Computer Science Journal of Moldova, vol.**4**, no.2(11), 1996

Parallel methods in problems of mathematical physics

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Abstract

The article deals with various methods of parallelization of algorithms of problems of mathematical physics. Parallel methods of solution of these problems on the basis of multiprocessor transputer based systems with distributed memory are considered.

The use of multiprocessor computer systems is one of the most perspective directions of development of computer facilities. Such systems are distinguished by high speed of information processing, high reliability, high productivity cost ratio. Besides, if for usual. processors of sequential action there exists the limit of computer performance of physical character, namely, it is the finitness of speed of propagation of an electromagnetic signal, the productivity of multiprocessor system can be increased by addition of new processors.

In reality, everything is a little bit more difficult. The increase of computation power of multiprocessor system is achieved only by an effectual parallelization of existing algorithms and programs or by construction of new parallel methods of solution. We shall consider parallel algorithms and methods for solution of equations of mathematical physics.

Equations of mathematical physics describe a plenty of physical, mechanical, biological and other processes. As a rule, for solution of these problems it is necessary to involve the computer of very high power. Equations of mathematical physics - are partial differential equations, and to solve them it is necessary to use numerical methods

on rather fine-structured grids. To estimate the volume of calculations, necessary to solve a particular problem we shall consider typical three-dimensional computation area, consisting of $100*100*100$ points [1], number of operations at one point 1000, number of iterative steps 1000. If it is necessary to solve the problem in a hour, the necessary computing capacity makes 300 Mflops. Total volume of memory, if it is necessary to store 10 real variables for one point of the differential scheme, makes 100 Mb. Until the recent times such computing capacities had only supercomputers of a CRAY type. The use of multiprocessor systems, consisting for example of 300 transputer IMST800 or of 30 modules TTM200 (consisting of T805 transputer + INTEL860), provides approximately the same productivity.

The efficiency of multiprocessor system can be determined as follows: $E = T_1/(pT_p)$. Here T_1 is the time of the problem solution at a single processor, t_p is the time of the problem solution using p processors. Acceleration factor can be determined by the following formula:

$$S = p * E.$$

The most essential factor, which determines the performance of multiprocessor system, is the choice of numerical algorithm and its adaptation for a particular system. For extra computation intensive problems the methods, which permit to obtain a linear growth of throughput with increase of a number of processors, are of the great interest. We shall limit ourselves by consideration of multiprocessor systems with distributed memory (MIMD - machines).

1 Basic methods of parallelization

As practice shows, practically important scientific and engineering problems described by equations of mathematical physics have, as a rule, their internal parallelism. Let us adduce the classification of decomposition levels of the original problem into hierarchically subordinated parts [2,3].

The first level of breaking up (decomposition) is the division of the original problem into some subtasks, which differ one of another by

initial data. To this level of decomposition, for example, the problem of multiphase gas dynamics pertains. If there is no heat transfer between phases, each phase can be calculated separately.

The second level of decomposition is a geometrical parallelism. In this case the computation area is broken into subareas, each of which is calculated by its own processor. It is necessary to organize exchange of the information between processors.

The third level of decomposition is the parallel execution of separate computation blocks, for example, calculation of Laplas transformations, matrix inversions, etc.

The fourth level of decomposition is the parallel execution of various stages of calcukation of arithmetic expressions.

In the problems of mathematical physics the second level of decomposition, namely, geometrical parallelism is the most natural and convenient one. Using this method it is necessary to fulfil a set of requirements to the used algorithm. It will much simplify organization of calculations in MIMD systems. These requirements can be formulated as follows [4]:

a. During calculations it is desirable, that the data exchange should occur between adjacent points of the differential grid. Therefore use of the explicit differential schemes are preferential.

b. It is better to break up the differential area into simple convex subareas. Thus the regularity of exchange between adjacent processors is achieved and the amount of the transmitted information is reduced.

c. It is necessary to choose such differential schemes, at which the ratio of the volume of calculations to the volume of the transmitted information is a certain value.

d. It is desirable to use differential schemes with predetermined amount of executed operations. The more predictable calculation time is thus achieved, and, accordingly, the more balanced load of a processor is obtained.

Therefore in the best way and with highest efficiency the uniform schemes of the comprehensive calculation respond to parallelization [5,6].

The important aspect of creation of effective parallel algorithms is the choice of topology of a processors network. A degree of connection of processors influences upon the two characteristics of algorithms: efficiency and degree of complexity of programming. The efficiency is influenced by the amount of communication lines between processors, through which the information is transmitted. The complexity of coding depends on how close the processors are one to another in topological sense. Thus, if the processor "n" has no direct connection with processor "n + 1", it is necessary to create buffer tasks for reception and retranslation of data at intermediate processors. As practice shows [3,7–10] the most easy to program are problems for linear networks; topology of a "lattice" type has additional efficiency; because while using transputers it is possible to perform calculations and to execute transfer of the information simultaneously.

2 Construction of parallel algorithms for solution of problems of mathematical physics

Ones of the most computation intensive problems, moreover, requesting a very long time of calculation, are problems of mechanics of continuous media. To this number problems of gas current calculations with allowance for influences of viscosity, problems of heat transfer, problems of radiation effects, multiphase problems, etc. pertain. Not less computation-power consumming are the problems of mechanics of a deformated solid body, which permit to calculate thermoelastoviskoplastic current of media in conditions of high-speed load. For this reason the large part of constructed algorithms pertains to the listed problems [1,2,4,7,9,10–14].

In work [2] construction of parallelization algorithm of problems of two- and three- dimensional aerodynamics with allowance for effects of viscosity using multiprocessor computing systems with distributed

memory is adduced. As a base known [15] kinetic-matched differential schemes (KMDS) were taken. The construction of such schemes is based on the use of models for particle distribution function or on the differencial approximation of Bolzman equation for one-particle distribution function. The obtained schemes are the comprehensive calculation schemes, which satisfy the conditions (a-d), mentioned above.

Constructed differential scheme was mapped onto the two-dimensional torus of the size $m*m$, constructed on the base of T800 transputer. The computational area was broken into rectangular subareas of minimal perimeter. Each subarea was calculated at a separate processor. Using a five-dot pattern the exchanges occur only between adjacent transputers. By use of a nine-dot pattern it is necessary to perform exchanges between transputers, located on a diagonal one of another and the information exchange is carried out in two stages.

Estimates of constructed parallelizing algorithm show, that it can effectively be used at multiprocessor computing systems, and the actual throughput is close to the total throughput of all processors.

In work [7] the flow of radiating gas is simulated. Such problems present increased requirements to computer facilities. Therefore parallel algorithm for use on a multiprocessor system with distributed memory was created. For parallelization the method of geometrical decomposition was used. The additional complicating factor was the necessity of calculation of a spectral energy flux. The complexity consists of that there it is necessary to use the values obtained in the whole computation area, i.e. in the large volume of information, transmitted through the transputer network.

It was marked, that the choice of parameters of differential scheme for multigroup diffusion equations is more rationally performed with application of iterative methods [6]. As an example the solution of an indicated problem on linear transputer chain, consisting of even number of processors was studied, and overlapping of computation areas of the adjacent processors was carried out. The indicated process is generalized and for a two-dimensional lattice of processors with even number of nodes in each direction. The calculations showed the high efficiency of the given algorithm, the losses for parallelization have not

exceeded 20

The similar problems of construction of algorithms of solution oftwo-dimensional problems of radiating gas dynamics (RGD) were discussed in work [11]. The solution of such problems is broken into several stages: solution of equations of gas dynamics (GD-block), solution of group equations of radiation diffusion (R - blocks), joint solution of an energy equation and of an averaged equation of radiation diffusion (T-block). Taking into account the radiation much complicates the solution of the problem, because the system of radiation equations, unlike other equations is three-dimensional owing to an additional dependence of radiating parameters on frequency ν.

The calculation of parameters of radiation fields is connected with the necessity of use of iterative methods for solution of difference equations. In given work, at construction of parallelization algorithm was used the fact, that with an multigroup approach the calculation of parameters of a radiation field can be broken into N weakly depended two-dimensional problems, the solution of each of which is possible to perform at a separate processor. Weak dependence consists in the fact that the information exchange is carried out only at the beginning and at the end of calculation on each temporal layer. Such parallelization method the authors have named parallelization by groups. The similar parallelization method for solution of one-dimentional problems of RGD was used in work [14]. Except parallelization by groups there was used also the principle of geometrical decomposition.

Let us examine in more detail this parallel algorithm. Let the rectangular area be broken into chain consisting of M subareas, where M is even. The multiprocessor system consists of $(N+1)*M$ of processors, connected as a lattice.

In a vertical direction of the lattice of processors functional parallelization of algorithm of solution of a problem and parallelization by groups are carried out t. On each temporal layer the solution of gas dynamics equations in the topmost horizontal line of the computing system and simultaneously the solution of all N group equations of radiation diffusion in its N lower lines are performed. After completion of these stages and calculation of averaged parameters the joint solu-

tion of an equation of radiation diffusion and an energy equation in the upper line of the computing system is performed.

In a horizontal direction of a processor lattice the geometrical parallelization ofthe algorithm of solution of the problem by one coordinate axis is carried out. It is thus possible to make a conclusion, that two-dimensional problems of RGD can be successfully solved using multiprocessor computing systems.

In work [4,9,10] a parallel algorithm for solution of two-dimensional problems of multiphase gas dynamics is constructed. The currents of multiphase media are of great represent theoretical and practical interest. In comparison with usual gas dynamical currents, the things are complicated by existence of additional phases, which can react with bearing gas flow and between each other. The original consecutive algorithm is based on a modified method of large particles (MMLP). As it is known, the algorithm MMLP consists of three stages: first, Eulerian stage, second, Lagrangian stage and third, final stage. The differential scheme is constructed using a 5-dot pattern that facilitates its parallelization.

Practically the algorithm is implemented for modelling of gas currents with polydispersive particles. Such problems are especially important for study of movement of dust-laden gas in gas-cleaning installations of complex geometry. A method of geometrical decomposition of area into subareas with minimal perimeter was applied. Each of such subareas was mapped on its own processor. As already was mentioned above, for explicit differential schemes the amount of exchanges between processors makes an insignificant part of time (10–15 %) in comparison with the time of calculation of one node of the scheme. The things are complicated by that for completion of one time step it is necessary to perform information exchange at each of indicated above stages. Thus the amount of exchanges between processors grows 3 times greater and comes nearer to 50 % of time of calculation of the node.

Besides that after each k steps it is necessary to receive some integrated characteristics of each node of differential scheme. These data are necessary for calculation the size of a time step, determined by Ku-

rant's condition of stability, and also for determination of discrepancies of energy, weight, etc. Thus a nontrivial problem of minimization of transfer pats trong the transputer network appears. It is necessary to transmit the information from each processor to a single allocated processor and there the calculation of necessary values must be carried out. In addition, it is not known beforehand, which processor to make allocated, so that to minimize time of information transfer. After finding of necessary values it is necessary to send them to all processors of system.

If the interaction between phases is absent, there appears the opportunity to carry out parallelization by phases. If there is sufficient number of processors it is possible to calculate each of phases + gas at its own processor lattice. The number of such two-dimentional lattices should coincide with the number of various phases of a firm fraction.

Existence of inter-phase heat and mass exchange results in necessity to organize information exchange between "layers" of such lattices. The constructed parallelization algorithm (in the absence of heat and mass exchange permits to obtain almost linear growth of performance of multiprocessor system.

To the construction of parallel algorithm of a two-dimensional problem of high-speed deformation of elastoplastic media work [4] is devoted. In it an impact of micrometeorite on a plating of a space vehicle is simulated. The substances of the striker and the target undergo significant deformations, therefore during calculations the part of cells is excluded from computation area, but there new computation areas appear, which it is necessary to enter into the computation zone. A method of geometrical decomposition of computation area into subareas was used, the form of this subareas depends on the initial form of the stuker and the target.

In work [13] the possibility of application of multiprocessor transputer systems for solution of a spatial two-dimensional problem of petroleum production process optimization was studied. However its solution, as well as the solution of other problems of mathematical physics using highly parallel computing systems, clash with serious difficulties, stipulated by the architecture of systems with distributed

memory. Currents of a liquid in the field of an oil deposit of a flat configuration are investigated.

It is necessary to place on the plan of a deposit some oil-producing wells, and also wells, injecting the water into the pool, so that with maximal efficiency expel the petroleum from the pool.

For calculations rectangular grids were used. In spite of the fact that the explicit differential schemes are well suited to process by multiprocessor systems with distributed memory, the nonlinear character of equations, underlying the accepted model claims additional iterations for determination of the solution at each time step. When performing the iterations their number depends on the set of conditions and can greatly vary for various subareas. When distributed processing of the information, the iteration at the part of processors can converge yet, while on others not yet. In such systems it is not clear, who should make a decision about termination of iterations, on the basis of which information, how this information gets in the processor and, finally, how processors of a the network can get know that the desigion is made.

In the work an decentralized algorithm of distribution of the global information and making of the decision is offered. For this purpose each processor in the network must be able to perform the following:

- To analyze the available information;
- In due time to inform a network about its own condition;
- Independently make the decision about termination of iterations.

The estimation of efficiency of offered algorithm (2) in comparison with algorithm transferring of the information to the single allocated transputer (1) gives:

$$T_1 = K(2RT_l + T_c), T_2 = (K + D)T_c,$$

Where T_l is the time, necessary for transmission of an indicator of the completion of iterations between adjacent processors, T_c is the time, necessary to calculate one iteration at a single transputer, K is the

number of iterations, necessary for obtaining the solution with given accuracy, R is the radius of the graph of the transputer network, D is the diameter of this graph, T_1, T_2 is the time, necessary for making the decision at the next temporae layer using the first method, and using decentralized algorithm accordingly. The offered approach is effective when $T_2 < T_1$, that it is true when $K > \frac{D}{2R}\frac{T_c}{T_l}$ from here follows, that if $T_l T_c$, then it is possible to assert, that it is true when $K > D/2R * L$, where L is the number of points, mapped onto one processor. From this follows, that the use of a decentralized algorithm is preferable.

As already was mentioned above, explicit differential schemes of problems of mathematical physics respond well to parallelization. Several works [16–18] are devoted to parallelization of separate stages of such algorithms. In them questions of parallelization of algorithms for matrix sweeping, matrix conversion, solution of some problems of linear algebra, etc. are studied.

The solution of ordinary differential equations is of great practical interest for modelling of various processes in mechanic, physics, etc. In work [19–20] algorithms parallelizing Runge-Kutta method for multiprocessor transputer systems are resulted.

When solving the indicated above problems of mathematical physics there arise the number of additional problems, concerned with balansing the load of multiprocessor system. In work [21–24] conditions of loading of a unit in a multiprocessor system are studied under preliminary definedf conditions of task arrival to the unit. In work [21] algorithms of dynamic loading with use of transition probabilities in $M|M|1$ using theory of quenes are investigated. (the notation of a kind $\alpha|\beta|\gamma$ was introduced by Kendall for brief description of service system. When $\alpha = M$ it is necessary, that the input flow is a Poisson one, $\beta = M$ means that the service is distributed exponentially, $\beta = G$ means that the service has an arbitrary distribution, $\gamma = 1$ means, that the system has a single processor consecutively processing the arriving requests.

In work [21] and in some others it is supposed, that the requests in the input flow are assumed homogeneous. In work [24] an attempt is made to take into account heterogeneity (priority character) of the

sequest flow. This approach is more natural and permits to describe more adequately real processes, taking place in the transputer network.

In conclusion it is necessary to note, that creation of the parallel versions of the application programs requires highly competent labour of experts, therefore the created packages of the applcation programs for multiprocessor complexes can exceed the cost of hardware of the complex. Besides this, not depending on just which types of processors will be formerly used for creation of parallel systems with distributed memory, computing algorithms and programs, developed for transputer systems, will not lose their actuality.

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