($i = \overline{1, I-1}$) category. The duration of a serving stage of a demand of i category by the QS j is considered to be of an exponential repartition with the average ν_{ji} for $i = \overline{1, I}$, $j = \overline{1, 2}$. The expressions for the calculation of some characteristics of this system are obtained, inclusively the average T_1 duration of response to the task:

$$T_1 = N\nu_1 + \left\{1 - \left(\sum_{j=0}^{N-1}(N_j - 1)C_j\right)^{-1}\right\}\left(\frac{\nu_1^2 + \sigma_1^2(\tau_1)}{2\nu_1} - \frac{\nu_1}{1 - \bar{f}_1(\alpha)}\right),$$

where:

$$C_j = \begin{cases} 1, & j = 0, \\ \prod\limits_{i=1}^{j}\left(\rho_{11} - \frac{\hat{\varphi}_1\rho_{21}}{1+i\rho_{21}}\right), & j = \overline{1, N-1} \end{cases}; \ \rho_{11} = \alpha\nu_{11};\ \rho_{21} = \alpha\nu_{21};$$

$$\sigma_i^2(\tau_i) = E[\tau_i^2] - \nu_i^2, \qquad i = \overline{1, I};$$

$$E[\tau_i^2] = \frac{\varphi_i}{\hat{\varphi}_i}(\nu_{1i} + \nu_{2i})^2\sum_{j=1}^{\infty} j^2\hat{\varphi}_i^j + \frac{\nu_{1i}^2 + \nu_{2i}^2}{\varphi_i}, \qquad i = \overline{1, I};$$

$$\bar{f}_i(\alpha) = \varphi_i\sum_{j=0}^{\infty}\frac{\hat{\varphi}_i^j}{\nu_{1i}^{j+1}\nu_{2i}^j\left(\frac{1}{\nu_{1i}} + \alpha\right)\left(\frac{1}{\nu_{2i}} + \alpha\right)}, \qquad i = \overline{1, I}.$$

For the average T_2 duration of response to the demands of the second category, the following takes place:

$$T_2 = \nu_2\left(1 + \frac{N\alpha_1E_1}{2\nu_2Q} + \frac{\lambda_2(E[\tau_2^2]Q^2 + N\alpha_1\nu_2E_{12})}{2(1 - \lambda_2\nu_2Q)}\right),$$

where $Q = 1 + N\alpha_1E_1$; $E_1 = \nu_1\sum\limits_{j=0}^{N-1}\begin{pmatrix} N-1 \\ j \end{pmatrix}C_j$,

$$E_{12} = \sum_{j=0}^{N-1}\begin{pmatrix} N-1 \\ j \end{pmatrix}C_j\left(E[\tau_1^2] - 2\nu_1^2\left(1 - \sum_{j=0}^{N-1}\begin{pmatrix} n \\ j+1 \end{pmatrix}C_j\right)\right) -$$

$$-\frac{2\nu_1}{\varphi_1^2}\sum_{j=1}^{N-1}\begin{pmatrix} N-1 \\ j \end{pmatrix}C_j\sum_{i=1}^{j}\frac{\nu_{11} + \frac{\hat{\varphi}_1\nu_{21}}{(1+i\rho_{21})^2}}{i\left(\rho_{11} + \frac{\hat{\varphi}_1\rho_{21}}{1+i\rho_{21}}\right)}.$$

If the incoming flow contains three or more categories of tasks ($I > 2$), we will distinguish two cases. The first one is determined by the equalities $\nu_{1i} = \nu_{12}$, $\nu_{2i} = \nu_{22}$, $\varphi_i = \varphi_2$, $i = \overline{3, I}$ and the second one – by the fact that some of the mentioned equalities of the first case don't take place. In the first case, for the determination of the average response T_i ($i = \overline{3, I}$) duration the recurrent formula similar to (9) can be used. For the second case, if the repartition of the duration of the serving of the $i = \overline{1, I}$ categories differs unessentially from the poisson one, the T_i ($i = \overline{3, I}$) durations can be determined in a recurrent way according to the approximate formula similar to (10).

3.4 Dual computer nodes with monodimensional flow

The examined computer nodes consist of N stations for users' interaction (the demands' source) with CN, l identical basic processors and $\hat{l}$ auxiliary processors. The tasks which are generated by the source, are preliminary processed in the auxiliary system, then in the basic system, and after the tasks' responses are transmitted to the source. From the point of view of the modeling, structure and operating, the systems of the basic processors and of the auxiliary ones are identical. The auxiliary system's parameters are noted in the same way as those of the basic system, but adding the " ˆ " sign above respective symbol.

The mutual independence of the functioning of the basic processors is assured. The operative memory as well as the extern memory are common for all the basic processors. The extern memory includes n units of one type and m units of the other type. The tasks are taken from the queue for their serving in the incoming order. The duration of preparation of one task by each of the source's stations is considered to have an exponential repartition, with the parameter α, and the duration of one stage of serving of one unit of the subsystem i ($i = 1, 2, 3$ – respectively: processor, "input-output channels – external memory of the first type", "input-output channels – external memory of the second type") – exponential repartition with the average $1/\mu_{ir}$ ($i = \overline{1, 3}$, $r = \overline{1, S}$).

The computer system is represented by a closed stochastic network which consists of 7 exponential multiunits QS: QS1 of N units, which represent the tasks' source; QS3.1 of l units, which represents the processors; QS3.2 of m units which represents the "input-output channels – external memory of the first type" system; QS3.3 of n units which represents the "input- output channels – external memory of the second type" system; three QS – QS2.1, QS2.2, QS2.3 – which represent the component parts of the auxiliary processors' system, which is similar to the QS for the basic processors.

Using the results concerning the monolevel MPCN, the fragment of the initial stochastic network, which contains the QS3.1– QS3.3 systems is represented by an exponential serving unit with the rate of the demands' serving

$$\mu_r = \pi_{0r}\mu_{1r}\left(l - \sum_{i=0}^{l-1}(l-i)\alpha_{ri}\right), \quad r = \overline{1,S},$$

where α_{ri} the busy state's probability of i processors of the basic system at the current degree r of multiprogramming. At the same time the initial stochastic network is replaced by an equivalent closed stochastic network, which consists of three QS: QS1- closed exponential tasks' source of N volume with the intensity of tasks preparation α; QS2 – monounity exponential system with the serving intensity of demands $\hat{\mu}_{\hat{r}}$ ($\hat{r} = \overline{1,\hat{S}}$) which is equivalent with the fragment of the initial stochastic network which represents the system of the auxiliary processors; QS3 – a monounity exponential system with the intensity of demands serving μ_r ($r = \overline{1,S}$) which is equivalent with the fragment of the initial stochastic network which represents the basic processors system.

For the calculation of this system's probabilities a_{ri} ($i = \overline{0,l-1}$, $r = \overline{1,S}$), the following expressions are obtained: $a_{ri} = \frac{t_1^i B_{ri}^{(n3)}}{i! G_r}$, $r = \overline{1,S}$, $i = \overline{0,l-1}$, where

$$B_{rz}^{(xy)} = t_y^{r-z}\left(\frac{x^{x+z-r}}{x!}\sum_{j=0}^{u}\frac{(xt_{2y})^j}{j!} + \frac{m^m}{m!}\sum_{j=\nu+1}^{r-z}\frac{t_{2ym}^j}{(r-z-j)!} + \right.$$

$$\left. + \begin{cases} \sum\limits_{j=u+1}^{\min(r-z,m)} \frac{t_{2y}^j}{j!(r-z-j)!}, & u = r-z-x < m = \nu, \\ \frac{m^m x^{x+z-r}\left((lt_{21m})^{u+1} - (lt_{21m})^{\nu+1}\right)}{m!x!(1-lt_{21m})}, & u = m < r-z-x+1 = \nu \end{cases}\right);$$

$$G_r = \sum_{k=0}^{\min(r,n)} \frac{t_3^k}{k!}B_{rk}^{(l1)} + \frac{n^n}{n!}\sum_{k=n+1,r>n}^{r}\frac{t_3^k}{n^k}B_{rk}^{(l1)};$$

$$t_1 = \frac{1}{\pi_{0r}\mu_{1r}}, \quad t_2 = \frac{\pi_{2r}}{\pi_{0r}\mu_{2r}}, \quad t_3 = \frac{\pi_{3r}}{\pi_{0r}\mu_{3r}}, \quad t_{ij} = \frac{t_i}{t_j}, \quad t_{ijk} = \frac{t_{ij}}{k}.$$

Knowing μ_r $(r = \overline{1,S})$, $\hat{\mu}_{\hat{r}}$ $(\hat{r} = \overline{1,\hat{S}})$, N and α, on the basis of the solution of Chapman-Kolmogorov equations system for the stationary state of the system, such characteristics of the network as the average response duration, the average tasks' number in the system can be determined.

Altogether the system of equations contains $(N+1)(N+2)/2$ equations and a same number of unknowns. For the not too great amounts of the index N, the solution of the later can be performed using traditional methods. For the great amounts of the index N, 2 ways are suggested: the CN detailing only to the level of the processors: the taking into consideration that for the great amounts of the N the output flow of the tasks' source is close to the poisson one.

In the first case the computer system is represented by a closed stochastic network, which consists of three multiunity exponential QS: QS1 of N units – the tasks' source; QS2 of $\hat{l}$ units – the auxiliary computer system; QS3 of l units – the basic computer system. The average response T duration to a task is determined, according to the formula: $T = \left(\hat{\tau} C_{\hat{l}2} + \tau C_{\hat{l}3}\right) / \; G_N$, where

$$\begin{aligned}
C_{u\nu} &= \sum_{i=1}^{u-1} \frac{\tau_{\nu 1}^{i}}{i!} B_{\nu i}^{(u)} + \frac{u^{u-1}}{u!} \sum_{i=u}^{N-1} (i+1) \tau_{\nu 1u}^{i} B_{\nu i}^{(u)} \cdot G_N = \\
&= \sum_{i=0}^{l-1} \frac{\tau_{31}^{i}}{i!} B_{2i}^{(\hat{l})} + \frac{l^l}{l!} \sum_{i=l}^{N} \tau_{31l}^{i} B_{2i}^{(\hat{l})}; \\
B_{\nu z}^{(u)} &= \sum_{j=0}^{\min(N-z,u-1)} \frac{\tau_{\nu 1}^{j}}{j!(N-z-j)!} + \frac{u^u}{u!} \sum_{j=u, u \le N-z}^{N-z} \frac{\tau_{\nu 1u}^{j}}{(N-z-j)!};
\end{aligned}$$

$\tau_1 = 1/\alpha$; $\tau_2 = \hat{\tau}$; $\tau_3 = \tau$; $\tau_{21} = \alpha\hat{\tau}$; $\tau_{31} = \alpha\tau$; $\tau_{iju} = \tau_{ij}/u$, and $\hat{\tau}$, τ – average serving duration of the tasks by an auxiliary processor and respectively – the basic one.

Usually the second variant is preferred due to a smaller error of the model. In this case the equivalent stochastic network is fulfilling the condition of the Jackson theorem division and it can be examined as an ensemble which consists of two independent completely accessible QS: QS2 and QS3 with an input flow of rate λ and the serving rate respectively: $\hat{\mu}_{\hat{r}}$ $(\hat{r} = \overline{1, \hat{S}})$ and μ_r $(r = \overline{1, S})$.

4 Macrosynthesis of the regional computer networks

The results of the analyses of the tasks' characteristics, of the computer nodes and data trunks modeling are used for the elaboration of the ensemble of algorithms for the regional computer networks macrosynthsis. The ensemble of the algorithms is stipulating the determination of set $Z \subseteq X$ of locations for placing the LPCN, of the Y_o set of locations for placing the BPCN, of the $r = \overline{1, |R|}$ rational variants of sets $Y(r) \subseteq Y_o$ $(r = \overline{1, |R|})$ of the locations for placing the PCN of the network. In a particular case, the R set can be unitary.

For each $r \in R$ variant, the problem's solution is obtained on the base of the general problem's decomposition in $m+1$ subproblems. One of them consists in the attachment of the X set's subscribers to the Z

set's PCN. Another subproblem is stipulating the MPCN $y_o \in Y(r)$ and the backbone trunks determination, which ensures the $C_{Y(r)}$ scantily expressed expenses' minimum with a backbone DT and the processing of the derived to MPCN information flow

$$C^*_{Y(r)} = \min_{y_o \in Y(r)} \left(\breve{C}_{\,yy'} + C_{y_o} \left(1 - \frac{U_{Xy_o}}{U_{y_o}} \right) \right)$$

at the (5), (6) restrictions. Here U_{Xy_o} is the MPCN capacity's quote which amount to the processing of the clients' tasks.

The other m subproblems consist from the rational addition of LPCN from the Z set to m BPCN of the $Y(r)$ set (finding the subsets Z_y, $y \in Y(r)$) and the redirection of the clients' tasks $x \in X$ between UN $x \in X$, LPCN $z \in Z$ and BPCN $y \in Y(r)$. As a criteria of the optimization is used the minimum of expressed expenditures C_{xy} with the processing of one part of clients' tasks at UN, the other one – at LPCN and the rest – at BPCN and the informational exchange between UN, LPCN, BPCN, i.e.

$$C^*_{xy^*} = \min_{y \in Y(r)} \left(C_x + \hat{C}_{xy} + \tilde{C}_{xy} \right), \qquad x \in X \tag{14}$$

at (2–7) restrictions. At the same time the $C^* = C^*_{Y(r)} + \sum_{x \in X} C^*_{xy^*}$ condition (see (1)) is respected. Afterwards the UN, LPCN, BPCN and DT characteristics of the network are made more precisely taking into consideration the results of LPCN attachment to BPCN.

To realize the totality of algorithms, at first, there are formulated and proposed different methods of solution of the some problems which could be used independent: the connection of a new client to the computer node; the synthesis of a videoprocessing system of the dated from the computer node and more clients' stations; macrosynthesis of the computer system which has the ierarhical structure with three levels. As a criteria is used the minimum of total expressed expenditures with the system. These problems foresee the rational distribution of the users' tasks differentiated by the operativity, laboriousity and processing complexity, between client nodes and computer nodes of the network. At the same time, if it is necessary, could be find the productivity of UN and PCN. The productivity U_x of UN could be choosed from a continua interval of the values or from the set of the discreet values.

The accomplished calculation confirmed the opportunity of the differentiation of the tasks and of the implementation of the LPCN in the topological-functional structure of the regional computer networks. With the costs reducing, the increase of the local informatic means becomes rational; with the increase of computer nodes performance, decrease of the expressed expenditures with BPCN and DT, increase the effect from the differentiation of the users' tasks by the processing complexity. The decrease of the computer nodes performance and the increase of the users' flows of tasks intensity and of the length of data channels, implies the increase of the number of tasks' categories, which is rational to be processed at the client computer nodes.

5 The capitalization of the RCN resources

5.1 Supervising the networks

There are essential factors that refer to the supervising of networks: configuration parameters, topology and characteristics of network's components; clients' need in information processing jobs (IPJ) and tasks' characteristics, including assistance requirements. One of the most efficient methods of supervising networks is the hybrid one [1]. This foresees the combination of centralized supervising with the spreaded one. The tables of routing the information regarding users' tasks are being calculated in a centralized way according to the statistics dealing with the state and the running of the network. However, every node can modify its own routing table to a certain extent on the basis or current local information. Centralized calculations are made periodically or whenever significant changes in network's state occur. Local modification are performed promptly whenever there's need.

The problem must be solved periodically at one of the network's CN, usually at MPCN, either given the expiration of some fixed time intervals or in case of some significant changes in network's state (fall or essential change of some network components' task etc.).

In this section we examine some aspects dealing with the centralized part of hybrid methods of supervising which include at the same time aspects of distributing IPJ among network's computer nodes. In accordance to the content, the examined problem consist in establishing IPJ's volume that can be carried out by network's every CN during the planning period as well as establishing the rational routes and the intensity of information's traffic for every CN, together with IPJ's composition for conveyance on each of the routes and processing by CN, and also, finding the average duration of restraint for tasks of different categories at network's elements that provide the searched extreme of the goal function to the respecting of certain restriction.

Taking into account the complexity, the problem is being examined as an ensemble of some relatively autonomous problems, which are only dependent with regard to their input and output; establishing the possible level of solving the needs of IPJ's clients; distribution of clients' IPJ among computer nodes; ascertaining routes of transfer and the allowed duration of restraint for information dealing with users' tasks toward network's elements; establishing the efficient time resource for given computer nodes and transfer trunks; gathering and analyzing the statistics referring to the functioning of the regional computer network.

The problem of estimating the possible level of satisfying subscribers' needs in IPJ and distributing IPJ among network's computer nodes are formulated as linear programming problems. *The first problem* foresees the fixing of such values of operative information of volume Y_k, ordinary information of volume Z_k and both operative and ordinary information of volume X_k that belong to CN k for $k = \overline{1, n}$, that would maximize the satisfaction of needs for operative information Y_k^o that must be processed in less than R_k times unit and of that ordinary information Z_k^o, for the period of planning T, which would provide

$$F = \sum_{k=1}^{n} \left(\gamma_k \sum_{i \in A'_k} y_{ik} + \nu_k \sum_{i=1}^{n} z_{iik} + \chi_k \sum_{i=1}^{n} x_{iik} \right) \to \max$$

certain initial data and a series of restrictions where: n – the number of network's CN; γ_k, ν_k, χ_k – importance coefficients of processing the information of CN k, inside the network, respectively operative, ordinary and both operative and ordinary; A_i – the set of network's CN connected through DT with CN k; $A'_k = A_k \cup k$; $y_{jk} = y_{ijk}$, z_{ijk}, x_{ijk} – the values of information volume, respectively operative, ordinary and both operative and ordinary information that belongs to CN k, transmitted by means of DT between CN i and j and processed by CN j, with $j \in A_i$, $j \neq i$, for all $k = \overline{1,n}$.

The second problem consists in establishing the values of the variables y_{ji}, z_{ijk}, x_{ijk} ($k = \overline{1,n}$, $i = \overline{1,n}$, $j \in A'_i$), which ensure the minimum level of summary expenditures with respect to weight (financial, time expenditures etc.) referring to the use of computer network's resources linked to the transfer and processing of information volumes Y_k, Z_k, X_k ($k = \overline{1,n}$), that is

$$F = \sum_{k=1}^{n} \left\{ \sum_{i \in A'_k} y_{ik} \left(\tfrac{c_i}{U_i} \eta_k + \sigma_{ki} t_{ki} (1 + r_k) \right) + \right.$$

$$+ \sum_{i=1}^{n} \left(\tfrac{c_i}{U_i} (\eta_k x_{iik} + \eta_{ok} z_{iik}) + \right.$$

$$\left. \left. \sum_{j \in A_i} \sigma_{ij} t_{ij} (1 + r_k)(z_{ijk} + x_{ijk}) \right) \right\} \rightarrow \min$$

for given restriction where $k = \overline{1,n}$, and c_i, σ_{ij} are the coefficients characterizing the weight (for example, the cost) of a working time unit for CN i and respectively DT between CN i and j.

The problem of establishing the information's transfer routes is also examined; the information deals with the tasks of different categories inside the computer networks according to the results of aggregate IPJ dispatcherising. Operative information includes information dealing with tasks of categories $i = \overline{1, I_1}$, and the ordinary one – information dealing with categories $i = \overline{I_1 + 1, I}$. Supposedly the following characteristics are known: the planning interval T; the tasks' characteristics

u_i, v_i, $i = \overline{1, I}$; for every network's CN $k = \overline{1, K}$ – the needs in IPJ, which are determined by intensities λ_{ki}^{Δ} $(i = \overline{1, I})$ of the tasks flows that must be served only by CN k, and the intensities λ_{ki}^{o} $(i = \overline{1, I})$ that can be served by other network's CN, the values of the operative information volume $y_{jk} = y_{ljk}$, ordinary information volume z_{ljk} and both ordinary and operative information volume x_{ljk} belonging to CN k, that are being transmitted through DT between CN i and j and being processed by CN j for $j \in A_l$, $k \neq j$.

We must determine: the set $P = \{p\}$ of transfer routes for information regarding the tasks in the network and the volume of operative information $\hat{Y}_p$, the ordinary on $\hat{Z}_p$ and the both operative and ordinary one $\hat{X}_p$, which is being transmitted on the route p for all the routes $p \in P$, that provide for

$$F_{Yk} = \sum_{p \in P_{Yk}} a_{Yp} \hat{Y}_p \to \min, \quad F_{Zk} = \sum_{p \in P_{Zk}} a_{Zp} \hat{Z}_p \to \min,$$

$$F_{Xk} = \sum_{p \in P_{Xk}} a_{Xp} \hat{X}_p \to \min, \ k = \overline{1, n}$$

the following restrictions

$$\sum_{p \in P_{Yjk}} \hat{Y}_p = y_{jk} > 0; \quad \sum_{p \in P_{Zljk}} \hat{Z}_p = z_{ljk} > 0; \quad \sum_{p \in P_{Xljk}} \hat{X}_p = x_{ljk} > 0$$

for all the indices' values $l = \overline{1, n}$, $j = \overline{1, n}$, $k = \overline{1, n}$, where: $P_{Yk} \subseteq P$, $P_{Zk} \subseteq P$, $P_{Xk} \subseteq P$ represent the totality of possible routes of transfering the information, respectively the operative, the ordinary, and both the operative and ordinary one from CN k to other network's CN with the volumes y_{jk}, z_{ljk}, x_{ljk}; $P_{Yjk} \subseteq P_{Yk}$, $P_{Zljk} \subseteq P_{Zk}$, $P_{Xljk} \subseteq P_{Xk}$ represent the totality of transfer routes for information regarding the tasks of the center k, which include DT between CN i and j; a_{Yp}, a_{Zp}, a_{Xp} being the coefficients that characterize the route p, for instance, the number of elements of route p. There are also accepted certain conditions referring to the passing of operative information into a merger of operative and ordinary information of dealing with the distribution of information among CN taking into account the complexity of their processing, etc. An algorithm of determining the structure of tasks flows for every CN on the routes of totality P is being proposed.

Knowing the data transfer routes, the tasks' and network's elements' characteristics, one can formulate and solve *the problem of assessment of effective time resources for both the given computer nodes and transfer trunks* in order to respond to users' tasks. Without reducing the problem's generality, it is considered that the duration of the restraint for both the tasks and the respective replying information at switching nodes can be neglected as compared to the transfer duration through DT and their processing by CN.

Here is the notation: L_1 – CN set; L_2 – DT set; $L = L_1 \cup L_2$; t_{dli} – the average allowed duration of restraint for tasks of category i at the l element of the network; P – the set of transfer routes for the tasks in the network; B_p – the set $p \in P$; P_l – the set of routes containing the l element of the network. The flows of input tasks having the category $i = \overline{1, I}$ are elementary having the rate λ_{li} $(i = \overline{1, I})$, and their serving duration has an exponential repartition with the intensity μ_{li} $(i = \overline{1, I})$.

Say we know the characteristics: $\alpha^o_{li} = \lambda^o_{li}/\Lambda^o_l$, $i = \overline{2, I}$, $l \in L_1$; $\alpha^o_{ji} = \alpha_{jki} = \lambda_{jki} / \sum\limits_{s=2}^{I} \lambda_{kks}$, $i = \overline{2, I}$, $j \in L_1$, $k \in L_1$ where: α^o_{li} – the share of the flow of tasks having the category i and the intensity λ^o_{li} in the summary flow of tasks having the category $i = \overline{2, I}$ and the intensity Λ^o_l, generated by CN l.

We must determine:

$$t_{dli} = \theta_l T_{di} \,/\, \tau_{p_l}, \qquad i = \overline{2, I},\ l \in L; \quad \Lambda_{lp},\ l \in L,\ p \in P, \tag{15}$$

that ensures $\sum\limits_{l \in B_p} \Lambda_{lp} \to \max$ $p \in P$ to the relations

$$\theta_l = \min_{p \in P_l} \theta_{lp}, \quad l \in L, \tag{16}$$

$$\Lambda_l = \min_{p \in P_l} \Lambda_{lp}, \quad l \in L, \tag{17}$$

$$\tau_p = \sum_{i=2}^{I} \alpha^o_{l_p i} T_{di}, \quad p \in P \tag{18}$$

and the restrictions $\sum\limits_{l \in B_p} \theta_{lp} = \tau_p$, $p \in P$, where l_p is the first element of route p.

The obtained solution:

$$\begin{aligned}
\Lambda_{lp} &= \frac{1}{r_l} - \frac{\frac{\sqrt{q_{j_p}+r_{j_p}s_{j_p}}}{r_{j_p}} + \sum\limits_{l \in (B_p \cap L_2)} \frac{\sqrt{q_l}}{r_l}}{r_l \left(\tau_p + \sum\limits_{l \in B_p} \left(r_l + \frac{q_l}{r_l}\right)\right)} \begin{cases} \sqrt{q_{j_p} + r_{j_p} s_{j_p}}, \ l = j_p, \\ \sqrt{q_l}, \ l \in (B \cap L_2). \end{cases} \\
\theta_{lp} &= r_l + \begin{cases} \frac{q_1 \Lambda_{lp}}{1 - r_l \Lambda_{lp}}, & l \in (B_p \cap L_2), \ p \in P, \\ \frac{s_l + q_1 \Lambda_{lp}}{1 - r_l \Lambda_{lp}}, & l = j_p, \ p \in P, \\ 0, & l \in (B_p \cap L_1), \ l \neq j_p, \ p \in P \end{cases}
\end{aligned} \tag{19}$$

$$r_l = \begin{cases} \sum\limits_{i=2}^{I} \frac{\alpha_{li}}{\mu_{li}}, & l \in (B_p \cap L_2), \ p \in P, \\ \frac{\mu_{l1}}{\mu_{l1} - \lambda_{l1}} \sum\limits_{i=2}^{I} \frac{\alpha_{li}}{\mu_{li}}, & l = j_p, \ p \in P \end{cases} \tag{20}$$

$$s_l = \frac{\lambda_{l1}}{(\mu_{l1} - \lambda_{l1})^2}, \quad l \in L;$$

$$q_l = \begin{cases} \sum\limits_{i=2}^{I} \frac{\alpha_{li}}{\mu_{li}^2}, & l \in (B_p \cap L_2), \ p \in P, \\ \frac{\mu_{l1}}{(\mu_{l1} - \lambda_{l1})^2} \left(\sum\limits_{i=2}^{I} \frac{\alpha_{li}}{\mu_{li}^2} - \frac{1}{2} \left(\sum\limits_{i=2}^{I} \frac{\alpha_{li}}{\mu_{li}} \right)^2 \right), & l = j_p, \ p \in P \end{cases},$$

$j_p \in (B_p \cap L_1)$ is the CN that process the tasks transfered on route p. Knowing Λ_{lp}, the capacity Λ_l of the element l is determined according to the formula (17), θ_{lp} ($l \in L$, $p \in P_l$) – according to the formula

(19), θ_l ($l \in L$) – according to the formula (16) and, finally, the allowed average duration t_{dli} ($i = \overline{2, I}$) of restraint for tasks having the category i at the element l of the network – according to the formula (15). The software packet DISTRIB has been made in order to solve the problem of supervising inside the regional network according to the given ensemble of algorithms.

5.2 Putting in order works' fulfillment

Research has begun referring to some aspects of putting in order works' fulfillment in sequential system (CN, SN etc.) which are basically reduced to solving the $M \times n$ Bellman-Johnson problem of putting in order n works at M consecutive units of processing [8]. With the condition that all works are implemented in the same order at all units, the problem consists in determining the order that would provide the minimum duration T of all the works' fulfillment:

$$\begin{aligned} T \quad = \quad & \max_{u_1=\overline{1,n}} \left(\sum_{k=1}^{u_1} \tau_{1i_k} + \max_{u_2=\overline{u_1,n}} \left(\sum_{k=u_1}^{u_2} \tau_{2i_k} + \ldots + \right.\right. \\ & \left.\left. + \max_{u_{M-1}=\overline{u_{M-2},n}} \left(\sum_{k=u_{M-2}}^{u_{M-1}} \tau_{M-1,i_k} + \sum_{k=u_{M-1}}^{n} \tau_{Mi_k} \right) \ldots \right)\right) \rightarrow \min, \end{aligned} \tag{21}$$

where τ_{ji_k} is the duration of execution of the work i_k, which is placed in the ordering R on the place k, by the unit j and $u_0 = 1$, $u_M = n$.

Let the schedule be $R_1 = R = \{i_1, i_2, \ldots, i_l, \ldots, i_{l+r-i}, \ldots, i_n\}$, and let its works be placed in the rising order of the index beside i, and the schedules R_j ($j = \overline{2, r!}$), which are distinguished among them and are different from the schedule R_1 only by placement of a part or of all works i_k ($k = \overline{l, l+r-1}$). Here r is the number that determines the interval composed from r works in the R schedule, where by changing their places inside the interval $[l, l+r-1]$ one can obtain $r!$ different schedules R_j $(j = \overline{1,r})$. Then, as it is shown in the essay, the necessary optimal condition is

$$\min_{j=\overline{1,M-1}} \left(z_{j-1} + \min_{p=\overline{0,M-j-1}} \left(Z_{j+p+1} + \right.\right.$$
$$\left.\left. + \min_{\substack{u_j \le u_{j+1} \le \ldots \le u_{j+p} < l+r-1, \\ u_j = \overline{l,l+r-1}}} \sum_{k=0}^{p} \sigma_{j+k,u_{j+k}+1} \right) \right) \ge$$
$$\ge \max_{k=\overline{2,r!}} \left(\min_{u_j=\overline{l,l+r-1}, j=\overline{1,M-1}} g_{ju_j}(R_k) \right), \tag{22}$$

where the components $g_{ju_j}(R_k)$, $j = \overline{1, M-1}$, $u_k = \overline{l, l+r-1}$, $k = \overline{2, r!}$ of the right part of the inequality (22) are calculated according to the formulas similar to the expression in square parentheses from the left part (for the schedule R_1) of this inequality taking into account works' placement in the schedule R_k; $\sigma_{j\nu} = \sum\limits_{k=l}^{\nu-2} \tau_{j+1,i_k} + \sum\limits_{k=\nu}^{l+r-1} \tau_{j,i_k}$. The magnitudes z_j, Z_{j+2} $(j = \overline{1, M-1})$ are the same for all the examined schedules R_k $(k = \overline{1, r!})$, but depend at $M > 2$, on the works' placement in these schedules on the place $\overline{1, l-1}$ and $\overline{l+r, n}$. They are calculated according to some particular formulas.

We must mention that for $M = r = 2$ the condition (22) is reduced to Johnson' works' ordering rule for sequential system having two processing units [8].

The veracity of some statements and consequences used for working out the algorithms of putting in order the works including the ones that follow is proved.

Statement 5.1. For the inopportuneness, in the sense $\min T$, of the switching of works' places i_α and i_β $(\alpha < \beta)$ for some arbitrary schedule R given the keeping of the placement for other works, it is only necessary for

$$\tau_{j+p+1,i_\alpha} + \sum_{k=0}^{\nu} \tau_{j+k,i_\beta} \ge \tau_{j+p+1,i_\beta} + \sum_{k=0}^{\nu} \tau_{j+k,i_\alpha},$$

$$\nu = \overline{0,p},\ p = \overline{0,M-j-1},\ \beta - \alpha > 1;$$

$$\min_{\nu=\overline{0,p+1}} \left(\sum_{k=0}^{\nu-1} \tau_{j+k,i_\beta} + \sum_{k=\nu}^{p} \tau_{j+k+1,i_\alpha} - \sum_{k=\alpha+1}^{\beta-1} \tau_{j+\nu,i_k} \right) \ge$$

$$\ge \min_{\nu=\overline{0,p+1}} \left(\sum_{k=0}^{\nu-1} \tau_{j+k,i_\alpha} + \sum_{k=\nu}^{p} \tau_{j+k+1,i_\beta} - \sum_{k=\alpha+1}^{\beta-1} \tau_{j+\nu,i_k} \right),$$

$$p = \overline{0,M-j-1}, \quad j = \overline{1,M-1}.$$

Consequence 5.1. It is unreasonable to switch the places for the neighboring work i_l and i_{l+1}, in the sense of $\min T$, if

$$\min_{\nu=\overline{0,p+1}} \left(\sum_{k=0}^{\nu-1} \tau_{j+k,i_{l+1}} + \sum_{k=\nu}^{p} \tau_{j+k+1,i_l} \right) \ge D(S), \tag{23}$$

$$j = \overline{1,M-1}, \quad p = \overline{0,M-j-1}.$$

where $D(S)$ is the expression for the right side which is different from the one of the left side of the inequality (23) only in regards to the transposition of the elements i_l, i_{l+r-1}.

Consequence 5.2. If for the pair of works (α,β) the following relations take place

$$\tau_{j+p+1,i_l} + \tau_{j,i_{l+r-1}} + \sum_{k=1}^{\nu} (q_{j+k-1,l+1} - \tau_{j+k,i_l}) \ge$$

$$\ge \tau_{j+p+1,i_{l+r-1}} + \tau_{j,i_l} + \sum_{k=1}^{\nu} \left(q_{j+k+1,l+1} - \tau_{j+k,i_{l+1}} \right),\ \nu = \overline{0,p},\ r > 2$$

$$\sum_{k=0}^{\nu-1} \tau_{j+k,i_\beta} + \sum_{k=\nu}^{p} \tau_{j+k+1,i_\alpha} \ge \sum_{k=0}^{\nu-1} \tau_{j+k,i_\alpha} + \sum_{k=\nu}^{p} \tau_{j+k+1,i_\beta},$$

$$\nu = \overline{0,p+1}, \quad p = \overline{0,M-j-1}, \quad j = \overline{1,M-1},$$

then it is timely to establish the places of these works, in the sense of $\min T$, in the schedule so that $\alpha \to \beta$.

Statement 5.2. If for the pair of works (α, β) takes place the relation

$$\min\left(\tau_{j+1,\alpha}; \tau_{j\beta}\right) > \min\left(\tau_{j\alpha}; \tau_{j+1,\beta}\right), \quad j = \overline{1, m-1}, \tag{24}$$

then for $k = \overline{2, m}$ at $\tau_{k\alpha} < \tau_{k\beta}$ then the following relation is also true:

$$\tau_{k\alpha} < \min\left(\tau_{p+1,\alpha}; \tau_{p\beta}\right), \quad p = \overline{1, k-1} \ldots \text{at}\, m = \overline{2, M}.$$

Statement 5.3. If for the works α and β the following inequalities take place

$$\min\left(\tau_{j+1,\alpha}; \tau_{j\beta}\right) \geq \min\left(\tau_{j\alpha}; \tau_{j+1,\beta}\right), \quad j = \overline{1, m-1}, \tag{25}$$

and, at the same time if for an arbitrary $\nu \in [2; m]$ the relation $\tau_{\nu\alpha} = \tau_{\nu\beta} < \tau_{\nu-1,\alpha}$, then $\tau_{\nu-1,\alpha} \leq \tau_{\nu-1,\beta}$, and for $k = \overline{2, m}$ at $\tau_{k\alpha} \leq \tau_{k\beta}$ the following inequality is true $\tau_{p\alpha} \leq \tau_{p\beta}$, $p = \overline{1, k-1}$ at $m = \overline{2, M}$.

Statement 5.4. If for an arbitrary $l \in [1; n-1]$ the following inequalities are true $\tau_{ji_l} \leq \tau_{ji_{l+1}}$, $j = \overline{1, m}$, then $T_{ji_l} \leq T'_{ji_l}$, $j = \overline{1, m}$, where $T_{ji_k} \leq T'_{ji_k}$, $(k = \overline{1, n})$ is the duration of finishing the execution of work i_k at the schedules, respectively, $R = \{i_1, i_2, \ldots, i_l, i_{l+1}, \ldots, i_n\}$ and R'_1, which is only different from R in regards to the transposition of the elements i_l, i_{l+1} at $m = \overline{1, M}$.

Statement 5.5. If for the works α and β the relation (25) takes place and at the same time if for an arbitrary $\nu \in [2, m]$

$$\tau_{\nu\alpha} < \tau_{\nu\beta} \tag{26}$$

and for an arbitrary $k \in [2, \nu - 1]$

$$\tau_{k\alpha} = \tau_{k\beta}, \tag{27}$$

then the following inequality takes place

$$\tau_{k\alpha} \geq \tau_{k-1,\alpha}, \tag{28}$$

then while placing these works close in the schedule, that is $\alpha = i_l$, $\beta = i_{l+1}$, $l \in [l, n-1]$, the following inequality takes place $T_{ji_{l+1}} \le T'_{ji_{l+1}}$, $j = \overline{1,m}$ for $m = \overline{2,M}$.

Consequence 5.3. If for any arbitrary works α and β the relation (24) or the relations (25)–(28) take place, then in placing of these works close in the schedule it is rational, in the sense $\min T$, that $\alpha \to \beta$.

The partial preliminary ordering of the pairs of work according to the consequence 5.3 demand a considerable lesser volume of calculations as compared to the partial preliminary ordering in a general case according to consequence 5.2.

Statement 5.6. The sufficient condition for satisfying the inequalities

$$\min_{\nu=\overline{0,p+1}} \left(\sum_{k=0}^{\nu+1} \tau_{j+k,\beta} + \sum_{k=\nu}^{p} \tau_{j+k+1,\alpha} \right) >$$

$$> \min_{\nu=\overline{0,p+1}} \left(\sum_{k=0}^{\nu+1} \tau_{j+k,\alpha} + \sum_{k=\nu}^{p} \tau_{j+k+1,\beta} \right),$$

$$j = \overline{1, M-1}, \quad p = \overline{0, M-j-1}$$

is their satisfaction at $p = 0$, that is the satisfaction of (24).

Statement 5.7. The sufficient condition for satisfying the inequality (23) at $i_l = \alpha$ and $i_{l+1} = \beta$ is the satisfaction of (23) at $p = 0$, that is the satisfaction of (25) at $m = M$, if the relations (26), (28) take place.

Consequence 5.4. If for any pair of works of the given totality the relations (24) or (25)–(28) take place at $m = M$, then the optimal schedule, in the sense $\min T$, can be obtained according to the rule: $\alpha \to \beta$, if (24) or (25)–(28) take place.

Statement 5.8. If $\tau_{1i} = \tau_{\nu i}$, $\tau_{\nu+1,i} = \tau_{ji}$, $\nu = \overline{1, \nu_i}$, $j = \overline{\nu+1, M}$, $i = \overline{1,n}$, then the optimal schedule, in the sense $\min T$, can be obtained according to the rule: $\alpha \to \beta$, if at $\nu_\alpha < \nu_\beta$ then the following inequalities are true $\tau_{1\beta} \ge \min(\tau_{M\alpha}, \tau_{M\beta})$, $\tau_{M\alpha} \ge \min(\tau_{1\alpha}, \tau_{1\beta})$, and in the other cases – $\min(\tau_{M\alpha}; \tau_{1\beta}) \ge \min(\tau_{1\alpha}; \tau_{M\beta})$.

Two particular cases of the problem $4 \times n$, defined by the statement 5.9 and by the consequence 5.5 are being researched.

Statement 5.9. At $M = 4$, in the sense $\min T$, the following are the sufficient suitability conditions for implementing the works in the same order at all system's processing units

$$\min\left(\tau_{3i_{l+1}}; \tau_{4i_l}\right) \geq \min\left(\tau_{3i_l}; \tau_{4i_{l+1}}\right), \quad l = \overline{1, n-1}.$$

Consequence 5.5. If there is an optimal schedule for the sequential system of two units 3 and 4 that coincide with the optimal schedule R at $M = 4$ obtained under the condition of the same implementing order at all the 4 processing units, then R is an optimal unconditional schedule.

Two particular cases of the problem $3 \times n$ defined by the statement 5.10, 5.11 are being examined.

Statement 5.10. If for the schedule $R = \{i_1, i_2, \ldots, i_n\}$ of implementing n works by a sequential system of 3 processing units the inequalities (25) are satisfied at $M = m = 3$ and at the same time at $\tau_{2i_l} + \tau_{2i_{l+1}} < \min(\tau_{1i_l} + \tau_{2i_l}; \tau_{1i_l} + \tau_{3i_{l+1}}; \tau_{2i_{l+1}} + \tau_{3i_{l+1}})$ for an arbitrary $l \in [1, n-1]$ the following inequality is also true

$$\min\left(\tau_{1i_{l+1}} + \tau_{2i_{l+1}}; \tau_{1i_{l+1}} + \tau_{3i_l}; \tau_{2i_l} + \tau_{3i_l}\right) \geq$$

$$\geq \min\left(\tau_{1i_l} + \tau_{2i_l}; \tau_{1i_l} + \tau_{3i_{l+1}}; \tau_{2i_{l+1}} + \tau_{3i_{l+1}}\right),$$

then the schedule R is optimal in the sense $\min T$.

Statement 5.11. If $\tau_{2i} \leq \min(\tau_{1i}; \tau_{3i})$, $i = \overline{1, n}$ then the optimal schedule of implementing n works by a sequential system of 3 processing units can be obtained while ordering the works according to the rule $i \rightarrow j$, if

$$\min\left(\tau_{1j} + \tau_{2j}; \tau_{2i} + \tau_{3i}\right) \geq \min\left(\tau_{1i} + \tau_{2i}; \tau_{2j} + \tau_{3j}\right).$$

On the base of the necessary optimal condition, the obtained statements and consequences an improvement is suggested regarding the laboriousity of the Johnson algorithm for the problem $2 \times n$ (algorithms

$A_{5.1}$, $A_{5.2}$); the algorithm $A_{5.3}$ is elaborated for the problem $3 \times n$ and the algorithm $A_{5.4}$ for the problem $M \times n$. The calculations for 620 variants of problems having the types: 3×6, 3×8, 3×20, 3×30, 3×50, 4×10 and 5×10 have confirmed the efficiency of the elaborated algorithms. The obtained assessments for the Johnson algorithms fits well with the similar results belonging to Baglio and Wagner [8]. The algorithms $A_{5.3}$, $A_{5.4}$ are more efficient than the known algorithms including the Johnson one. The exact solution using the algorithms $A_{5.3}$, $A_{5.4}$, has been found for 619 out of 620 variants, although the maximum allowed number of iterations ($\leq$200) and the breadth r of the window ($\leq$4) have been the upper limits.

Conclusion

The obtained results are partially applied while creating, developing and using some computer networks in Moldova. The elaborated methods, algorithms, models and also the respective program products can be used at the creation, development and the efficient turning to account of computer networks' resources of the data trunks of data teleprocessing systems and of regional computer networks as a whole. The results referring to the queuing systems, sequential systems of works implementation can have a larger corresponding application.

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