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Construction of a Parallel Algorithm to Solve the Multiphase Gas Dynamics Problem

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Abstract

This paper considers questions of an effective use of multiprocessor computing system to implement a parallel algorithm solving the multiphase gas dynamics problem. A technique is offered to parallelize the two-dimensional explicit differential scheme to implement it on multiprocessor systems with distributed memory (MIMD architecture).

MIMD is the architecture used in transputer systems. It has its advantages and shortcomings. As its advantages one should classify its relative cheapness (concerning the productivity/cost ratio), its compactness and reliability, as well as the possibility to adjust the topology of processor arrays to a specific application [1,2]. However the architecture of such systems requires serious revision of existing algorithms and software. In many cases there arises a need to develop new approaches to construction of numerical methods solving the multiphase gas dynamic taking into account structure of multiprocessor systems.

Let us estimate the number of processor units necessary to solve a specific problem described below. Let the computational area for a two-dimensional (2D) problem contains 100*100 points and the number of floating point operations per a point makes about 1000. Let number of iterations is about 1000, and number of phases is 100. If we want to solve such a problem in an hour then necessary computing power should make about 30 Mflops. If at each computational point of the area we should be store about 50 real numbers with double accuracy, then the necessary volume of memory is about 40 Mbt. The corresponding

computational capacities could be provided by a processor array of about 30 transputers of T800 type or by three of TTM 200.

To find the estimation of multiprocessor system efficiency one should know the productivity of one processor unit in the system and the character of growth of system productivity when the number of processor units in it grows. As a base effectivity factor the characteristics of transputer are used shown with optimized test programs which work utilizing only internal memory area of transputer. Access time to the operative memory makes 3...6 cycles of processor, so the repeated access to the memory reduces the productivity. Taking into account characteristic properties of command set and of processor architecture lets us to increase several times the productivity of each processor unit.

The efficiency of multiprocessor system can be determined as the following ratio:

$$E = T_1/pT_p,$$

where T_1 is solution time of the problem using one processor unit, and T_p - solution time of the problem using p processors [3]. The acceleration factor is determined as: $S = p * E$.

In numerical simulation not only the peak productivity of the system which is achieved on the carefully optimized examples is important but the real one which can be achieved on a specific class of problems. Analysis of effective productivity on specific application problems is an independent goal difficult to achieve.

The most essential factor which determines the real power of multiprocessor system is choice of algorithm and the technique of its implementation using the system you dispose. For computation - consuming problems in the area of mathematical simulation, the algorithms which let us to obtain linear growth of productivity of system when the number of processor units in it increases are of special interest. Numerical methods of solution of problems in mathematical physics, and specifically in gas dynamics, posses, as a rule, its internal parallelism. However the character and the degree of its parallelisation depends on the stage of problem solution when the parallelisation is made. These

stage correspond to levels of repeated decomposition of a problem into more and more small hierarchically subordinated parts [4,5].

The solution of the number of important applied problems in the field of ecology requires to use the methods of mathematical experiment, and subsequently to resort to the numerical experiment using computers. Research of currents in multiphase dispersion flows of gas containing solid particles is a very urgent problem of mechanics of continuous media. Especially acute it is at gas cleaning installations for thermal power stations, at cement and ferro-concrete factories etc. Increase of degree of purification of gas flow results in essential improvement of ecological conditions. Let us consider statement of a problem, main equations and method of solution. The flow of polydispersion gas through an installation of complex shape is simulated.

For the simulation usual assumptions of mechanics of continuous media are used: each phase is considered to be ideal, viscosity and heat conductivity of phases are taken into account only for inter-phase friction and heat and mass transfer processes; collisions of solid particles, their splitting or coagulation shall be neglected [6]. Let as also limit to the situations where one can assume the particles and the distances between them very small in comparison with characteristic size scale of the flow.

We shall represent the part of volume of the suspension occupied by phase i by the value of its volumetric contents $\alpha_i \quad (i = 1, 2)$. To each point the relative phase densities ρ_i, characterizing their masses in volume unit of the suspension, and the real densities, characterizing the densities of substances they consist ρ_i^u, will be put in correspondence.

$\rho_1 = \alpha_1 \rho_1^u$, $\rho_2 = \alpha_2 \rho_2^u$.

The volumetric content of the suspended phase of particles will be determined by the volume of an individual particle θ and the number of particles in the volume unit of the suspension, which can be called the numerical density of the suspension $\alpha_2 = \theta n$. We shall study the movement of the suspension media using the two-phase example (with two speed values and two temperature values), i.e. let us assume that all particles in the solid phase are identical.

Note that in this case the number of differential equations to be

solved is two times more than in the case of usual one phase gas. Therefore the questions of an optimal construction of the program implementing the parallel algorithm of their solution become especially important.

Proceeding from aforesaid, the system of differential equational describing unsteady spatial currents of gas suspension could be written down in the following form:

$$\frac{\partial \rho_1}{\partial t} + \nabla \rho_1 \mathbf{W}_1 = 0, \quad \frac{\partial \rho_2}{\partial t} + \nabla \rho_2 \mathbf{W}_2 = 0,$$

$$\rho_1 \frac{d_1 \mathbf{W}_1}{dt} = -\alpha_1 \nabla p - n\mathbf{f}, \quad \rho_2 \frac{d_2 \mathbf{W}_2}{dt} = n\mathbf{f} - \alpha_2 \nabla p,$$

$$\frac{\partial}{\partial t}(\rho_1 E_1 + \rho_2 E_2) + \nabla(\rho_1 E_1 \mathbf{W}_1 + \rho_2 E_2 \mathbf{W}_2) + \nabla p(\alpha_1 \mathbf{W}_1 + \alpha_2 \mathbf{W}_2) = 0,$$

$$\frac{\partial \rho_2 J_2}{\partial t} + \nabla(\rho_2 J_2 \mathbf{W}_2) = ng, \tag{1}$$

$$(E_i = J_i + \mathbf{W}_i^2/2).$$

Here $\mathbf{W}_i, J_i, E_i$ are the speed, the specific internal and the specific full energy of phase i, p is the pressure in the suspension, $\mathbf{f}$ is the total force effecting upon a separate particle from of the gas, g is the intensity of heat inflow to the surface of a separate particle. Through ∇ and $\frac{d_i}{dt}$ the symbolical operators are designated:

$$\nabla \equiv \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k},$$

$$\frac{d_i}{dt} \equiv \frac{\partial}{\partial t} + (\mathbf{W}\nabla)$$

To close the system of differential equations (1) one should exactly specify the laws of phase interaction and formulate the equations of state.

$$P = P(\rho_1^u, T_1), \quad J_1 = J_1(\rho_1^u, T_1), \quad \rho_2^u = const, \quad J_2 = J_2(T_2), \tag{2}$$

Here $T_i \quad (i = 1, 2)$ - is the temperature of phase i - of the suspension.

To choose the physical model we shall follow works [6,7]. Specifying the laws of phase interaction we shall presume that suspended particles are spheres of diameter δ.

Assume, besides that, the intensities of thermal and force interaction of a particle with the gas do not depend on presence of another particles at its nearest vicinity. Then to determine the intensity of heat exchange one can use usual formulae:

$$g = \pi\delta^2\beta(T_1 - T_2) = \pi\delta\lambda_1 Nu((T_1 - T_2), \quad Nu = Nu(Re, Pr, M),$$

$$Re = \rho_1^u\delta|\mathbf{W}_1 - \mathbf{W}_2|/\mu_1, \quad Pr = c_{p1}\mu_1/\lambda_1, \quad M = |\mathbf{W}_1 - \mathbf{W}_2|/C_2. \tag{3}$$

Here β is the heat transfer factor, Nu and Pr are Nusselt and Prandtl numbers. Re and M are Reynolds and Mach numbers for the relative flow about the particle. Trough λ_1 and μ_1 the heat conductivity and the dynamic viscosity of the gas are designated, c_{p1} is the heat capacity of the gas at constant pressure C_1 is the local sound speed in the gas.

The resulting force of interaction between phases $\mathbf{f}$ we shall determine in view of non-stationary effects in the flow of the gas about the particle in its relative movement. For this purpose we shall approximately represent it as the sum:

$$\mathbf{f} = \mathbf{f}_\mu + \mathbf{f}_\mathbf{A} + \mathbf{f}_\mathbf{m} \tag{4}$$

Where $\mathbf{f}_\mu$ is the force of viscous friction, $\mathbf{f}_\mathbf{A}$ is the Archimedes force, $\mathbf{f}_\mathbf{m}$ is the force of the appended mass:

$$\mathbf{f}_\mu = \frac{1}{8}\pi\delta^2\rho_1^u C_\delta|\mathbf{W}_1 - \mathbf{W}_2|(\mathbf{W}_1 - \mathbf{W}_2), \quad C_\delta = C_\delta(Re, M)$$

$$\mathbf{f}_A = \theta\rho_1^u\frac{d\mathbf{W}_1}{dt}, \quad \mathbf{f}_m = \frac{1}{2}\theta\rho_1^u(\frac{d\mathbf{W}_1}{dt} - \frac{d\mathbf{W}_2}{dt}), \tag{5}$$

here C_δ is the aerodynamic drag of the particle.

Note, that because of essential distinction of the sizes of suspended particles and the sizes of suspended particles and the body flown about, the Reynolds number of an external flow is in many orders more than the Reynolds number for interaction between the gas and particles. Therefore, for enough large class of problems one may take into account the viscosity and the heat conductivity only for phase interaction, counting the gas in the gas dynamical sense as non-viscous and non-heat-transferable.

If we take into account the forces of phase interaction (4) and (5) from phase acceleration, then it is reasonable to solve the equations for the system moment relative to the derivatives: $\frac{d\mathbf{W}_1}{dt}$, $\frac{d\mathbf{W}_2}{dt}$. When we fulfill this procedure concerning the gas suspensions with enough small pressures $\rho_1^u/\rho_2^u \ll 1$ and concentrations of the suspended phase $\alpha_2 \ll 1$, neglecting the terms with the order $O(\alpha_2, \rho_1^u/\rho_2^u)$ it gives us:

$$\rho_1 \frac{d\mathbf{W}_1}{dt} = -(1 - \frac{3}{2}\alpha_1)\nabla p - \chi n \mathbf{f}_\mu,$$

$$\rho_2 \frac{d\mathbf{W}_2}{dt} = -\frac{3}{2}\alpha_2 \nabla p + \chi n \mathbf{f}_\mu, \tag{6}$$

$$\chi = (1 - \frac{3}{2}\alpha_2 - \rho_1^u/2\rho_2^u).$$

Equations (6) could be conveniently rewritten in the quasidivergent form:

$$\frac{\partial \rho_1 \mathbf{W_1}}{\partial t} + \nabla \rho_1 \mathbf{W_1}(\mathbf{W_1}, \mathbf{l}) + (1 - \frac{3}{2}\alpha_2)\nabla p = -\chi n \mathbf{f}_\mu,$$

$$\frac{\partial \rho_2 \mathbf{W_2}}{\partial t} + \nabla \rho_2 \mathbf{W_2}(\mathbf{W_2}, \mathbf{l}) + \frac{3}{2}\alpha_2 \nabla p = \chi n \mathbf{f}_\mu. \tag{7}$$

Here $\rho_i \mathbf{W_i}(\mathbf{W_i}, \mathbf{l})$ is the momentum vector fluxes for phase i through surface, which is perpendicular to the unit vector $\mathbf{l}$, $(\mathbf{W_i}, \mathbf{l})$ is the scalar product of vectors $\mathbf{W_i}$ and $\mathbf{l}$.

The effects in (6) and (7) have the order of value of volumetric gas content α_2. In a number of practically important cases of an external

aerodynamical flow of dusted gas about the body the value of volumetric contents $\alpha_2 < 1\%$ of the suspend phase is low, although the mass contents of the particles can even exceed the mass content of the gas because of its low pressure. Therefore we shall further neglect the terms of order α_2. Correspondingly we shall limit ourselves to the cases of moderate pressures, when $\rho_1^u/\rho_2^u \ll 1$.

Resistance factor for the particle C_δ is found using the empirical dependence.

$$C_\delta = \begin{cases} \frac{24}{Re} + \frac{4}{Re^{0.33}}, & \text{if} \quad 0 < Re \leq 700, \\ 4.3(lgRe)^{-2}, & \text{if} \quad 700 < Re \leq 2000. \end{cases}$$

Nusselt number Nu, Prandtl number Pr and Reynolds number Re are interconnected as follows:

$$Nu = 2.0 + 0.6Re^{1/2}Pr^{1/3}.$$

The differential scheme calculating the flows of heterogeneous media is the developed one calculating the movement of the single-phase gas [8,9] . At Eulerian stage of the traditional formulae are used:

$$\begin{aligned} \frac{\tilde{u}_{i,j}^n - u_{i,j}^n}{\triangle t} &= -\frac{p_{i+1,j}^n - p_{i-1,j}^n}{2\triangle x}\frac{\alpha_{i,j}^n}{\rho_{i,j}^n} \\ \frac{\tilde{v}_{i,j}^n - v_{i,j}^n}{\triangle t} &= -\frac{p_{i,j+1}^n - p_{i,j-1}^n}{2\triangle y}\frac{\alpha_{i,j}^n}{\rho_{i,j}^n} \\ \frac{\tilde{E}_{i,j}^n - E_{i,j}^n}{\triangle t} &= -\frac{\alpha_{i,j}^n}{\rho_{i,j}^n}\Big\{\frac{p_{i+1,j}^n u_{i+1,j}^n - p_{i-1,j}^n u_{i-1,j}}{2\triangle x} \\ &\quad + \frac{p_{i,j+1}^n v_{i,j+1}^n - p_{i,j-1}^n v_{i,j-1}^n}{2\triangle y}\Big\} \end{aligned} \tag{8}$$

Values $\tilde{u}_{i,j}^n$ and $\tilde{v}_{i,j}^n$, $\tilde{E}_{i,j}^n$ are determined using equations (8). All values concern to the gas phase.

At Lagrangian stage the mass transfer through cell borders for each phase is calculated. The fluxes of values for the gas phase are calculated using the formulae:

$$\triangle(M_1\phi_1)^n_{i+1/2,j}$$

$$= \begin{cases} \rho^n_{(1)i,j}\phi^n_{(1)i,j}\frac{\tilde{u}^n_{i,j}+u^n_{i,j}}{2}\triangle y\triangle t, & \text{if } \tilde{u}^n_{i,j}+u^n_{i,j}>0, \\ \rho^n_{(1)i+1,j}\phi^n_{(1)i+1,j}\frac{\tilde{u}^n_{i,j}+u^n_{i,j}}{2}\triangle y\triangle t, & \text{if } \tilde{u}^n_{i,j}+u^n_{i,j}<0, \end{cases} \tag{9}$$

$$\phi_1 = (1, u, v, E).$$

The fluxes of solid phase parameters are determined similarly (9). At a final stage, on the basis of laws of conservation, the values of parameters for both phases at a new temporal level are found ρ^{n+1}_ω, E^{n+1}_ω, u^{n+1}_ω, v^{n+1}_ω, $\omega = 1, 2$.

Doing this, the phase interaction $\mathbf{f}$ and heat flow q are taken into account. The differential formulae for the final stage have the following form:

$$\begin{aligned} \rho^{n+1}_{(1)i,j} &= \rho^n_{(1)i,j} + \Big\{\triangle(M_1)^n_{i,j-1/2} + \triangle(M_1)^n_{i-1/2,j} \\ &\quad -\triangle(M_1)^n_{i,j+1/2} - \triangle(M_1)^n_{i,j-1/2}\Big\} \\ &= \rho^{n+1}_{(1)i,j} + \sum \triangle(M_1)/(\triangle x\triangle y), \\ u^{n+1}_{(1)i,j} &= (\rho^n_{(1)i,j}/\rho^{n+1}_{(1)i,j})\tilde{u}^n_{(1)i,j} + \sum \triangle(M_1)\tilde{u}_{(1)}/(\triangle x\triangle y\rho^{n+1}_{(1)i,j}) \\ &\quad -\triangle t f_x/\rho^{n+1}_{(1)i,j} \end{aligned}$$

The formulae determining the parameters of a suspended solid phase are similar to ones adduced above.

The differential scheme shown above belongs to an explicit type and is implemented using a rectangular differential grid. The method of geometrical decomposition is the best of methods mentioned above. The differential grid must be broken into rectangular subareas in such a way, that the amount of information transmitted between the adjacent processor units will be minimal. The effectiveness of the system will be maximal in the case if information exchange time between the adjacent

processor units will not exceed the computation time of these units. As is known, transputers can execute operations of computation and data exchange simultaneously.

The number of subareas into which the initial differential grid is broken, must be equal to the number of transputers in the system. Besides that, it is necessary, that the number of computational nodes in each [3,5] subarea were identical. The differential scheme used is homogeneous, i.e. formulae, used in each computational node, are the same. These conditions, namely, the homogeneity of scheme used, and the equal number of nodes computed by each transputer, should basically provide a uniform load (balance) of the multiprocessor system.

Let us map the computational grid onto the transputer array. The differential scheme for the method of large particles uses an 5-point pattern. In boundary points of computational subarea calls to the memory of an adjacent transputer are required. It is not necessary for computations in internal nodes of the subarea.

Let us study multiprocessor implementation of the problem solution. Let the computation for nodes in an upper subarea is carried out at the transputer number n , while the computation for a lower subarea - at the transputer number $n-1$. To obtain the solution in a boundary node of a subarea one must dispose the information about gas dynamical parameters stored in the $n-1$ th transputer. So it is necessary to organize an exchange channel to transmit data between adjacent transputers.

The physical connection between transputers is to be made through a sole bidirectional link. Thus it is necessary to implement reception of messages from each boundary node in the correspondent boundary node at the adjacent transputer.

For computation of multiphase gas dynamical flows in the conditions of low phase interaction one can organize parallelization of calculations on the basis of phases. Because transputers have only 4 link channels each, the corresponding topology can not be built. It is necessary either to use more advanced transputers of TMS320C40 type or to keep in the memory of each transputer of usual type n-dimensional vector containing data describing n phases of the flow.

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