Computer Science Journal of Moldova, vol.**28**, no.3(84), 2020

On groupoids with Bol-Moufang type identities

Grigorii Horosh, Victor Shcherbacov,
Alexandru Tcachenco, Tatiana Yatsko

Abstract

We present results about groupoids of small order with Bol-Moufang type identities both classical and non-classical which are listed in [7], [9].

2000 Mathematics Subject Classification: 08A35, 68R05, 20N05

Key words and phrases: groupoid, quasigroup, Bol-Moufang type identity.

1 Introduction

Binary groupoid $(G, *)$ is a non-empty set G together with a binary operation "$*$" which is defined on the set G.

An identity based on a single binary operation is of Bol-Moufang type if "both sides consist of the same three different letters taken in the same order but one of them occurs twice on each side" [9]. We use list of 60 Bol-Moufang type identities given in [13].

There exist other (more general) definitions of Bol-Moufang type identities and, therefore, other lists and classifications of such identities [1], [7]. An identity based on a single binary operation is of generalized Bol-Moufang type if "both sides consist of the same three different letters but one of them occurs twice on each side" [1], [7]. In this paper we use both classifications.

We shall name here classical Bol-Moufang type identities as identities from Fenyves's list. We shall name here non-classical type identities as generalized Bol-Moufang type identities. It is clear that any classical type identity is also of non-classical type, but inverse is not true.

Quasigroups and loops, in which Bol-Moufang type identity is true, are central and classical objects of Quasigroup Theory. We recall, works of R. Moufang, G. Bol, R. H. Bruck, V. D. Belousov, K. Kunen, S. Gagola and of many other mathematicians are devoted to the study of quasigroups and loops with Bol-Moufang type identities [1]–[3], [7], [9]–[11], [14], [16], [19].

We continue the study of groupoids with Bol-Moufang type identities [17], [6] [4], [5], [12], [22].

Notice, research of groupoids of small order with some identities is a well-known problem in mathematical literature.

For groupoids the following natural problem is researched: how many groupoids with some identities of small order there exist? A list of numbers of semigroups of orders up to 8 is given in [20], of order 9 – in [8]; a list of numbers of quasigroups up to 11 is given in [15], [18].

2 Results

2.1 Some results on groupoids of order two

It is clear that there exist 16 groupoids of order 2 and there exist n^{n^2} of groupoids of order n. For example, $3^{3^2} = 19\,683$, $4^{4^2} = 4\,294\,967\,296$.

We list isomorphic pairs of groupoids of order two. If a groupoid does not have a pair, then this groupoid has an automorphism group of order two.

Below, the quadruple 22 12 means a groupoid of order 2 with the following Cayley table:

$*$	1	2
1	2	2
2	1	2

and so on. In such a record, a groupoid is commutative if and only if two elements of a quadruple, the second and the third, are equal.

Groupoid $(G, \cdot)$ is isomorphic to groupoid $(G, \circ)$ if there exists a permutation α of symmetric group S_G such that $x \circ y = \alpha^{-1}(\alpha x \cdot \alpha y)$ for all $x, y \in G$.

Groupoid $(G, \cdot)$ is anti-isomorphic to groupoid $(G, \circ)$ if there exists a permutation α of symmetric group S_G such that $x \circ y = \alpha^{-1}(\alpha y \cdot \alpha x)$ for all $x, y \in G$.

Remark 1. *If groupoid $(G, \cdot)$ is anti-isomorphic to groupoid $(G, \circ)$, then groupoid $(G, \circ)$ is anti-isomorphic to groupoid $(G, \cdot)$. Really $x \cdot y = \alpha(\alpha^{-1} y \circ \alpha^{-1} x)$ for all $x, y \in G$.*

Remark 2. *In commutative groupoid $(G, \cdot)$ any anti-isomorphism coincides with isomorphism.*

It is easy to check that the following propositions are fulfilled.

Proposition 1. *Only the following groupoids of order two are isomorphic in pairs:*
11 11 *and* 22 22*;*
11 12 *and* 12 22*;*
11 21 *and* 21 22*;*
11 22*;*
12 11 *and* 22 12*;*
12 12*;*
12 21 *and* 21 12*;*
21 11 *and* 22 21*;*
21 21*;*
22 11.

Proposition 2. *Only the following groupoids of order two are anti-isomorphic in pairs:*
11 21 *and* 22 12*;*
21 22 *and* 12 11*;*
11 22 *and* 12 12*;*
21 21 *and* 22 11.

Proposition 3. *Only the following groupoids of order two are isomorphic or anti-isomorphic:*

11 11 *and* 22 22*;*
11 12 *and* 12 22*;*
11 21 *and* 21 22 *and* 22 12 *and* 12 11*;*
11 22 *and* 12 12*;*
12 21 *and* 21 12*;*
21 11 *and* 22 21*;*
21 21 *and* 22 11.

Corollary 1. *The following groupoids of order two are non isomorphic and non anti-isomorphic in pairs:* <u>11 11</u>*;* <u>11 12</u>*;* 11 21*;* <u>11 22</u>*;* <u>12 21</u>*;* 21 11*;* 21 21.

Using the list of groupoids which is presented in Proposition 3, we can compose other lists of groupoids for Corollary 1. For example, instead of groupoid <u>11 11</u> we can write groupoid <u>22 22</u> and so on.

In the list presented in Corollary 1 semigroups of order two are underlined [23].

2.2 (12)-parastrophes of identities

We recall, (12)-parastrophe of groupoid $(G, \cdot)$ is a groupoid $(G, *)$ in which operation "$*$ " is obtained by the following rule:

$$x * y = y \cdot x. \tag{1}$$

It is clear that for any groupoid $(G, \cdot)$ there exists its (12)-parastrophe groupoid $(G, *)$.

Cayley table of groupoid $(G, *)$ is a mirror image of the Cayley table of groupoid $(G, \cdot)$ relative to main diagonal. Notice, for any binary quasigroup there exist five its parastrophes [2], [18], [22] more.

Suppose that an identity F is true in groupoid $(G, \cdot)$. Then we can obtain a (12)-parastrophic identity F^* of the identity F, replacing the operation "$\cdot$" with the operation "$*$" and changing the order of variables by using rule (1).

Remark 3. *In quasigroup case, similarly to* (12)*-parastrophe identity other parastrophe identities can be defined. See [21] for details.*

It is clear that an identity F is true in groupoid $(G, \cdot)$ if and only if in groupoid $(Q, *)$ the identity F^* is true.

Proposition 4. *The number of groupoids of a finite fixed order in which the identity F is true coincides with the number of groupoids in which the identity F^* is true.*

Example 1. *[12]. Way 1. We find* (12)*-parastrophe of the Bol-Moufang type identity F_1: $xy \cdot zx = (xy \cdot z)x$.*

We have $(x*z)*(y*x) = x*(z*(y*x))$. After renaming of variables $(y \leftrightarrow z)$ and operation $(* \to \cdot)$ we obtain the following Bol-Moufang type identity F_3: $xy \cdot zx = x(y \cdot zx)$.

Therefore $(F_1)^* = F_3$. The vice versa, $(F_3)^* = F_1$, is also true.

Way 2. We recall, left translation of a groupoid $(G, \cdot)$ is defined as follows: $L_a x = a \cdot x$ for all $x \in G$; right translation of a groupoid $(G, \cdot)$ is defined similarly: $R_a x = x \cdot a$ for all $x \in G$ and a fixed element $a \in G$.

Then we can re-write identity F_1 in the following form: $L_x y \cdot R_x z = R_x(L_x y \cdot z)$.

There exists the following connections between left and right translations of a groupoid $(G, \cdot)$ and its (12)*-parastrophe [18], [22]:*

$$L_a^* = R_a^{\cdot}, R_a^* = L_a^{\cdot}. \tag{2}$$

Further, using rules (1) and (2), we have $L_x z \cdot R_x y = L_x(z \cdot R_x y)$, $xz \cdot yx = x(z \cdot yx)$. After renaming variables $(y \leftrightarrow z)$, we obtain the following Bol-Moufang type identity F_3: $xy \cdot zx = x(y \cdot zx)$, i.e., $(F_1)^* = F_3$.

Theorem 1. *For classical Bol-Moufang type identities over groupoids the following equalities are true:*

$(F_1)^* = F_3$, $(F_2)^* = F_4$, $(F_5)^* = F_{10}$, $(F_6)^* = F_6$, $(F_7)^* = F_8$, $(F_9)^* = F_9$, $(F_{11})^* = F_{24}$, $(F_{12})^* = F_{23}$, $(F_{13})^* = F_{22}$, $(F_{14})^* = F_{21}$, $(F_{15})^* = F_{30}$, $(F_{16})^* = F_{29}$, $(F_{17})^* = F_{27}$, $(F_{18})^* = F_{28}$, $(F_{19})^* = F_{26}$, $(F_{20})^* = F_{25}$, $(F_{31})^* = F_{34}$, $(F_{32})^* = F_{33}$, $(F_{35})^* = F_{40}$, $(F_{36})^* = F_{39}$, $(F_{37})^* = F_{37}$, $(F_{38})^* = F_{38}$, $(F_{41})^* = F_{53}$, $(F_{42})^* = F_{54}$, $(F_{43})^* = F_{51}$, $(F_{44})^* = F_{52}$, $(F_{45})^* = F_{60}$, $(F_{46})^* = F_{56}$, $(F_{47})^* = F_{58}$, $(F_{48})^* = F_{57}$, $(F_{49})^* = F_{59}$, $(F_{50})^* = F_{55}$.

For quasigroups, the analogue of Theorem 1 is given in [14].

Proposition 5. *Any of the following groupoids* 11 11*,* 22 22*,* 11 12*,* 12 22*,* 11 22*,* 12 12 *satisfies any of the identities* F_1*–*F_{60}*.*

Proof. It is possible to use direct calculations. □

2.3 Number of groupoids

Original algorithm is elaborated, the corresponding program is written for generating the groupoids of small (2, 3, and 4) orders with generalized Bol-Moufang identities.

The developed algorithm consists of two parts. In the first part we generate a groupoid. In the second part we check, if this groupoid satisfies a Bol-Moufang identity. And so on.

Usually we present a groupoid as a string or two-dimensional array.

Notice, number of groupoids of order 3 with mentioned in Table 2 identities are also given in [5].

We count number of groupoids of order two with classical Bol-Moufang type identities given in [13], including also the number of non-isomorphic ones and number of non-isomorphic and non-anti-isomorphic groupoids of order 2 (see Table 1). Notice, in some places Table 1 coincides with the corresponding table from [12].

Table 1 is organised as follows: in the first column there is given the name of identity in Fen'vesh list; in the second – the abbreviation of this identity, if this identity has a name; in the third – the identity is given; in the fourth column there is indicated the number of groupoids of order 2 with the corresponding identity; in the fifth column – the number of non-isomorphic groupoids; and, in the sixth column – the number of non-isomorphic and non-anti-isomorphic groupoids with the corresponding identity is given.

Table 1: Number of groupoids of order 2 with classical Bol-Moufang identities

Name	**Abb.**	**Ident.**	**2**	**n.-is.**	**n.-is., an.**
F_1		$xy \cdot zx = (xy \cdot z)x$	10	6	5
F_2		$xy \cdot zx = (x \cdot yz)x$	9	6	5
F_3		$xy \cdot zx = x(y \cdot zx)$	10	6	5
F_4	middle Mouf.	$xy \cdot zx = x(yz \cdot x)$	9	6	5
F_5		$(xy \cdot z)x = (x \cdot yz)x$	11	7	6
F_6	extra ident.	$(xy \cdot z)x = x(y \cdot zx)$	10	7	5
F_7		$(xy \cdot z)x = x(yz \cdot x)$	9	6	5
F_8		$(x \cdot yz)x = x(y \cdot zx)$	9	6	5
F_9		$(x \cdot yz)x = x(yz \cdot x)$	10	6	5
F_{10}		$x(y \cdot zx) = x(yz \cdot x)$	11	7	6
F_{11}		$xy \cdot xz = (xy \cdot x)z$	8	5	4
F_{12}		$xy \cdot xz = (x \cdot yx)z$	9	7	6
F_{13}	extra ident.	$xy \cdot xz = x(yx \cdot z)$	9	6	5
F_{14}		$xy \cdot xz = x(y \cdot xz)$	10	6	5
F_{15}		$(xy \cdot x)z = (x \cdot yx)z$	11	7	6
F_{16}		$(xy \cdot x)z = x(yx \cdot z)$	11	7	6
F_{17}	left Mouf.	$(xy \cdot x)z = x(y \cdot xz)$	10	7	5
F_{18}		$(x \cdot yx)z = x(yx \cdot z)$	8	5	4

F_{19}	left Bol	$(x\cdot yx)z = x(y\cdot xz)$	9	6	5
F_{20}		$x(yx\cdot z) = x(y\cdot xz)$	9	6	5
F_{21}		$yx\cdot zx = (yx\cdot z)x$	10	6	5
F_{22}	extra ident.	$yx\cdot zx = (y\cdot xz)x$	9	6	5
F_{23}		$yx\cdot zx = y(xz\cdot x)$	9	6	5
F_{24}		$yx\cdot zx = y(x\cdot zx)$	8	5	4
F_{25}		$(yx\cdot z)x = (y\cdot xz)x$	9	6	5
F_{26}	right Bol	$(yx\cdot z)x = y(xz\cdot x)$	9	6	5
F_{27}	right Mouf.	$(yx\cdot z)x = y(x\cdot zx)$	10	7	5
F_{28}		$(y\cdot xz)x = y(xz\cdot x)$	8	5	4
F_{29}		$(y\cdot xz)x = y(x\cdot zx)$	11	7	6
F_{30}		$y(xz\cdot x) = y(x\cdot zx)$	11	7	6
F_{31}		$yx\cdot xz = (yx\cdot x)z$	8	5	4
F_{32}		$yx\cdot xz = (y\cdot xx)z$	9	6	5
F_{33}		$yx\cdot xz = y(xx\cdot z)$	9	6	5
F_{34}		$yx\cdot xz = y(x\cdot xz)$	8	5	4
F_{35}		$(yx\cdot x)z = (y\cdot xx)z$	9	6	5
F_{36}	RC ident.	$(yx\cdot x)z = y(xx\cdot z)$	9	6	5
F_{37}	C ident.	$(yx\cdot x)z = y(x\cdot xz)$	10	7	5
F_{38}		$(y\cdot xx)z = y(xx\cdot z)$	8	5	4
F_{39}	LC ident.	$(y\cdot xx)z = y(x\cdot xz)$	9	6	5
F_{40}		$y(xx\cdot z) = y(x\cdot xz)$	9	6	5

F_{41}	LC ident.	$xx \cdot yz = (x \cdot xy)z$	9	6	5
F_{42}		$xx \cdot yz = (xx \cdot y)$ z	12	7	5
F_{43}		$xx \cdot yz = x(x \cdot yz)$	8	5	4
F_{44}		$xx \cdot yz = x(xy \cdot z)$	9	6	5
F_{45}		$(x \cdot xy)z = (xx \cdot y)z$	9	6	5
F_{46}	LC ident.	$(x \cdot xy)z = x(x \cdot yz)$	11	7	6
F_{47}		$(x \cdot xy)z = x(xy \cdot z)$	8	5	4
F_{48}	LC ident.	$(xx \cdot y)z = x(x \cdot yz)$	10	7	5
F_{49}		$(xx \cdot y)z = x(xy \cdot z)$	9	6	5
F_{50}		$x(x \cdot yz) = x(xy \cdot z)$	11	7	6
F_{51}		$yz \cdot xx = (yz \cdot x)x$	8	5	4
F_{52}		$yz \cdot xx = (y \cdot zx)x$	9	6	5
F_{53}	RC ident.	$yz \cdot xx = y(zx \cdot x)$	9	6	5
F_{54}		$yz \cdot xx = y(z \cdot xx)$	12	7	6
F_{55}		$(yz \cdot x)x = (y \cdot zx)x$	11	7	6
F_{56}	RC ident.	$(yz \cdot x)x = y(zx \cdot x)$	11	7	6
F_{57}	RC ident.	$(yz \cdot x)x = y(z \cdot xx)$	10	7	6
F_{58}		$(y \cdot zx)x = y(zx \cdot x)$	8	5	4
F_{59}		$(y \cdot zx)x = y(z \cdot xx)$	9	6	5
F_{60}		$y(zx \cdot x) = y(z \cdot xx)$	9	6	5

and L3, M1 and M3, M2 and M4, T1 and T3, T4 and T5, are (12)-parastrophic identitities. Therefore the numbers of groupoids of fixed order with these (12)-parastrophic identitities coincide.

Table 2: Number of groupoids of order 2, 3 and 4 with Bol-Moufang identities.

Name	**Abbr.**	**Ident.**	**#2**	**#3**	**#4**
Extra	EL	$x(y(zx)) = ((xy)z)x$	10	239	18744
Moufang	ML	$(xy)(zx) = (x(yz))x$	9	196	25113
Left Bol	LB	$x(y(xz)) = (x(yx))z$	9	215	22875
Right Bol	RB	$y((xz)x) = ((yx)z)x$	9	215	22875
C-loops	CL	$y(x(xz)) = ((yx)x)z$	10	209	26583
LC-loops	LC	$(xx)(yz) = (x(xy))z$	9	220	26583
RC-loops	RC	$y((zx)x) = (yz)(xx)$	9	220	26583
Middle Nuclear Square	MN	$y((xx)z) = (y(xx))z$	8	350	122328
Right Nuclear Square	RN	$y(z(xx)) = (yz)(xx)$	12	932	2753064
Left Nuclear Square	LN	$((xx)y)z = (xx)(yz)$	12	932	2753064
Comm. Moufang	CM	$(xy)(xz) = (xx)(zy)$	8	297	111640
Comm. C-loop	CC	$(y(xy))z = x(y(yz))$	8	169	12598
Comm. Alternative	CA	$((xx)y)z = z(x(yx))$	6	110	10416
Comm. Nuclear square	CN	$((xx)y)z = (xx)(zy)$	9	472	1321661
Comm. loops	CP	$((yx)x)z = z(x(yx))$	8	744	1078744

Continued on next page

Table 2 – *Continued from previous page*

Name	**Abbr.**	**Ident.**	**#2**	**#3**	**#4**
Cheban, 1	C1	$x((xy)z) = (yx)(xz)$	8	219	19846
Cheban, 2	C2	$x((xy)z) = (y(zx))x$	6	153	12382
Lonely, I	L1	$(x(xy))z = y((zx)x)$	6	117	6076
Cheban, I, Dual	CD	$(yx)(xz) = (y(zx))x$	8	219	19846
Lonely, II	L2	$(x(xy))z = y((xx)z)$	7	157	11489
Lonely, III	L3	$(y(xx))z = y((zx)x)$	7	157	11489
Mate, I	M1	$(x(xy))z = ((yz)x)x$	6	111	11188
Mate, II	M2	$(y(xx))z = ((yz)x)x$	7	196	26785
Mate, III	M3	$x(x(yz)) = y((zx)x)$	6	111	11188
Mate, IV	M4	$x(x(yz)) = y((xx)z)$	7	196	26785
Triad, I	T1	$(xx)(yz) = y(z(xx))$	6	162	67152
Triad, II	T2	$((xx)y)z = y(z(xx))$	6	180	53832
Triad, III	T3	$((xx)y)z = (yz)(xx)$	6	162	67152
Triad, IV	T4	$((xx)y)z = ((yz)x)x$	6	132	42456
Triad, V	T5	$x(x(yz)) = y(z(xx))$	6	132	42456
Triad, VI	T6	$(xx)(yz) = (yz)(xx)$	8	1419	9356968
Triad, VII	T7	$((xx)y)z = ((yx)x)z$	12	428	2914658
Triad, VIII	T8	$(xx)(yz) = y((zx)x)$	6	120	11580
Triad, IX	T9	$(x(xy))z = y(z(xx))$	6	102	6192
Frute	FR	$(x(xy))z = (y(zx))x$	6	129	16600
Crazy Loop	CR	$(x(xy))z = (yx)(xz)$	7	136	12545
Krypton	KR	$((xx)y)z = (x(yz))x$	9	268	93227

About computer. The corresponding program has run at 4 nodes with the following properties: System – Virtual machine; OS – Ubuntu

18.04; CPU – Virtual CPU, up to 2.40GHz; Memory – 8GB.

Acknowledgement. The authors thank Prof. C.V. Gaindric for his valuable suggestions.

References

[1] Reza Akhtar, Ashley Arp, Michael Kaminski, Jasmine Van Exel, Davian Vernon, and Cory Washington, "The varieties of Bol-Moufang quasigroups defined by a single operation," *Quasigroups Related Systems*, vol. 20, no. 1, pp. 1–10, 2012.

[2] V.D. Belousov, *Foundations of the Theory of Quasigroups and Loops*, Moscow: Nauka, 1967. (in Russian).

[3] R.H. Bruck, "Some theorems on Moufang loops," *Math. Z.*, vol. 73, pp. 59–78, 1960.

[4] Vladimir Chernov, Alexander Moldovyan, and Victor Shcherbacov, "On some groupoids of order three with Bol-Moufang type of identities," in *Proceedings of the Conference on Mathematical Foundations of Informatics MFOI2018, July 2-6, 2018, Chisinau*, (Chisinau, Moldova), 2018, pp. 17–20.

[5] Vladimir Chernov, Alexander Moldovyan, and Victor Shcherbacov, "On some groupoids of small orders with Bol-Moufang type of identities," 2018, arXiv:1812.02511.

[6] Vladimir Chernov, Nicolai Moldovyan, and Victor Shcherbacov, "On some groupoids of small order," in *The Fourth Conference of Mathematical Society of the Republic of Moldova dedicated to the centenary of Vladimir Andrunachievici (1917-1997), June 28 - July 2, 2017, Chisinau, Proceedings CMSM4'*, (Chisinau, Moldova), 2017, pp. 51–54.

[7] B. Cote, B. Harvill, M. Huhn, and A. Kirchman, "Classification of loops of generalized Bol-Moufang type," *Quasigroups Related Systems*, vol. 19, no. 2, pp. 193–206, 2011.

[8] Andreas Distler and Tom Kelsey, "The semigroups of order 9 and their automorphism groups," *Semigroup Forum*, vol. 88, no. 1, pp. 93–112, 2014.

[9] F. Fenyves, "Extra loops. II. On loops with identities of Bol-Moufang type," *Publ. Math. Debrecen*, vol. 16, pp. 187–192, 1969.

[10] Stephen M. Gagola, "Hall's theorem for Moufang loops," *J. Algebra*, vol. 323, no. 12, pp. 3252–3262, 2010.

[11] Stephen M. Gagola, "How and why Moufang loops behave like groups," *Quasigroups Related Systems*, vol. 19, no. 1, pp. 1–22, 2011.

[12] Grigorii Horosh, Victor Shcherbacov, Alexandru Tcachenco, and Tatiana Yatsko, "On some groupoids with Bol-Moufang type identities," in *Proceedings of the 5-th Conference on Mathematical Foundations of Informatics, MFOI 2019, July 3-6*, (Iasi, Romania), July 2019, pp. 149–158.

[13] T.G. Jaíyeola, E. Ilojide, M. O. Olatinwo, and F. Smarandache, "On the Classification of Bol-Moufang Type of Some Varieties of Quasi Neutrosophic Triplet Loop (Fenyves BCI-Algebras)," *Symmetry*, vol. 10, pp. 1–16, 2018. DOI:10.3390/sym10100427.

[14] K. Kunen, "Quasigroups, loops and associative laws," *J. Algebra*, vol. 185, no. 1, pp. 194–204, 1996.

[15] Brendan D. McKay and Ian M. Wanless, "On the number of latin squares," *Ann. Comb.*, vol. 9, no. 3, pp. 335–344, 2005.

[16] R. Moufang, "Zur Structur von Alternativ Körpern," *Math. Ann.*, vol. 110, pp. 416–430, 1935.

[17] B. V. Novikov, "On decomposition of Moufang groupoids," *Quasigroups and related systems*, vol. 16, no. 1, pp. 97–101, 2008.

[18] H.O. Pflugfelder, *Quasigroups and Loops: Introduction*, Berlin: Heldermann Verlag, 1990.

[19] J. D. Phillips and Petr Vojtechovsky, "The varieties of loops of Bol-Moufang type," *Algebra Universalis*, vol. 54, no. 3, pp. 259–271, 2005.

[20] S. Satoh, K. Yama, and M. Tokizawa, "Semigroups of order 8," *Semigroup forum*, vol. 49, pp. 7–29, 1994.

[21] A.V. Scerbacova and V.A. Shcherbacov, "On spectrum of medial T_2-quasigroups," *Bul. Acad. Ştiinţe Repub. Mold. Mat.*, no. 2, pp. 143–154, 2016.

[22] Victor Shcherbacov, *Elements of Quasigroup Theory and Applications*, Boca Raton: CRC Press, 2017.

[23] Wikipedia. "Semigroup with two elements," 2015. https://en.wikipedia.org/wiki/Semigroup_with_two_elements.

Grigorii Horosh[1], Victor Shcherbacov[2], Alexandr Tcachenco[3], Tatiana Yatsko[4]

[1]Vladimir Andrunachievici Institute of Mathematics and Computer Science, Moldova
E–mail: grigorii.horos@math.md

[2]Vladimir Andrunachievici Institute of Mathematics and Computer Science, Moldova
E–mail: victor.scerbacov@math.md

[3]Vladimir Andrunachievici Institute of Mathematics and Computer Science, Moldova
E–mail: atkacheno405@gmail.com

[4]Shevchenko Transnistria State University
E–mail: yaczkot@bk.ru