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Experimental results on quadratic assignment problem

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Abstract

The paper presents experimental results on quadratic assignment problem. The "scanning area" method formulated for radioelectronic equipment design is applied. For all more complex tests ours results are better or coincident with the ones known in literature. Conclusion concerning the effectiveness of method are given.

1 Introduction

The quadratic assignment problem belongs to the discrete programming for which still there are no exact effective algorithms. Its exact solution is connected with explicit or implicit full numbering and evaluation of all variants for assignment of objects to locations. At present the optimum displacement of n object on n positions is possible in case $n \leq 15$ [3]. The increase of productivity of computing systems shifts unsignificantly up this limit. Most of the real problems have significantly large area and for the solution, heuristic (suboptimal) algorithms are used.

2 Statement of the problem

A set of objects $E = \{e_1, e_2, \ldots, e_m\}$, is given which should be located on a set of fixed positions $P = \{p_1, p_2, \ldots, p_n\}$ where $m > 1$, $n > 1$, $m \leq n$. Any object could take one position.

An area of the distances between elements of P is given. The distance between objects is computed usually as Euclid's (euk)

$$D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

where x and y are coordinates of positions p_i and p_j.

The orthogonal distance (ort) is also used

$$D_{ij} = |x_i - x_j| + |y_i - y_j|$$

There are also other more complex systems for evaluations of distances among three or more objects.

The matrix for binary relations between objects from E is given F_{ij} $(j = 1, 2, \ldots, m;\ j = 1, 2, \ldots, m)$. It is accepted that the two matrices D and F are symmetric, i.e.

$$D_{ij} = D_{ji} \quad \mathit{and} \quad F_{ij} = F_{ji}.$$

In the general case the relations F could involve more then two objects.

A correspondence $\alpha(E \rightarrow P)$ should be found for which the evaluation for quality of solution

$$S(\alpha) = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} F_{ij} D_{kl}$$

gets optimal value.

Here k and l are indexes of positions taken from the object e_i and e_j, resp. In most of problems a minimum value for $S(\alpha)$ is looked for.

3 A "scanning area" method

In the solution of problem the "scanning area" method is used, applied in radioelectronic equipment design [1]. The essence of method consists in separation of initial problem to a number of smaller problems (areas) in which the same problem is solved using exact or sufficiently exact methods. The dimensions of these areas are not fixed and are chosen depending on the computing resources available. In practice, areas of four or six positions are used.

Important peculiarity of method is overlapping of areas. Initially separated optimization problems are solved subsequently for neighbouring overlapping areas due to which the process is called scanning (Fig. 1). The total scanning of all areas is one iteration. The iterations are repeated until better solutions at least in one area are obtained. The final solution is a local optimum. The method has potentialities for getting out the local optimum and continuing the optimizations [1].

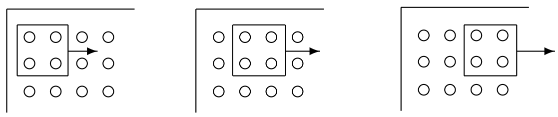

Fig. 1. Forming of overlapping areas (scanning)

Sequence for scanning of areas is called strategy of scanning. Part of possible strategy of scanning is shown in Fig. 2. It is established later on that the process is not strongly sensitive to strategies of scanning. Much better results are obtained also in the case of random scanning of areas.

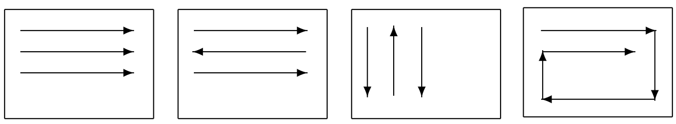

Fig. 2. Different strategies of scanning

The overlapping of areas in separated steps of scanning permits objects to optimize their location not only in the area but also to move out of it. This peculiarity expands considerably the potentialities of method for scanning of better local optimum.

4 Method and results of experiments

The "scanning area" method is realized in Pascal. For experiments computer Compac with Pentium 166 MHz processor is used.

The data for tests examples are taken from publications by Nugent and Steinberg [4,5]. Checking of starting solution shown by Nugent [4] is performed. The results obtained coincide with the ones given in Nugent [4], which guarantees the correctness of data introduced.

For each of tests by Nugent [4], 100 random initial solutions are generated which are optimized by the "scanning area" method. It is supposed that the potentialities of method will permit to find deep enough local optimum.

The scanning is performed in area of four positions. For getting out of the local optima in more complex test examples a change of system for evaluation of distances [1] and scanning with six positions is used which influences the time for obtaining results. The evaluation of solutions obtained is shown in Table 1.

Table 1

N	Example	a	b	c	d	e
1.	Nugent 12	12	Ort	289	289 (1)	1 sec
2.	Nugent 15	15	Ort	575	563 (2)	3 sec
3.	Nugent 20	20	Ort	$*$ 1285	1287	13 min
4.	Nugent 30	30	Ort	$*$ 3064 (3)	3079	5 min (4)
5.	Steinberg	36	Euk	$*$ 4119.7 (5)	4124.97	–
6.	Steinberg	36	Euk^2	7926 (6)	7926	–
7.	Steinberg	36	Ort	$*$ 4799 (6)	4802	–

a = dimension, b = distance, c = best value obtained by the "scanning area" method, d = best published value, e = time on Compac 2000 with Pentium 166 MHz processor, $*$ = a result better than the ones published in literature

Comment to the table:

(1) The evaluation shown is exact solution of problem.

(2) The result can not be checked by the solution shown by Burkard [3]. Sharp local optimum or inexact data by Burkard are supposed.

(3) A better result is found of evaluation 3062 (shown in Appendix) but the time is larger.

(4) The time for optimization by “scanning area” method depends on the character of relations between objects in different tests and is not always connected with the scale of test.

(5) The result is published in 1983 [2]. It is obtained through optimization of initial solution.

(6) The result is obtained through optimization of solution of test 5 in Table 1.

Remarks:

1. All evaluations for the existing best result (colon ’d’ in Table 1) are taken from Burkard [3] whereas the evaluation is transformed in the semi-perimeter of rectangle. The solutions given by Burkard for the tests of Nugent do not contain enough data for their checking.

2. The test problems of Nugent [4] containing five, six, seven and eight objects are left. They do not make a problem and confirm the results obtained.

3. In the test by Nugent [4] with subsequent numbers from 1 to 4 in the above Table, series of 100 random initial solutions are optimized. The average time for obtaining the better results shown is computed by division of the total time for all 100 solutions and the number of optimum solution obtained.

4. The solution of test for subsequent numbers 3, 4, 5 and 7 in above Table whose evaluations are better that the known in the publications referred are given in the Appendix.

5 Conclusions

The test examples studied by Nugent and Steinberg [4,5] are characterized by random element of relations between objects. The new better results are obtained from a series of random initial placements. Consequently the potentialities of “scanning area” method allow to scan effectively large area of neighbouring local optima. It can be supposed that a set of random initial solutions is enough to obtain sufficiently satisfying local optimum.

For certain problems, having element of organization of relations between objects (i.e. “planar”) it is worthwhile to find an initial solution which to be optimized later on.

Appendix

Best solutions

Test example Nugent 20 [4]
Dimension: 20, $D_{ij} = |x_i - x_j| + |y_i - y_j|$
Best published value: 1287 [3]
New value: 1285

6	1	7	5	17
13	8	20	15	19
16	11	12	2	4
9	3	10	14	18

Test example Nugent 30 [4]
Dimension: 30, $D_{ij} = |x_i - x_j| + |y_i - y_j|$
Best published value: 3079 [3]
New values: 3064,3067,3068,3071,3073,3076,3077
Best new value: 3062

21	2	13	6	12	5
28	29	9	10	24	26
25	19	7	8	1	17
4	30	16	11	22	23
20	14	3	27	18	15

Test example Steinberg [5]
Dimension: 36, $D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$
Best published value: 4124.97 [3]
New value: 4119.74 [2]

24	25	26	27	11	6	5	3	
22	21	23	14	12	13	4	8	2
33	34	32	19	20	7	10	18	17
	31	30	29	28	15	1	9	16

Test example Steinberg [5]
Dimension: 36, $D_{ij} = |x_i - x_j| + |y_i - y_j|$
Best published value: 4802 [3]
New value: 4799

24	25	26	27	11	6	5	3	
22	21	23	14	12	13	4	8	2
33	34	32	19	20	7	1	10	18
	31	30	29	28	15	9	16	17

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