The structural robustness of multiprocessor computing system

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Abstract

The model of the multiprocessor computing system on the base of transputers which permits to resolve the question of valuation of a structural robustness (viability, survivability) is described.

Introduction

We consider the model of the multiprocessor computing system on the base of transputers (CT) which permits to resolve the question of valuation of a structural robustness (viability, survivability) of CT [1, 2].

For research we chose a class of managing computing systems. The examples of such systems are in [3, 4]. This class is simulated by a communication network. As a main component of CT for this class its fragment FCT is considered. FCT is a microstructure connecting the output into external environment ((I/OP — input/output point) with data processing unit (DPU) or one entrance point (I/OP1) in the microstructure with other entrance point (I/OP2). The difference of FCT from other components of CT is insignificant. All significant points (DPU, I/OP or DHP — data handling points) in FCT or in another structures and microstructures we shall designate as DP (data point) if it is insignificant what point is it.

At simulation FCT by a communication network, the transputer is considered as a complex element, consisting of four simple communication elements, possessing one common point, in which concentrates

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a computing element (DPU). An idea of a rank is suggested. The rank is a quantitative characteristic, determining the minimum quantity of faults of simple elements, causing the fault of a communication network between first DP and second DP.

We also use a structural probability as a measure of keeping the rank of FCT at the first single fault. By using these two characteristics we suppose a measure for evaluating a structural robustness of multiprocessor systems.

The examples of some typical microstructures and ways of account of the structural robustness for them are submitted.

1 A rank of elements and structure elements

For research we have chosen a class of managing computing systems. This class is simulated by a communication network.

All elements are considered only as communication elements. The elements can be simple or compound. At failure a simple element fails as a whole, so all communications provided to it are broken off. In the compound element one of the simple elements fails only. Other elements continue to act.

The state of an element of communication network is defined by its rank. The rank shows, how many of failures should take place before the element would be left out of operation completely and its rank would become equal to zero. The simple communication elements have ranks equal to one, the rank of the compound one is determined by a quantity of the simple elements containing in it.

We consider the following compound elements:

- kind A — the element is made of independent simple elements. Each simple element executes all functions of communication. Failure of one simple element lowers the rank of the compound one and its productivity, but fulfillment of functions does not infringe. Only after the rank of the element becomes equal to zero the element terminates the fulfillment functions;

- kind B — the element is made of dependent simple elements;
Each simple element executes a part of functions of communication. Failure of a simple element lowers the rank of the compound one and infringes fulfillment functions of its. In an elementary case all entering elements are identical, therefore the reduction of quantity of connected elements occurs proportionally to reduction of quantity of serviceable simple elements;

- kind C — the combination of the kinds A and B. Failure of the one simple element can lower the whole rank of the element C and can lower the rank of one of entering in C the elements B. The compound element has complex rank, which consists of the rank of the kind A (the whole part) and the rank of the kind B (the fractional part).

We introduce the following designations:

- path (P) — a chain of consistently connected communication elements, connecting one of the outputs with the particular item of data processing (I/O with DP);

- path element (PE) — one simple communication element, separate or entering in the compound communication element;

- path branching (PBr) — a point, where more than two path elements are connecting. Really in each separate time moment the connection passes only through two connected to this point elements, and other elements do not participate in the implementation of the communication at this moment. They can be used in the other time moment. The path branching is formed by several simple elements of one compound communication element;

- path site (PS) — some consecutive path elements between two adjacent path branchings. Two external path elements of a path site are the simple elements of the compound communication elements;

- path branch (PB) — one or some consecutive path sites between two conjugated path branchings. The path branchings are conjugated, if they limit two or more the longest chosen branches;
• path crossing (PC) — branching, which unites a few of paths;

• path section (PSc) — a few consistently connected path elements between two adjacent path crossings.

The rank of a path site is equal to quantity of parallel path sites between given branchings. It determines influence of failures in elements on implementability of communication between path branchings. The rank of a part of a path between conjugated path branchings is equal to quantity of path branches of maximum length between these branchings. At “correct” structure of whole path it is possible to decompose it into parts without the rest. Each of them is limited by conjugated path branchings. The rank of a path will be equal to the minimum rank among all ranks of parts of a path. At a “wrong” structure it is trouble to choice conjugated path branchings.

To decompose a path into parts in this case, it is necessary to reject the “interfering” parallel sites of a path.

All parts of a path, which have a rank equal to one, are critical. At the first failure in these parts the communication on the given path is infringed. The element failure on noncritical part of a path does not lower the rank and the path is not destroyed.

The rank of a path sections between conjugated path crossings is equal to quantity of path sections of maximum length. The connections, which are path sections of the maximum length for chosen path crossings, are simultaneously path branches for each path and conjugated path crossings are simultaneously conjugated path branchings. For determination of the rank the total set of paths between a data processing unit and an output is divided on the parts by allocation of conjugated crossings. For a “correct” structure the rank is equal to the minimum rank of all its parts.

For a “wrong” structure the rank is determined similarly, but decomposition is made on an adjusted structure.

At consideration of structural robustness we suppose the reliability of all simple elements identical. The probability of failure of a simple communication element does not depend on its place in the network or component. We do not consider failures of transputer as a computing
element. Thus the robustness counted is the robustness of a switching network.

We consider the transputer as an element of a kind B, which consists of four simple elements of communication. They have one common point. The transputer as a computing element (DPU) is concentrated into this point. We suppose that simple elements of transputer are independent. The failure of the first simple element of communication does not infringe work of other three ones, the second one of another two ones etc.

On Fig.1 an example of a fragment of a structure is shown. We see 3 independent paths from an output (I/OP) to data processing unit (DPU): right (four simple elements), left-hand (six simple elements) and central (six simple elements). To allocate a central part of a fragment, it is necessary to separate the left-hand and the right-hand path. It is necessary to throw away 3 of the compound elements (two at the left and one from the right). A smaller fragment from eight components with one entrance on the part of an I/OP and two entrances on the part of DPU will be formed.

Five components of this minifragment have one connection with elements of right-hand and left-hand paths. The appropriate simple elements of these connections “have dropped out” of an allocated fragment. The components, which include these simple elements, are transformed from fourvalent to threevalent. The valency at such allocation of a fragment can be reduced up to two. The compound communication component which has a data processing unit can have valency equal to 2.

The application of the method of allocation of microstructures by separating of “another’s” connections and “another’s” simple elements of connection enables in details to investigate the structural robustness of information systems on the basis of models of fragments.

We use a fragment on Fig. 2 for the analysis of a structure. The blocks of a structure 01, 02, 03, 04, 11, 12, 13, 14, 21, 22, 23, 24, 31, 41, 42, 51, 52, 61, 71, 81, 82, 91, 92 represent path sites without points of branching. Each PS is determined by length. Length is equal to quantity of elements entering in it. On Fig.3A the simple elements
FIG. 1. The extensive equivalent circuit of connection of the output with the item of data processing for the three-level filled hierarchical structure.
FIG. 2. The equivalent structure of the symmetric graph in the convolution image.
A. Extensive image.

B. Convolution image

FIG.3. The equivalent structure consisting of one main and one additional chains — $S(1/1)$.
on path sites are represented obviously. Their quantity is indicated above or under an appropriate site by the latin letter with indexes. On Fig. 3B these sites are represented conditionally. We suppose the probability of failure for all elements identical. Path sites on Fig. 2 are designated only by indexes. There are allocated three main paths:
- path (01) - (02) - (03) - (04);
- path (11) - (12) - (13) - (14);
- path (21) - (22) - (23) - (24).

We suppose the first path is the main one. The chain of elements, implementing this path is also main. There are three parallel branches:
- branch (02) - (03);
- branch (31);
- branch (41) - (42).

As a main branch we accept the first branch. If the branch is a uniform PS, it be determined by summary length. Summary length is equal to a sum of lengths of sites entering in a branch. The similar approach is fair for other paths.

On an example of a fragment on Fig. 2 we determine characteristic microstructures, existing in the structure of a fragment:

1) We consider the chain (01) - (02) - (03) - (04) and one branch (31).

We obtain a microstructure on Fig.3 (microstructure on Fig.3 and 4, variants B and C). In this microstructure the chain (01) - (02) - (03) - (04) is main, the chain (31) is additional;

2) We choose two paths: a path (01) - (02) - (03) - (04) and a path (21) - (81) - (82) - (24). Between average points of these paths there are two connections: (91) and (92). Thus, there are two chains, main and additional, implementing two paths, and connection between average points of these chains (microstructure on Fig. 5A);

3) As a main chain we choose a path (01) - (41) - (42) - (04), as an additional chains we choose (11) - (51) and (02) - (92) - (82)
A. The main image.

B. The equivalent image.
C. Variants of the structure after the first failure.

Variant 1

\[ n_1 + n_{03} \]

\[ n_01 + n_2 \]

Variant 2

\[ n_1 + n_{02} \]

\[ n_{01} \]

\[ n_2 \]

Variant 3

\[ n_{02} + n_{03} \]

\[ n_{01} \]

\[ n_2 \]

Variant 4

\[ n_{01} + n_{02} \]

\[ n_1 \]

\[ n_{03} \]

Variant 5

\[ n_{02} + n_2 \]

\[ n_1 \]

\[ n_{03} \]

FIG. 4. The elementary structure consisting of one main and two overlapping additional chains — \( S(1/2) \).
A. The structure $S(1/3)$ after the first failure for variants 1,2,3,4 in the equivalent image.

B. The structure $S(1/3)$ after the first failure for variants 5,6,7,8 in the equivalent image.

C. The structure $S(1/3)$ after the second failure for subvariants 51, 61, 71, 81 in the equivalent image.

FIG.5. Variants of the structure $S(1/3)$ after first and second failures.
A. The main image.

B. The equivalent image — variant 1.

B. The equivalent image — variant 2.

FIG.6. The elementary structure consisting of one main and three overlapping additional chains — $S(1/3)$. 
The structural robustness ...

- (24). A microstructure of the main chain and two overlapping ones represented on Fig. 4A is formed.

To these microstructures we can add another one:

4) main and three overlapping additional chains (microstructure on Fig. 6A).

From the point of view of robustness the construction of the information system model includes the two stages:

1) construction of the model structure;

2) calculation of the robustness for the constructed structure of model.

For model structure construction the following algorithm can be used:

1. The classification of the purposes, tasks, works of the information system.

2. The construction of the minimal variants of subsystems. For each separate purpose sufficient quantity of data processing unit are chosen and connections with an output to external environment are organized. Characteristic functional points (data handling points — DHP) are compared to elements of a structure.

3. Formation of the maximum variants of subsystems. Required level of reservation is provided.

4. Fragments of a communication network for all kinds of DHP are allocated in the variants of structures.

5. Allocation in the structures of fragments of standard microstructures is done.

As a result of fulfillment of this algorithm, the structure of information-computing system will be transformed from the form, constructed on the basis of elements, into the form, constructed on the basis of
microstructures. If microstructures to replace by blocks, it is possible to try to allocate microstructures of the second level. The consecutive transition to structures of more and more high level is process of assembly of a structure.

The process of assembly of a structure is shown on Fig.7. On Fig.7A a fragment includes path sites, consisting from simple communication elements. The path sites of a path are represented in the form of squares. One can see three microstructures: a microstructure $S(1/3)$ (main chain with three overlapping ones) and two microstructures $S(1/2)$ (main chain with two overlapping additional chains). As a result of fulfillment of the first stage of assembly a structure on Fig.7B will be formed. Designations of assembled microstructures contain one stroke. It indicates the first stage of process.

The obtained structure contains the two parallel chains of microstructures of the first order. It can be considered as a standard microstructure of the kind: the main chain and the additional one. As a result of its assembly a structure on Fig.7C will be formed. In the designation two strokes (second stage) are contained.

This way it is possible to build any structure. The difficulties are connected with availability of “superfluous” connections. They need to be broken off. It can be made after the careful analysis of importance of connections for valuation of robustness of an information system.

After assembly a structure is ready for fulfillment of robustness calculation procedures.

We expand the concept of the rank to a structure. We delete functional aspects from definition of a fragment and preserve the structural one. A minifragment is a structure of any complexity, connecting two objects. All external connections of the minifragment are united into three groups: breaking off, connected with the first object (the first point — first entrance) and connected with the second object (the second point — the second entrance). So there are two entrances in the given structure. Quantity of connections of the first entrance may differ from another one.

Let us determine the rank of minifragment, including two consecutive components, be equal a rank of an element, which has lesser rank.
A. The extensive image.

B. The structure of an assembled image — the first level.

C. The structure of an assembled image — the second level.

FIG. 7. A simple structure combined of elementary structures.
The rank of the minifragment built of two components, connected in parallel, is equal to a sum of ranks of these elements. The rank of a chain of elements is equal to the rank of an element, which has the minimal rank. The rank of a minifragment, consisting of several chains, connected in parallel is equal to a sum of ranks of these chains. The rank of a minifragment, made of a sequence of some ministructures, is equal to the rank of a ministructure, which has the minimal rank. The rank of a minifragment, consisting of some microstructures, connected in parallel is equal to the sum of ranks of these microstructures.

2 A structural probability and calculation of structural robustness

We determine on a set of communication elements, forming structure of minifragment, failure of one of elements as an event. Total ensemble of events is a failure of all elements of communication of the minifragment. We determine as a structural probability the probability of failure of one of path sites in the minifragment structure. It equals to the ratio of the summary failure probability of all elements of a chosen PS to the summary probability of failure of all elements of the minifragment.

The calculated structural probability at the same time specifies failure probability of some PS, as a part of a structure, and preservation probability of another part of a structure. It can be used for quantitative valuation of preservation probability of the rank of the considered structure. The procedure of calculation of the preservation probability of the rank of a structure of a minifragment is described in the following algorithm:

1. Allocation minifragments of the structures.

2. Partition of the minifragments into standard microstructures.

3. Allocation in each microstructure of paths and determination the rank of the microstructure.
4. Allocation in the microstructure of all those path sites, which do not lower the rank of the structure.

5. Count the preservation probability and the rank for a microstructure.

Availability of the two concepts, the rank and the structural probability, enables us to construct a quantitative expression for calculation of the structural robustness of the minifragment (microstructure):

$$J_{F_k} (N, S, f^{in1}, f^{in2}, R, P^{sr}) = (r + P^{sr}),$$

where $F_k$ is a chosen minifragment (microstructure), $N$ is a composition of elements of the minifragment, $S$ — structure of the minifragment, $f^{in1}$ is the first point of an entrance into the minifragment, $f^{in2}$ is the second point of an entrance into the minifragment, $R$ is a set of ranks of path branches (path sections), $P^{sr}$ is a structural preservation probability of rank of the minifragment, $r$ is a rank of the minifragment.

As a result of calculation under this formula a real number is obtained, the whole part of which indicates the rank of a minifragment (microstructure), and the fractional one the preservation probability for this rank at the first single failure.

Each next failure occurs in new conditions: as a result of the previous failure the PS, in which the failure has been occurred, is deleted from a structure. All characteristics of a minifragment, except for its designation and the points of entrance, can change.

3 Examples

Earlier the following microstructures were considered:

1) consecutive microstructure of elements or microstructures (microstructure $S(1/0)$ — for elements);

2) parallel microstructure of chains (microstructures) — microstructure $S(k/0)$ — for chains of elements ($k$ — quantity of chains).
If these microstructures are made of elements, the preservation probability of rank for them is equal to zero. Quantitative valuation of robustness of such microstructures is equal to their rank. The example of the second microstructure with rank which equals two, is represented on Fig. 4C, Variant 1.

There may be allocated very many microstructures. Below as an example account of structural probability for three of microstructures is considered:

3) microstructure consisting of one main and one additional chains (microstructure $S(1/1)$ — for elements);

4) microstructure consisting of one main and two additional overlapping chains (microstructure $S(1/2)$ — for elements);

5) microstructure consisting of one main and three additional overlapping chains (microstructure $S(1/3)$ — for elements).

### 3.1 A microstructure consisting of one main and one additional chains

In the most general case the microstructure is represented on Fig.3. The preservation probability of a rank is equal to

$$P^{sr} = (n_{02} + n_1)/(n_{01} + n_{02} + n_{03} + n_1).$$

The probability of reduction of a rank is equal to

$$P^{pr} = (n_{01} + n_{03})/(n_{01} + n_{02} + n_{03} + n_1).$$

We accept for example, that every site have the length, equal to two: $n_{01} = n_{02} = n_{03} = n_1 = 2$, then $P^{sr} = 0.5$, and $J = 1.5$ for the first failure. After the first failure the structure is transformed into a chain (microstructure 1), the robustness of which is equal to 1. The second failure destroys the microstructure.
3.2 A microstructure consisting of one main and two overlapping additional chains

We consider a structure including two additional chains with overlapping (Fig. 4A). We transform the structure to a form shown at Fig. 4B. Two total paths, connected one with another by two intermediate connection are formed. The rank of a path is equal to two. The single failure cannot break off the connection. At the first failure the probability of destruction of a microstructure is equal to zero. The preservation probability for the rank of a path is equal to 
\[ P^{pr} = \frac{n_{01}}{n_{01} + n_{02} + n_{03} + n_{1} + n_{2}} \], and the probability of reduction of the rank is equal 
\[ P^{pr} = \frac{n_{01} + n_{03} + n_{1} + n_{2}}{n_{01} + n_{02} + n_{03} + n_{1} + n_{2}} \].

As a result of the first failure the structure would change and would be reduced to the two total paths not possessing intermediate connections (Fig. 4C, variant 1 is a variant of preservation of the rank, at which in the structure there are paths \((n_{1} - n_{2})\) and \((n_{1} - n_{03})\), or to one path, including the main and the additional chain (variants 2,3,4,5 are the variants of reduction of rank).

Thus, after the first failure the microstructure \(S(1/2)\) is transformed into microstructure \(S(2/0)\), or into microstructure \(S(1/1)\). The second failure is considered now for these structures. The third single failure of an element causes destruction of the path in all cases.

3.3 A microstructure consisting of one main and three overlapping additional chains

We consider a structure containing three additional chains with overlapping (Fig. 6A).

In the case of three additional overlapping chains the two equivalent structures (Fig. 6B and 6C) can be constructed:

1) total path \((m_{1} - m_{04} - m_{05})\), total path \((m_{01} - m_{02} - m_{3})\) and intermediate connections \(m_{2}\) and \(m_{03}\);

2) total path \((m_{1} - m_{03} - m_{3})\), total path \((m_{01} - m_{2} - m_{05})\) and intermediate connections \(m_{02}\) and \(m_{04}\).
The rank of the path is equal to two. The analysis of equivalent structures shows that there are four variants of preservation of rank and four variants of reduction of rank for a path at the first single failure:

1) Site \((m_2)\) is destroyed. The rank of the path is preserved. The probability of the given variant is equal to

\[
P_1 = \frac{m_2}{(m_0 + m_{02} + m_{03} + m_{04} + m_{05} + m_1 + m_2 + m_3)}.
\]

As a result we have the following structure: a total path \((m_1 - m_{04} - m_{05})\), a total path \((m_{01} - m_{02} - m_3)\) and an intermediate connection \(m_{03}\);

2) Site \((m_{02})\) is destroyed. The rank of the path is preserved. The probability of the given variant is equal to

\[
P_2 = \frac{m_{02}}{(m_0 + m_{02} + m_{03} + m_{04} + m_{05} + m_1 + m_2 + m_3)}.
\]

As a result we have the following structure: a total path \((m_1 - m_{03} - m_3)\), a total path \((m_{01} - m_{02} - m_{05})\) and an intermediate connection \(m_{04}\);

3) Site \((m_{03})\) is destroyed. The rank of the path is preserved. The probability of the given variant is equal to

\[
P_3 = \frac{m_{03}}{(m_0 + m_{02} + m_{03} + m_{04} + m_{05} + m_1 + m_2 + m_3)};
\]

As a result we have the following structure: a total path \((m_1 - m_{04} - m_{05})\), a total path \((m_{01} - m_{02} - m_3)\) and an intermediate connection \(m_2\);

4) Site \((m_{04})\) is destroyed. The rank of the path is preserved. The probability of the given variant is equal to

\[
P_4 = \frac{m_{04}}{(m_0 + m_{02} + m_{03} + m_{04} + m_{05} + m_1 + m_2 + m_3)};
\]

As a result we have the following structure: a total path \((m_1 - m_{03} - m_3)\), a total path \((m_{01} - m_2 - m_{05})\) and an intermediate connection \(m_{02}\);

232
5) Site \((m_1)\) is destroyed. The rank of the path is lowered. The probability of the given variant is equal to

\[ P_5 = \frac{(m_1)}{(m_{01} + m_{02} + m_{03} + m_{04} + m_{05} + m_1 + m_2 + m_3)}; \]

The result is the following structure: a total path \((m_{01} - m_{02} - m_3)\) and an additional chain \((m_2 - m_{05})\), duplicating site \((m_{02} - m_3)\), with an intermediate connection between the central point of a chain and the central point of a site \((m_{04} - m_{03})\);

6) Site \((m_{01})\) is destroyed. The rank of the path is lowered. The probability of the given variant is equal to

\[ P_6 = \frac{(m_{01})}{(m_{01} + m_{02} + m_{03} + m_{04} + m_{05} + m_1 + m_2 + m_3)p}; \]

As a result we have the following structure: a total path \((m_1 - m_{04} - m_{05})\) and an additional chain \((m_{03} - m_3)\), duplicating site \((m_{04} - m_{05})\), with an intermediate connection between the central point of a chain and the central point of a site \((m_{02} - m_2)\);

7) Site \((m_3)\) is destroyed. The rank of the path is lowered. The probability of the given variant is equal to

\[ P_7 = \frac{(m_3)}{(m_{01} + m_{02} + m_{03} + m_{04} + m_{05} + m_1 + m_2 + m_3)}; \]

The result is the following structure: a total path \((m_1 - m_{04} - m_{05})\) and an additional chain \((m_{01} - m_2)\), duplicating site \((m_{01} - m_2)\), with an intermediate connection between the central point of a chain and the central point of a site \((m_{02} - m_{03})\);

8) Site \((m_{05})\) is destroyed. The rank of the path is lowered. The probability of the given variant is equal to

\[ P_8 = \frac{(m_{05})}{(m_{01} + m_{02} + m_{03} + m_{04} + m_{05} + m_1 + m_2 + m_3)}. \]

The result is the following structure: a total path \((m_1 - m_{03} - m_3)\) and an additional chain \((m_2 - m_{02})\), duplicating site \((m_{01} - m_{02})\), with an intermediate connection between the central point of a chain and the central point of a site \((m_2 - m_{04})\).
Thus, after the first failure in the four first variants the rank of a microstructure is preserved and the resulting equivalent microstructure represented on Fig. 6A will be formed which shows the microstructure $S(1/2)$. The second and third failures can be represented by application of a procedure of decomposition for the indicated microstructure by means of substitution as length of sites the values $n_{01}$, $n_{02}$, $n_{03}$, $n_1$, $n_2$ of appropriate expressions, describing lengths of sites in each of four listed variants:

1) **Variant 1**:  
   \[ n_{01} = m_1; \]
   \[ n_{02} = m_{03}; \]
   \[ n_{03} = m_3; \]
   \[ n_1 = m_{01} + m_{02}; \]
   \[ n_2 = m_{04} + m_5; \]

2) **Variant 2**:  
   \[ n_{01} = m_{01} + m_2; \]
   \[ n_{02} = m_4; \]
   \[ n_{03} = m_{03} + m_3; \]
   \[ n_1 = m_4; \]
   \[ n_2 = m_{05}; \]

3) **Variant 3**:  
   \[ n_{01} = m_{01}; \]
   \[ n_{02} = m_2; \]
   \[ n_{03} = m_{05}; \]
   \[ n_1 = m_1 + m_{04}; \]
   \[ n_2 = m_{02} + m_3; \]

4) **Variant 4**:  
   \[ n_{01} = m_{01}; \]
   \[ n_{02} = m_{02}; \]
   \[ n_{03} = m_3; \]
   \[ n_1 = m_{1} + m_{03}; \]
   \[ n_2 = m_2 + m_{05}. \]

The summary probability of preservation of the rank after the first single failure of a communication element is equal to $P^{sr} = (m_{02} + m_{03} + m_4 + m_2)/(m_{01} + m_{02} + m_{03} + m_4 + m_5 + m_1 + m_2 + m_3)$;

The probability of reduction of the rank is equal to $P^{pr} = (n_{01} + m_{05} + m_1 + m_3)/(m_{01} + m_{02} + m_{03} + m_4 + m_5 + m_1 + m_2 + m_3)$.

We consider general case of a structure for variants 5,6,7,8 (Fig. 5B). We designate it as $S(1/1/1)$. We introduce the following designations. The structure consists of six sites:

- $n_{01}$, $n_{02}$, $n_{03}$ — main path; $n_1$, $n_2$ — additional chain, overlapping the sites $n_{02}$, $n_{03}$; $n_{001}$ — additional connection between the middle of a additional chain and a point of connection of overlapped sites.

At the first single failure the probability of preservation of the path rank is $P^{sr} = (n_{02} + n_{03} + n_1 + n_2 + n_{001})/(n_{01} + n_{02} + n_{03} + n_1 + n_2 + n_{001})$, the probability of reduction of the rank is $P^{pr} = (n_{01})/(n_{01} + n_{02} +$
There are the following variants of changes in a structure:

1. Variant 1 — site \( (n_{001}) \) is destroyed. The resulting structure is: the main chain \( (n_{01}) - (n_{02} + n_{03}) \); an additional chain \( (n_1 + n_2) \). The probability of the given variant is equal to \( P_1 = (n_{001})/(n_{01} + n_{02} + n_{03} + n_1 + n_2 + n_{001}) \);

2. Variant 2 — site \( (n_1) \) is destroyed. The resulting structure is: the main chain \( (n_{01} + n_{02}) - (n_{03}) \); an additional chain \( (n_{001} + n_2) \). The probability of the given variant is equal to \( P_2 = (n_1)/(n_{01} + n_{02} + n_{03} + n_1 + n_2 + n_{001}) \);

3. Variant 3 — site \( (n_2) \) is destroyed. The resulting structure is: the main chain \( (n_{01} + n_{02}) - (n_{03}) \); an additional chain \( (n_1 + n_{001}) \). The probability of the given variant is equal to \( P_3 = (n_2)/(n_{01} + n_{02} + n_{03} + n_1 + n_2 + n_{001}) \);

4. Variant 4 — site \( (n_{02}) \) is destroyed. The resulting structure is: the main chain \( (n_{01} + n_{02}) - (n_2) \); an additional chain \( (n_{001} + n_{03}) \). The probability of the given variant is equal to \( P_4 = (n_{02})/(n_{01} + n_{02} + n_{03} + n_1 + n_2 + n_{001}) \);

5. Variant 5 — site \( (n_{03}) \) is destroyed. The resulting structure is: the main chain \( (n_{01} + n_{03}) - (n_2) \); an additional chain \( (n_{02} + n_{001}) \). The probability of the given variant is equal to \( P_5 = (n_{03})/(n_{01} + n_{02} + n_{03} + n_1 + n_2 + n_{001}) \);

6. Variant 6 — site \( (n_{01}) \) and all path in whole are destroyed. The probability of the given variant is equal to \( P_6 = (n_{01})/(n_{01} + n_{02} + n_{03} + n_1 + n_2 + n_{001}) \).

Calculation of particular valuations of probability of preservation of the rank is executed by substitution of valuations:
1) **Variant 5**:  
\[ n_{01} = m_{01}; \]
\[ n_{02} = m_{02}; \]
\[ n_{03} = m_{03}; \]
\[ n_1 = m_2; \]
\[ n_2 = m_05; \]
\[ n_{001} = m_{04} + m_{03}; \]

2) **Variant 6**:  
\[ n_{01} = m_{01}; \]
\[ n_{02} = m_{03}; \]
\[ n_{03} = m_{03}; \]
\[ n_1 = m_04; \]
\[ n_2 = m_{05}; \]
\[ n_{001} = m_2 + m_{02}; \]

3) **Variant 7**:  
\[ n_{01} = m_{05}; \]
\[ n_{02} = m_{02}; \]
\[ n_{03} = m_{01}; \]
\[ n_1 = m_{04}; \]
\[ n_2 = m_1; \]
\[ n_{001} = m_{02} + m_{03}; \]

4) **Variant 8**:  
\[ n_{01} = m_{03}; \]
\[ n_{02} = m_{02}; \]
\[ n_{03} = m_{01}; \]
\[ n_1 = m_{03}; \]
\[ n_2 = m_1; \]
\[ n_{001} = m_{04} + m_2; \]

After the first failure of an element in the structure \( S(1/1/1) \) in the five variants the structure represented on Fig. 5C will be formed, — a microstructure \( S(1/1) \), the calculations for which are made using procedure considered earlier.

**References**


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