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Numerical implementation of mechanical quadratures method for solving singular integral equations given on closed smooth contours

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Abstract

In the paper we give numerical implementation of computational schemes for the method of mechanical quadratures for solving singular integral equations, given on the closed smooth contours.
The posibility of graphic construction of the 3-dimensional domain for the initial equation is provided, as well as the graphic interpretation of results.
This program contains the dialogue with the users.

The purpose of the paper is the numerical implementation of mechanical quadratures method for solving singular integral equations (SIE), given on closed smooth contours of integration. The theoretical foundation of these numerical schemes is given in [1,2]. But the problem of numerical examination and of implementation of such computing schemes is not solved. In this work we propose the algorithms of numerical implementation of two computing schemes of mechanical quadratures method for solving the complete SIE, given on arbitrary closed smooth contours Γ bounding an arbitrary simply connected region containing the point $t = 0$.

The numerical procedure GRAF.PAS gives the posibility for grafical representation of three-dimentsional region of initial SIE. With the help of computing procedure ARANJARE we can change the point of observation of the volume figure as necessary, which is very convenient for research.

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The dialogue with the user with necessary commentary is organized, all rezults are prezented in grafical form .

1 The computing scheme of mechanical quadratures method

In the Banach space of Hölder functions $H_{\beta}(\Gamma)(0<\beta<1)$ consider a complete SIE of the form

$$(A\varphi \equiv)a(t)(P\varphi)(t)+b(t)(Q\varphi)(t)+\frac{1}{2\pi i}\int_{\Gamma}K(t,\tau)\varphi(\tau)d\tau=f(t), t\in\Gamma, \quad (1)$$

where $a(t),b(t),f(t)\in H_{\alpha}(\Gamma)(0<\beta<\alpha\leq 1)$, $K(t,\tau)\in H_{\alpha,\alpha}(\Gamma),\varphi(t)$ is an unknown function, $P=\frac{1}{2}(I+S),Q=I-P,I$ is the identity operator and S is the operator of singular integration in the meaning of Cauchy principal value.

The approximate solution of SIE (1) we seek for the form

$$\varphi_n^{(r)}(t)=\sum_{k=-n}^{n}\alpha_k^{(r)}\cdot t^k, \quad (2)$$

where the constants $\{\alpha_k^{(r)}\}_{k=-n}^{n}$ $(r=0,1)$ are defined by mechanical quadratures method from the following system of linear algebric equations (SLAE):

$$a(t_j)\sum_{k=0}^{n}\alpha_k^{(r)}\cdot t_j^k+b(t_j)\sum_{k=-n}^{-1}\alpha_k^{(r)}\cdot t_j^k+$$

$$+\sum_{k=-n}^{n}\alpha_k^{(r)}\sum_{m=0}^{2n}K(t_j,t_m)t_m^{1+r(k-1)}\Lambda_{-k+r(k-1)}^{(m)}=f(t_j), \quad (3)$$

$j=\overline{0,2n}$; $\Lambda_{-k+r(k-1)}^{(m)}$ are the quadratures coefficients determined from the relations

$$\prod_{\substack{p=0;\\ p\neq m}}^{2n}\frac{(t-t_p)}{(t_m-t_p)}\left(\frac{t_m}{t}\right)^n=\sum_{k=-n}^{n}\Lambda_k^{(m)}t^k.$$

The theoretical foundation of the computing scheme (3) in Hölder spaces for arbitrary closed smooth contours ($r = 0$) is obtained in [2, .76] and for arbitrary closed Lyapunov contours ($r = 1$) in [1].

The conditions of convergence for computing scheme (3) for the case of arbitrary closed smooth contour Γ are given below.

Theorem 1 *Let the following condition be fulfilled:*

1) $a(t) \cdot b(t) \neq 0 (t \in \Gamma)$;

2) $ind[a(t) \cdot b^{-1}(t)] = 0 (t \in \Gamma)$;

3) $dim Ker A = 0$;

4) $t_j = \psi(w_j)$, $w_j = exp(\frac{2\pi i}{2n+1}(j-n))$, $j = \overline{0, 2n}, i^2 = -1$, $\psi(w) = c \cdot w + c_0 + \frac{c_1}{w} + \frac{c_2}{w^2} + \cdots$ *is the function which implements the conform mapping of* $|w| \geq 1$ *on the exterior of* Γ *such that* $\psi(\infty) = \infty$, $\psi'(\infty) = c > 0$.

Then the SLAE (3) has a unique solution $\{\alpha_k^{(r)}\}_{k=-n}^{n}, r = 0, 1$ *for suficiently large* $n (n \geq n_0)$. *The approximate solutions* $\varphi_n^{(r)}(t)$ *(2) converge as* $n \to \infty$ *in the norm of space* $H_\beta(\Gamma)$ *to the exact solution* $\varphi(t)$ *of SLAE (1) for any function* $f(t) \in H_\alpha(\Gamma)$.

For the rate of convergence the following conditions hold:

$$\| \varphi - \varphi_n^{(r)} \|_\beta = O\left(\frac{ln^2 n}{n^{\sigma(\alpha) - \beta + r[\delta - \sigma(\alpha) + \beta]}} \right) H(f; \sigma(\alpha)),$$

where $\sigma(\alpha) = \begin{cases} \alpha & for \;\; \alpha < 1 \\ 1 - \varepsilon & for \;\; \alpha = 1 \end{cases}$; $\delta = min(\beta; \alpha - \beta)$; $H(f, \sigma(\alpha))$ *is the smallest constant with which the function* $f(t)$ *satisfies the Hölder conditions with exponent* σ *on contour* Γ.

2 The numerical implementation of computing schemes

The numerical algoritm of mechanical quadratures method is composed by the following logical scheme:

Step 1. The procedures of complex aritmetics: **sc** is addition, **dc** is subtraction, **pc** is multiplication,**cc** is division, **cexp** is exponential function, **putere** is raising to power.

Step 2. The initial data: procedures **fcont, fc, fd, fk, ff** corespond to the description of the functions $\psi(w), c(t), d(t), k(t,\tau), f(t)$.
The introduction of ε, the numerical precision of the condition SLAE or the conditions 1) and 2) of the theorem.
The introduction of $n = \overline{1,N}$; $2n+1 = \overline{3, 2N+1}$, the numbers of points on contour Γ with help of which the coefficients $\alpha_k^{(r)}$ of the approximate solution $\varphi_n^{(r)}(t) = \sum_{k=-n}^{n} \alpha_k^{(r)} t^k$ are calculated.
$N \leq 8$ for the PS IBM with procesor INTEL 386.
The introduction of nt, the number of calculational points on contour Γ.

Step 3. The modelling of initial volume region (the procedure **Constr**) by its sections . These sections represent the region bounded by arbitrary closed smooth contours that are situated on parallel planes on arbitrary distance.The quantity of these sections ns must be fixed by user.

If the sectors are similar,parallel and equidistant, then the user must introduce the parametres of the first and the last contours and the step(in two directions) of displacement between them. If the sectors are arbitrary parallel, then the user must introduce the parametres and the step of displacement for every section.

Note that the checking of the constructed figure can be done before the beginning of calculations.

Step 4. The choice of the point of observation with reference to the constructed figure is implemented by means of the procedure **Aranjare**.

Step 5. The control of the conditions of solving for initial SLAE on every section $\Gamma_p (1 \leq p \leq ns)$ i.e.:

1) $a(t)b(t) \neq 0 (t \in \Gamma_p)$ with precision ε using the procedure **CalcProdAB**;
2) $ind[a(t)b^{-1}(t)] = 0 (t \in \Gamma_p)$ with precision ε using the procedure **CalcInd**.

Step 6. The calculation of quadratures nodes on contour $\Gamma_p (1 \leq p \leq ns)$ using the procedure **CalcNod** with help of the relations $t_j = \psi(w_j),\ \ j = \overline{0, 2n+1}$, where $\psi(w_j) = \psi(exp(\frac{2\pi i}{2n+1}(j-n)))$. Here the function $\psi(w) \equiv cw + c_0 + \frac{c_1}{w} + \frac{c_2}{w^2} + \cdots$ gives the equation of contour. We use the ellipse for the test example:

$$\psi(w) = \frac{1}{2}[(R_1 + R_2)w + \frac{(R_1 - R_2)}{w}],$$

$$R_1, R_2 \in Z^+ - the\ \ radiuses\ \ of\ \ the\ \ ellipse.$$

Step 7. The calculation of the elements of quadratures coefficients of the SLAE (3) (the procedure **CalcLambda**): $\Lambda_{-1}^{(m)}, m = \overline{0, 2n}$ for $r = 1$ and $\Lambda_{-k}^{(m)}, k = \overline{-n, n}, m = \overline{0, 2n}$ for $r = 0$.

Step 8. The calculation of the complex matrix C of dimention $(2n+1) \times (2n+1)$ (the procedure **ForMatComp**) by relations:

$$c_{j,n+k}^{(r)} = b(t_j) \cdot t_j^k + \sum_{m=0}^{2n} K(t_j, t_m) \cdot t_m^{1+r(k-1)} \Lambda_{-k+r(k-1)}^{(m)}, k = \overline{-n, -1},$$

$$c_{j,n+k}^{(r)} = a(t_j) \cdot t_j^k + \sum_{m=0}^{2n} K(t_j, t_m) \cdot t_m^{1+r(k-1)} \Lambda_{-k+r(k-1)}^{(m)}, k = \overline{0, n},$$

$j = \overline{0, 2n}, r = 0, 1.$

Step 9. The calculation of the right of the SLAE, that is the complex vector of dimension $(2n+1)$ (procedure **ff**) by formula $f_j = f(t_j), j = \overline{0,2n}$. We obtain the SLAE $C \cdot \vec{\alpha} = \vec{f}$ as a rezult.

Step 10. The solution of SLAE. We implement the transition to the matrix $S(i,k)$ and vector of right parts $fp(i)(i,k = \overline{1,4n+2})$ in the real aritmetics. This transition permits to use the large number of standart programs for solving SLAE. The procedure **ForMatReal** uses the following correlations of transition:

$$c^{(r)}_{j,m} = a^{(r)}_{j,m} + i \cdot b^{(r)}_{j,m}, \quad \alpha^{(r)}_j = x^{(r)}_j + i \cdot y^{(r)}_j, \quad f_j = f_{1j} + i \cdot f_{2j},$$

$$j,m = \overline{0,2n}, \quad r = 0,1.$$

As a rezult we obtain SLAE $S \cdot \vec{z} = \vec{fp}$, where

$$S = \begin{pmatrix} \{a^{(r)}_{j,m}\}^{2n}_{j,m=0} & \{-b^{(r)}_{j,m}\}^{2n}_{j,m=0} \\ \{b^{(r)}_{j,m}\}^{2n}_{j,m=0} & \{a^{(r)}_{j,m}\}^{2n}_{j,m=0} \end{pmatrix}, \quad \vec{z} = \begin{pmatrix} \{x^{(r)}_j\}^{2n}_{j=0} \\ \{y^{(r)}_j\}^{2n}_{j=0} \end{pmatrix},$$

$$\vec{fp} = \begin{pmatrix} \{f_{1j}\}^{2n}_{j=0} \\ \{f_{2j}\}^{2n}_{j=0} \end{pmatrix}.$$

Step 11. The calculation of the approximate solution $\varphi^{(r)}_n(t) = \sum\limits_{k=-n}^{n} \alpha^{(r)}_k t^k, r = 0,1$ using the Horner scheme by means of the procedure **Horner** .

Step 12. Numerical and grafical representation of rezults of the initial volume region and on its sectors $\Gamma_p, p = \overline{1,ns}$.

3 The description of main procedures

The index of the function $G(t), t \in \Gamma$ is calculated in the procedure **CalcInd** by formula [3, c.104]

$$ind[G(t)] \cong \frac{1}{2\pi} \sum_{i=1}^{q} \frac{\xi_i \cdot \eta_{i-1} - \xi_{i-1} \cdot \eta_i}{\sqrt{\xi^2_{i-1} + \eta^2_{i-1}} \cdot \sqrt{\xi^2_i + \eta^2_i}},$$

where ξ_i *and* η_i are the real and imaginary parts of the function $G(t)$ in points $t = t_i, i = \overline{1,q}$. The precision ε is obtained in following manner: at first we calculate $Indice1 = ind[G(t)]$ for $q = 8$, then $Indice2 = ind[G(t)]$ for $q = 16$. If $|Indice1 - Indice2| \leq \varepsilon$, then $ind[G(t)] = Indice2$. Otherwise we double the numbers of points on contour Γ and repeat the calculation. The user must determine the maximum number of points on contour Γ during the initial dialogue .

The quadratures coefficients $\Lambda_{-1}^{(m)}, m = \overline{0,2n}$ for $r = 1$ and $\Lambda_{-k}^{(m)}$, $k = \overline{-n,n}$, $m = \overline{0,2n}$ for $r = 0$ are calculated by procedure **CalcLambda** .These coefficients are determined from the fundamental Lagrange polinomials $l_m(t), m = \overline{0,2n}$, constructed by the system of nodes $\{t_m\}_{m=0}^{2n}$:

$$l_m(t) = \frac{(t-t_0)\cdots(t-t_{m-1})(t-t_{m+1})\cdots(t-t_{2n})}{(t_m-t_0)\cdots(t_m-t_{m-1})(t_m-t_{m+1})\cdots(t_m-t_{2n})} \cdot \left(\frac{t_m}{t}\right)^n =$$

$$= \frac{1}{(t_m-t_0)\cdots(t_m-t_{m-1})(t_m-t_{m+1})\cdots(t_m-t_{2n})} \cdot \left(\frac{t_m}{t}\right)^n \times$$

$$\times[t^{2n} + \gamma_1^{(m)} t^{2n-1} + \gamma_2^{(m)} t^{2n-2} + \cdots + \gamma_{n+1}^{(m)} t^{n-1} + \cdots + \gamma_{2n}^{(m)}] =$$

$$= \frac{t_m^n}{(t_m-t_0)\cdots(t_m-t_{m-1})(t_m-t_{m+1})\cdots(t_m-t_{2n})} \times$$

$$\times[\gamma_0^{(m)} t^n + \gamma_1^{(m)} t^{n-1} + \cdots + \gamma_{n+1}^{(m)} t^{-1} + \cdots + \gamma_{2n}^{(m)} t^{-n}] = \sum_{k=-n}^{n} \Lambda_k^{(m)} t^k. \quad (4)$$

Where

$$\begin{cases} 1 = & \gamma_0^{(m)}, \\ t_0 + t_1 + \cdots + t_{m-1} + t_{m+1} + \cdots + t_{2n} = & -\gamma_1^{(m)}, \\ t_0t_1 + t_0t_2 + \cdots + t_{m-1}t_{m+1} + \cdots + t_{2n-1}t_{2n} = & \gamma_2^{(m)}, \\ \cdots & \cdots \\ t_0t_1 \cdots t_n + t_0 \cdots t_{n-1}t_{n+1} + \cdots + t_n \cdots t_{2n} = & (-1)^{n+1}\gamma_{n+1}^{(m)}, \\ \cdots & \cdots \\ t_0t_1 \cdots t_{n+k-1} + t_0 \cdots t_{n+k-2}t_{n+k} + \cdots & \\ +t_{n-k+1} \cdots t_{2n} = & (-1)^{n+k}\gamma_{n+k}^{(m)}, \\ \cdots & \cdots \\ t_0t_1 \cdots t_{m-1}t_{m+1} \cdots t_{2n} = & \gamma_{2n}^{(m)}. \end{cases} \tag{5}$$

Note, that in (5) the node t_m is excluded.

So we obtain coefficients $\gamma_{n+1}^{(m)}$ for r=1 and $\gamma_{n+k}^{(m)}$, $k = \overline{-n,n}$, $m = \overline{0,2n}$ for r=0. The quadratures coefficients $\Lambda_{-k}^{(m)}$, $k = \overline{-n,n}$, $m = \overline{0,2n}$ and $\Lambda_{-1}^{(m)}$ for $k = 1$ are calculated from (4) as follows:

$$\begin{aligned} &\Lambda_{-k}^{(0)} = \frac{t_0^n}{(t_0 - t_1)\cdots(t_0 - t_m)\cdots(t_0 - t_{2n})} \cdot \gamma_{n+k}^{(0)}, \\ &\cdots \\ &\Lambda_{-k}^{(m)} = \frac{t_m^n}{(t_m - t_0)\cdots(t_m - t_{m-1})(t_m - t_{m+1})\cdots(t_m - t_{2n})} \cdot \gamma_{n+k}^{(m)}, \\ &\cdots \\ &\Lambda_{-k}^{(2n)} = \frac{t_{2n}^n}{(t_{2n} - t_0)\cdots(t_{2n} - t_m)\cdots(t_{2n} - t_{2n-1})} \cdot \gamma_{n+k}^{(2n)}. \end{aligned} \tag{6}$$

As a rezult we obtain the matrix of quadratures coefficients $\{\Lambda_{-k}^{(m)}\}$, $k = \overline{-n,n}, m = \overline{0,2n}$ $(r = 0)$ of the form:

$$\begin{bmatrix} \Lambda_{-n}^{(0)}\cdots & \Lambda_{-1}^{(0)}\cdots & \Lambda_{n}^{(0)} \\ \cdots\cdots & \cdots\cdots & \cdots \\ \Lambda_{-n}^{(m)}\cdots & \Lambda_{-1}^{(m)}\cdots & \Lambda_{n}^{(m)} \\ \cdots\cdots & \cdots\cdots & \cdots \\ \Lambda_{-n}^{(2n)}\cdots & \Lambda_{-1}^{(2n)}\cdots & \Lambda_{n}^{(2n)} \end{bmatrix}$$

The n-column of this matrix corresponds to case $r = 1$ in SLAE (3).

The coefficients $\gamma_{n+k}^{(m)}$ are calculated by algoritm that generates itself the necessary quantity of enclosed $n+k$ cycles. We use the procedure **REC** with the scheme :

```
for l[0]:=0 to n-k do
   for l[1]:=l[0]+1 to n-k+1 do
      for l[2]:=l[1]+1 to n-k+2 do
         ⋱
            for l[i]:=l[i-1]+1 to n-k+i do
               ⋱
                  for l[n+k-1]:=l[n+k-2]+1 to 2n-1 do.
```

The calculation of coefficients $\gamma_{n+1}^{(m)}, m = \overline{0, 2n}$ is the same with help of procedure **REC** for $k = 1$.

The procedure **Horner** gives the possibility to calculate the approximate solution $\varphi_n^{(r)}(t)$ by formula:

$$\varphi_n^{(r)}(t) = \sum_{k=-n}^{n} \alpha_k^{(r)} \cdot t^k = \sum_{k=-n}^{-1} \alpha_k^{(r)} \cdot t^k + \sum_{k=0}^{n} \alpha_k^{(r)} \cdot t^k, \quad r = 0, 1,$$

where

$$\sum_{k=0}^{n} \alpha_k^{(r)} \cdot t^k = ((\cdots(\alpha_n^{(r)} \cdot t + \alpha_{n-1}^{(r)}) \cdot t + \alpha_{n-2}^{(r)}) \cdot t + \cdots + \alpha_1^{(r)}) \cdot t + \alpha_0^{(r)},$$

$$u_n = \alpha_n^{(r)}, u_{n-1} = u_n \cdot t + \alpha_{n-1}^{(r)}, \cdots u_0 = u_1 \cdot t + \alpha_0^{(r)};$$

$$\sum_{k=-n}^{-1} \alpha_k^{(r)} \cdot t^k = (((\cdots(\frac{\alpha_{-n}^{(r)}}{t} + \alpha_{-n+1}^{(r)})\frac{1}{t} + \alpha_{-n+2}^{(r)})\frac{1}{t} + \cdots + \alpha_{-2}^{(r)})\frac{1}{t} + \alpha_{-1}^{(r)})\frac{1}{t},$$

$$v_n = \alpha_{-n}^{(r)}, v_{n-1} = \frac{v_n}{t} + \alpha_{-n+1}^{(r)}, \cdots v_1 = \frac{v_2}{t} + \alpha_{-1}^{(r)}.$$

Hence $\varphi_n^{(r)}(t) = u_0 + v_1$.

The complete description of all procedures of algorithm and the test calculations are implemented in the program **EisCuadr.Pas** in **PASCAL**.The program is presented in the Republican Fund of Algorithms and Programs.

4 Rezults

The calculations were tested for the following initial data:

$$t = \psi(w) = \frac{1}{2}[(R_1+R_2)\cdot w + \frac{(R_1 - R_2)}{w}], R_1 = 3.75 \div 2.8, R_2 = 1.2 \div 1.0,$$

$$a(t) = t^2 + 8\cdot t + 7, \quad b(t) = 6\cdot t + 6,$$

$$K(t,\tau) = \frac{2}{\tau^2} + \frac{3\cdot t+1}{\tau} - \frac{6}{t} + \tau + \tau^2 \cdot t,$$

$$f(t) = t^4 + 16\cdot t^3 + 75\cdot t^2 + 100\cdot t + 75 + \frac{54}{t} + \frac{54}{t^2}.$$

The exact solution of the test example is the function

$$\varphi(t) = t^2 + 8\cdot t + 4 + \frac{3}{t} + \frac{9}{t^2}.$$

The computing rezults (the case $r = 1$ in the SLAE (3)):

The nodes $t_j, j = \overline{0, 2n}$ on contour $\Gamma_p, p = \overline{1, ns}$ and the coefficients of approximate solution $\alpha_k, k = \overline{-n, n}$, for $n = 4, ns = 8, p = 7$.

$t_j, j = \overline{0, 2n}$	$\alpha_k, k = \overline{-n, n}$
$t_0 = -2.758 - i \cdot 0.351$	$\alpha_{-4} = 2.4 \cdot 10^{-7} - i \cdot 1.2 \cdot 10^{-7}$
$t_1 = -1.467 - i \cdot 0.890$	$\alpha_{-3} = -1.3 \cdot 10^{-7} - i \cdot 4.1 \cdot 10^{-8}$
$t_2 = 0.509 - i \cdot 1.012$	$\alpha_{-2} = 9.0000000 - i \cdot 2.4 \cdot 10^{-9}$
$t_3 = 2.248 - i \cdot 0.661$	$\alpha_{-1} = 2.9999999 - i \cdot 3.5 \cdot 10^{-8}$
$t_4 = 2.935 + i \cdot 0.000$	$\alpha_0 = 4.0000001 + i \cdot 2.4 \cdot 10^{-8}$
$t_5 = 2.248 + i \cdot 0.661$	$\alpha_1 = 7.9999999 - i \cdot 9.7 \cdot 10^{-10}$
$t_6 = 0.509 + i \cdot 1.012$	$\alpha_2 = 1.0000001 - i \cdot 6.9 \cdot 10^{-10}$
$t_7 = -1.467 + i \cdot 0.890$	$\alpha_3 = -7.4 \cdot 10^{-11} - i \cdot 1.8 \cdot 10^{-10}$
$t_8 = -2.758 + i \cdot 0.351$	$\alpha_4 = 4.0 \cdot 10^{-11} + i \cdot 6.2 \cdot 10^{-11}$

Table 1.

The calculated points $t_s, s = \overline{0, nt}$, approximate solution $\varphi_n(t_s)$ and the error of approximate solution $\Delta_n(t_s)$, for $n = 4, nt = 6$:
$t_0 = -2.64 - i \cdot 0.4$; $t_1 = -0.65 - i \cdot 1.0$; $t_2 = 1.83 - i \cdot 0.8$;
$t_3 = 2.93 + i \cdot 0.0$; $t_4 = 1.83 + i \cdot 0.8$; $t_5 = -0.65 + i \cdot 1.0$;
$t_6 = -2.64 + i \cdot 0.4$.

$\varphi_n(t_s)$	$\Delta_n(t_s) = \lvert\varphi(t_s) - \varphi_n(t_s)\rvert$
$\varphi_n(t_0) = -10.2 - i \cdot 1.43$	$\Delta_n(t_0) = 6.2 \cdot 10^{-8} + i \cdot 3.6 \cdot 10^{-8}$
$\varphi_n(t_1) = -5.71 - i \cdot 10.3$	$\Delta_n(t_1) = -8.5 \cdot 10^{-8} + i \cdot 8.6 \cdot 10^{-8}$
$\varphi_n(t_2) = 24.2 - i \cdot 7.11$	$\Delta_n(t_2) = 2.7 \cdot 10^{-8} + i \cdot 1.8 \cdot 10^{-8}$
$\varphi_n(t_3) = 38.2 + i \cdot 0.00$	$\Delta_n(t_3) = 9.9 \cdot 10^{-9} + i \cdot 4.2 \cdot 10^{-10}$
$\varphi_n(t_4) = 24.2 + i \cdot 7.11$	$\Delta_n(t_4) = -6.6 \cdot 10^{-9} - i \cdot 1.2 \cdot 10^{-8}$
$\varphi_n(t_5) = -5.71 + i \cdot 10.3$	$\Delta_n(t_5) = -2.3 \cdot 10^{-7} + i \cdot 3.4 \cdot 10^{-8}$
$\varphi_n(t_6) = -10.3 + i \cdot 1.43$	$\Delta_n(t_6) = 6.4 \cdot 10^{-8} + i \cdot 4.5 \cdot 10^{-8}$

Table 2.

Grafical rezults:

Initial volume region.

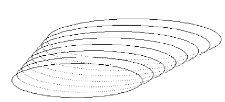
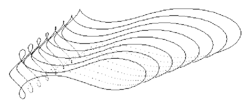


Figure 1.



ıtions on volume region.

Figure 2.

The section $\Gamma_p, p = 7$ of the volume region.

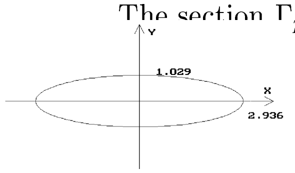


Figure 3.

The solution $\varphi_n(t)$ on $\Gamma_p, p = 7$.

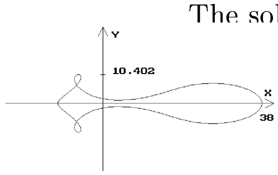


Figure 4.

The index of the function $a(t)b^{-1}(t)$ on $\Gamma_p, p = 7$.

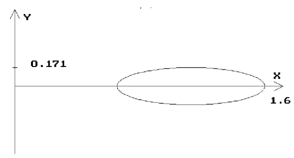


Figure 5.

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