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"The absence of the difference from a pot is potness" – Axiomatic Proofs of Theorems Concerning Negative Properties in Navya-Nyāya

Eberhard Guhe

Abstract

The present paper deals with an aspect of the Navya-Nyāya "logic of property and location" (Matilal) in classical Indian philosophy, namely the so-called "absences" (*abhāva*). Following George Bealer (*Quality and Concept*, Oxford 1982) we may regard these negative properties as the result of applying certain algebraic operations to property terms, which Bealer names after their corresponding propositional or first-order operations ("negation of a property", "conjunction of properties", "existential generalization of a property" etc.). Bealer introduces these operations in his property theories in order to explain how the denotation of a complex property term can be determined from the denotation(s) of the relevant syntactically simpler term(s). An interesting case in Navya-Nyāya is the "conjoint absence" (*ubhayābhāva*), which can be regarded as the Sheffer stroke applied to property terms.

We will show that an extension of Bealer's axiomatic system T1 may serve to prove some of the Navya-Naiyāyikas' intuitions concerning iterated absences, such as "the relational absence of the difference from a pot", "the relational absence of the relational absence of a pot" or "the relational absence of the relational absence of the relational absence of a pot". The former, e.g., was claimed to be identical to the universal "potness".

Keywords: Indian logic, Navya-Nyāya, intensionality, property theories, negation.

1 Introduction

The present paper is about late Indian logic. More specifically, we will deal with a type of logic which originated in a school of classical Indian philosophy called "Navya-Nyāya" ("New Logic"?). Its early beginnings date back to the 12th or 13th century with authors such as Śaśadhara and Maṇikaṇṭha Miśra.[1] Gaṅgeśa's magnum opus Tattvacintāmaṇi (14th century) was seminal for the development of the typical style of the Navya-Naiyāyikas' approach to logical and epistemological issues. In order to define their concepts with utmost precision they designed an ideal language, a kind of Leibnizian characteristica universalis based on a canonical form of Sanskrit, which serves to explicate the objective content of verbalized and unverbalized cognitions and to disambiguate sentences formulated in ordinary Sanskrit. The school reached its peak in the works of authors such as Raghunātha Śiromaṇi (16th century), Jagadīśa and Gādādhara (17th century) and remained active through to the 19th century.

Navya-Nyāya logic was dubbed a "logic of property and location" by Matilal. In order to demonstrate what this means, let us take an empty pot as an example. Even though the pot is empty, Navya-Naiyāyikas claimed that there are lots of items in this pot and also all around it. The universals (*jāti*) "substanceness" (*dravyatva*) and "potness" (*ghaṭatva*), e.g., are in the pot and all around it. There are some other properties attached to the pot, such as "being created" (*kṛtatva*) and "being non-eternal" (*anityatva*), which were unlike universals not counted as elementary constituents of empirical reality in Navya-Nyāya. As nominal properties (*upādhi*) they were nevertheless considered to be part of the actual world. Navya-Naiyāyikas reified universals and nominal properties, i.e., they treated them as individuals.

In the present paper we will focus on negative properties, the so-called "absences" (*abhāva*), which were also regarded as individuals. There are two types of absence. In order to illustrate them we can

[1] There is good reason to believe that the works of Udayana (11th century) already mark the advent of Navya-Nyāya (cf. [13], p. 9f).

again refer to our example of an empty pot. Since the pot is different from a cloth, it is a locus of the "mutual absence" (*anyonyābhāva*) *of* or the "difference" (*bheda*) *from* a cloth. Since there is no food in the pot, it is also a locus of the "absence of food". This is another type of negative property. More accurately, it is called "relational absence" (*saṃsargābhāva*), since this property characterizes a locus as being unrelated to the absentee in terms of one of the relations enunciated in the Navya-Nyāya system of ontological categories. Since food has no "contact" (*saṃyoga*) with the pot, the latter is a locus of the "absence of food having contact with the pot".[2] The specification of the relation whereby an absentee fails to reside in a locus was regarded as crucial: Although potness resides in a pot via a relation called "inherence" (*samavāya*), a pot is a locus of the absence of potness having contact with a pot. A difference is construed as a denial of a further type of relation, namely "identity" (*tādātmya*).

Let us look again at the properties "being created" and "being non-eternal". Navya-Naiyāyikas assumed that whatever is created is non-eternal and vice versa. Therefore both properties were believed to share the same loci, i.e., they were regarded as equi-locatable. Nevertheless, they were considered to be distinct properties. This coincides with our intuition, because we can imagine a logically possible world in which something is created, but eternal.

What about potness and the absence of the difference from a pot? Differences were assumed to be related to their loci via the so-called "peculiar relation" (*svarūpasaṃbandha*). So, we may understand this absence as an absence whose absentee, i.e., the difference from a pot, is unrelated to a locus in the sense that it has no peculiar relation to that locus. But no matter in what way we specify the relation here, the difference from a pot is always absent from every pot, since every pot is not different from some pot ($\forall x(Px \rightarrow \exists y(Py \wedge \neg(x \neq y)))$). So, the relational absence of the difference from a pot resides in every pot. On the other hand, every pot is a locus of potness. Hence, the relational absence of the difference from a pot is equi-locatable with

[2]To be more precise, such an absence was said to be "limited" (*avacchinna*) by contact.

potness. In this case we might have the feeling that the expressions “the relational absence of the difference from a pot” and “potness” refer to the same property. The former expression seems to be a kind of logically equivalent circumlocution of the latter. Actually, the Navya-Nyāya logician Mathurānātha equates “potness” with “the relational absence of the difference from a pot”.

The example of the properties “being created” and “being non-eternal” shows that in the Navya-Nyāya logic of property and location properties do not conform to an extensionality principle. Unlike sets, which are identical if they have the same members, properties need not be identical if they have the same loci.

On the other hand, the example of the properties “potness” and “the relational absence of the difference from a pot” shows that an appropriate formal reconstruction of the Navya-Nyāya logic of property and location should be equipped with a criterion for the identification of properties. An obvious idea is to model the Navya-Nyāya intuitions about the identity of properties by regarding identity as tantamount to necessary equivalence. This is expressed in [2] in the form of axiom A8 as part of Bealer’s intensional property theory T1, as we will see below.

2 Towards a formal reconstruction of the logic of Navya-Nyāya

2.1 G. Bealer’s calculus T1 as a basic framework

T1 harmonizes well with the reification of properties in Navya-Nyāya, since Bealer denotes properties by means of terms. For that purpose he adds square brackets to a first-order language along with the following formation rule:

If A is a formula and $v_1, \ldots, v_m$ $(0 \leq m)$ are distinct variables, then $[A]_{v_1 \ldots v_m}$ is a term.

A term of the form $[A]_{v_1 \ldots v_m}$ denotes ...

a) a proposition, if $m = 0$ ("that A").
b) a property, if $m = 1$ ("being a v_1 of which A is true").
c) an m-ary relation, if $m \geq 2$ ("the relation which holds between $v_1, \ldots, v_m$ *iff* A applies to them").

If Px translates into "x is a pot", then $[Px]_x$ (which can be read as "being an x such that x is P") is an analytical expression for "potness". The function of the index variable x is to bind the free occurrence of x in Px. The square brackets around Px indicate an intensional context. If one substitutes an expression within the bracketed part by another one which is extensionally equivalent, one might change the reference of the property term. Such restrictions concerning the substitutability of extensionally equivalent expressions generally distinguish intensional from extensional logical systems.

The language of T1 includes also the modal operators $\Box$ and $\Diamond$, but as defined symbols. An expression of the form $\Box A$ is adopted as a convenient abbreviation of expressions such as $N[A]$, where N is a one-place predicate expressing "...is necessary". The semantic model structure for T1 (cf. [2], p. 49) contains a condition which ensures that there is only one necessary truth (cf. [2], p. 52f). Since $[x = x]$ is a trivial necessary truth for any proposition x, $[A]$ can be identified with it if A is necessarily true. Therefore it is possible to define the modal operator $\Box$ simply by means of the square brackets: $\Box A :\leftrightarrow [A] = [[A] = [A]]$ (A is necessarily true *iff* the proposition "that A" is identical to a trivial necessary truth.) As usual, $\Diamond A :\leftrightarrow \neg \Box \neg A$.

Bealer shows that we obtain a sound and complete calculus by axiomatizing T1 in the following way (cf. [2], p. 58f):

A1: Truth-functional tautologies

A2: $\forall v_i A(v_i) \to A(t)$, where t is free for v_i in A, i.e., no free occurrence of v_i in A lies within the scope of a quantifier or a sequence of index variables in a term $[\ldots]_{v_1 \ldots v_m}$ which would bind a variable occurring in t.

A3: $\forall v_i(A \to B) \to (A \to \forall v_i B)$, where v_i is not free in A.

A4: $v_i = v_i$

A5: $v_i = v_j \to (A(v_i, v_i) \leftrightarrow A(v_i, v_j))$, where $A(v_i, v_j)$ is a formula that arises from $A(v_i, v_i)$ by replacing some (but not necessarily all) free occurrences of v_i by v_j, and v_j is free for the occurrences of v_i that it replaces.

A6: $[A]_{u_1 \dots u_p} \neq [B]_{v_1 \dots v_q}$, where $p \neq q$.

A7: $[A(u_1, \dots, u_p)]_{u_1 \dots u_p} = [A(v_1, \dots, v_p)]_{v_1 \dots v_p}$, where these two terms are alphabetic variants.

A8: $[A]_{u_1 \dots u_p} = [B]_{u_1 \dots u_p} \leftrightarrow \Box \forall u_1 \dots \forall u_p (A \leftrightarrow B)$

A9: $\Box A \to A$

A10: $\Box(A \to B) \to (\Box A \to \Box B)$

A11: $\Diamond A \to \Box \Diamond A$

R1: If $\vdash A$ and $\vdash (A \to B)$, then $\vdash B$.

R2: If $\vdash A$, then $\vdash \forall v_i A$.

R3: If $\vdash A$, then $\vdash \Box A$.

A1 – A5 along with R1 and R2 constitute an axiomatization of first-order predicate logic including identity. A6 – A8 determine how to deal with the intensional abstracts in T1. A8 furnishes a criterion for the identification of intensional abstracts. It captures the idea that identity is tantamount to necessary equivalence. A9 – A11 and R3 are the modal part of the axiomatic system S5 of propositional modal logic.

2.2 Extensions of T1 which function as alternatives to set theories

2.2.1 The naive property abstraction in Navya-Nyāya

In order to prove some of the Navya-Naiyāyikas' intuitions about negative properties we need an extension of this calculus. More specifically, we need a kind of comprehension principle for properties. The

Navya-Naiyāyikas themselves formulated such a principle in the following way: *tattvavat tad eva.* – "Anything which possesses the property 'being that' is that." (Cf. [9], p. 36)

In order to see how this rule works one might replace the Sanskrit word *tat* ("that"), which has the same function as a schematic variable here, by words like *ghaṭa* ("pot"). *ghaṭatvavān ghaṭa eva* means: "Anything which possesses the property 'potness' is a pot." Thus, the *tattvavat tad eva*-rule can be regarded as a kind of counterpart of the naive class abstraction in set theory:

$a \in \{x|A(x)\} \leftrightarrow A(a)$, where a is free for x in A and vice versa.

This equivalence can be transformed into a formal version of the naive property abstraction rule in Navya-Nyāya by replacing $\{x|A(x)\}$ by the corresponding property term in T1, i.e., $[A(x)]_x$. In order to express that something possesses or is a locus of $[A(x)]_x$ we can use Bealer's Δ-relation, which functions as a counterpart of the $\in$-relation in set theory (cf. [2], p. 96). Thus, if we understand the *tattvavat tad eva*-rule in the sense of an equivalence, we can formalize it in the following way[3]:

$(*)\ a \, \Delta \, [A(x)]_x \leftrightarrow A(a)$, where a is free for x in A and vice versa.

2.2.2 A property-theoretic variant of Zermelo-Russell's antinomy and its Sanskrit equivalent

Navya-Nyāya logicians were not aware that a variant of Zermelo-Russell's antinomy can be derived from the *tattvavat tad eva*-rule (cf. [5], p. 109 and [6], p. 144f):

Let us replace the word *tat* ("that") in the *tattvavat tad eva*-rule by *asvavṛttitva* ("being not resident in itself"). This property can easily

[3]The present interpretation of the *tattvavat tad eva*-rule as an equivalence is confirmed by Matilal, who characterizes the specific style of Navya-Nyāya texts in the following way: "Simple predicate formulations, such as 'x is F' are noted, but only to be rephrased as 'x has *F-ness*' (where '*F-ness*' stands for the property derived from 'F')." ([11], p. 115).

be formalized. If one admits $x \Delta x$ as a formal equivalent of "x resides in itself", "being not resident in itself" can be expressed as $[\neg x \Delta x]_x$. Let r be an abbreviation of this property.

(a) If r is resident in itself (i.e., if it is *svavṛtti*), then the property "being not resident in itself" (*asvavṛttitva*) resides in r. Therefore (according to the *tattvavat tad eva*-rule) r is not resident in itself (i.e., it is *asvavṛtti*). (Contradiction!)

This is the formal counterpart of the argument:

$$r \Delta r \Rightarrow \underbrace{r \Delta [\neg x \Delta x]_x}_{\text{can be substituted for } a \Delta [A(x)]_x \text{ in } (*)} \Rightarrow \neg r \Delta r$$

(b) If r is not resident in itself (i.e., if it is *asvavṛtti*), then (according to the *tattvavat tad eva*-rule) the property "being not resident in itself" (*asvavṛttitva*) resides in r. Therefore r is resident in itself (i.e., it is *svavṛtti*). (Contradiction!)

This is the formal counterpart of the argument:

$$\underbrace{\neg r \Delta r}_{\text{can be substituted for } A(a) \text{ in } (*)} \Rightarrow r \Delta [\neg x \Delta x]_x \Rightarrow r \Delta r$$

(a) and (b) together yield the following variant of Zermelo-Russell's antinomy:

$r \Delta r \leftrightarrow \neg r \Delta r$

2.2.3 An ST_2-style extension of T1 ("T1+") as an appropriate framework for a formal reconstruction of Navya-Nyāya logic

In order to modify (∗) in such a way that its paradoxical consequence disappears we can try to imitate the strategies which were pursued by the founders of set theories in order to safeguard the naive class abstraction rule against Zermelo-Russell's antinomy.

Certain restrictions in standard systems of set theory would, however, interfere with ontological commitments in Navya-Nyāya. In ZF (Zermelo-Fraenkel set theory), e.g., sets are the only objects in the domain of models of this system. However, since Navya-Naiyāyikas also talk about non-class-like objects, such as, e.g., pots, one needs a system which is similar to set theories with urelements.

Moreover, some logical arguments in Navya-Nyāya involve universal properties such as nameability, which can be regarded as the analogue of a proper class in set theory. Talking about proper classes like, e.g., $\{x \mid x = x\}$ ("the universal class") is admissible in NBG (Neumann-Bernays-Gödel set theory), but not in ZF. Therefore a property adaptation of NBG with urelements is preferable as a system which may serve to model logical inquiries concerning properties in Navya-Nyāya.

Mendelson incorporates urelements into the framework of NBG (cf. [12], p. 297f). He uses lower-case Latin letters (x, y, z) as restricted variables for sets, capital Latin letters (X, Y, Z) as restricted variables for classes (i.e., for sets and proper classes) and lower-case boldface Latin letters $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ as variables for classes and urelements alike (cf. [12], p. 297). In the present property adaptation of set-theory the same kinds of variables stand for set-like properties, class-like properties (i.e., set-like and properly class-like properties) and urelements, respectively. Lower-case boldface Latin letters as index variables in property terms refer to urelements and set-like individuals. Thus, $[A(\mathbf{x})]_{\mathbf{x}}$ has to be understood in the sense of "being an urelement or a set-like individual $\mathbf{x}$ such that A is true of $\mathbf{x}$". Without this restriction $[\mathbf{x} = \mathbf{x}]_{\mathbf{x}}$ might pass for a property of all properly class-like properties. However, the presumptive existence of such a property invokes a variant of Zermelo-Russell's antinomy in the present logical framework, as we will see below.

A property version of the NBG comprehension axiom seems to be still too restrictive, because it does not include impredicative instantiations, which a Navya-Naiyāyika might not want to rule out (cf. the example given in fn. 4). Since impredicative comprehension is admissible in QM (Quine-Morse set theory, also known as "Morse-Kelley set theory"), but not in NBG, the modification of $(*)$ should be patterned after the QM comprehension axiom. By means of the predicates $P_s\mathbf{x}$ (for "$\mathbf{x}$ is a set-like property") and $U\mathbf{x}$ (for "$\mathbf{x}$ is an urelement") it can be expressed in the following way:

(C) $\forall\mathbf{x}(P_s\mathbf{x} \lor U\mathbf{x} \to (\mathbf{x}\Delta[A(\mathbf{y})]_\mathbf{y} \leftrightarrow A(\mathbf{x})))$, where $\mathbf{x}$ is free for $\mathbf{y}$ in A and vice versa.[4]

There is still another constraint in standard systems of set theory which should not be reproduced in a formal reconstruction of Navya-Nyāya logic: It is commonly assumed that proper classes can never be elements of classes, i.e., (even finite) collections of proper classes do not exist.

In Navya-Nyāya, however, it is possible to apply the *-tva*-abstraction technique repeatedly, so that we might create an expression like *abhidheyatvatva* ("nameabilityness"), which denotes a property of nameability. The analogue of such a property in set theory would be the singleton of the universal class, something which does not exist according to standard systems of set theory. One might call it a "hyper-class" ([4], p. 142).[5]

An appropriate set-theoretic system on which one can model a formal reconstruction of Navya-Nyāya logic should endorse the existence of hyper-classes, hyper-hyper-classes (i.e., classes of hyper-classes) etc. In [4] (cf. p. 142f) the authors design such a system by combining

[4]Since (C) is impredicative, we can use it to formalize substitution instances of the *tattvavat tad eva*-rule, such as: "$\mathbf{x}$ is a locus of the property 'being a locus of some property which is equi-locatable with nameability' (*abhidheyatvasamaniyatakiṃciddharmādhikaraṇatva*) iff $\mathbf{x}$ is a locus of some property which is equi-locatable with nameability." The symbolization key ...

$N\mathbf{x}$: "$\mathbf{x}$ is nameable"
$\mathbf{x}L\mathbf{y}$: "$\mathbf{x}$ is a locus of $\mathbf{y}$"
$\mathbf{x} \doteqdot \mathbf{y}$: "$\mathbf{x}$ is equi-locatable with $\mathbf{y}$", i.e., $\forall\mathbf{z}(\mathbf{z}L\mathbf{x} \leftrightarrow \mathbf{z}L\mathbf{y})$

... yields the following instantiation of (C):

$\forall\mathbf{x}(P_s\mathbf{x} \lor U\mathbf{x} \to (\mathbf{x}\Delta[\exists\mathbf{z}(\mathbf{x}\Delta\mathbf{z} \land \mathbf{z} \doteqdot [N\mathbf{y}]_\mathbf{y})]_\mathbf{x} \leftrightarrow \exists\mathbf{z}(\mathbf{x}\Delta\mathbf{z} \land \mathbf{z} \doteqdot [N\mathbf{y}]_\mathbf{y}))$

[5]The concept of a hyper-class should not be confounded with that of a hyperset (i.e., a non-wellfounded set) in non-wellfounded systems of set theory (cf. [1], p. 6).

the set theories of QM and ZF. The resulting system ST_2 can serve as a set-theoretic prototype of the Navya-Nyāya logic of property and location if we additionally take into account urelements. In ST_2 with urelements $S\mathbf{x}$ (read: "**x** is a set") functions as a primitive monadic predicate. The system comprises the following axioms:

(a) A sethood axiom: Every member of a set is a set or urelement.

(b) All the axioms of QM with urelements (with due regard to the above-mentioned notational convention for variables).

(c) The axioms of ZF with all variables replaced by upper case variables.

This is a two-tier set theory with sets and urelements in the bottom tier and classes in the upper tier. (c) warrants the existence of hyper-classes, hyper-hyper-classes etc. in ST_2. Due to the ZF-axiom of pairing with upper case variables we can, e.g., pair the universal class V with itself in order to obtain the hyper-class $\{V\}$. However, a hyper-class which contains all proper classes does not exist in ST_2. Since there is no universal set in ZF, there is also no way to obtain a corresponding universal hyper-class by means of the axioms in (c).[6]

Proper classes can be elements in ST_2, but they should still be distinguishable from sets. This is achieved by adding (a), which ensures that proper classes cannot be elements of sets.

A property-theoretic counterpart of ST_2 with urelements can be obtained by transforming (a), (b) and (c) into the corresponding property versions. Only the variants of the axiom of extensionality in (b) and (c) have to be exlcuded, because there is already a criterion for the identification of intensional abstracts in T1, namely A8.[7] The extension of T1 which includes the above-mentioned axioms of a property

[6]If $V = \{x \mid x = x\}$ were a set, then $\{x \in V \mid x \notin x\}$ would also be a set according to the ZF-comprehension axiom, i.e., $\{x \mid x = x \wedge x \notin x\} = \{x \mid x \notin x\} = Ru$ would be a set. Hence, $Ru \in Ru \leftrightarrow Ru \notin Ru$.

[7]The following list takes its cue from the property versions of NBG and ZF in [2] (cf. p. 265). For the sake of completeness we have included all the axioms ensuing from the property adaptation of ST_2 with urelements, although some of them might

adaptation of (a), (b) and (c) (exluding the variants of the axiom of extensionality in (b) and (c)) will be called "T1+" hereafter.[8]

be irrelevant to a formal reconstruction of Navya-Nyāya logic. A notable exception is a regularity axiom for properties, which does play an important role in logical inquiries concerning properties in Navya-Nyāya (cf. [8]).

(a)′ $\forall x \forall \mathbf{y}(\mathbf{y}\Delta x \rightarrow (P_s\mathbf{y} \vee U\mathbf{y}))$

(b)′ (Urelements) $\forall \mathbf{x}(U\mathbf{x} \rightarrow \forall \mathbf{y}(\mathbf{y} \not\Delta \mathbf{x}))$

(Comprehension) $\forall \mathbf{x}(P_s\mathbf{x} \vee U\mathbf{x} \rightarrow (\mathbf{x}\Delta[A(\mathbf{y})]_{\mathbf{y}} \leftrightarrow A(\mathbf{x})))$, where $\mathbf{x}$ is free for $\mathbf{y}$ in A and vice versa.

(Null) $\exists x \forall \mathbf{y}(\mathbf{y} \not\Delta x)$

(Pairing) $\forall \mathbf{x} \forall \mathbf{y}((P_s\mathbf{x} \vee U\mathbf{x}) \wedge (P_s\mathbf{y} \vee U\mathbf{y}) \rightarrow \exists z \forall \mathbf{w}(\mathbf{w}\Delta z \leftrightarrow (\mathbf{w} = \mathbf{x} \vee \mathbf{w} = \mathbf{y})))$

(Union) $\forall x \exists y \forall \mathbf{z}(\mathbf{z}\Delta y \leftrightarrow \exists \mathbf{w}(\mathbf{w}\Delta x \wedge \mathbf{z}\Delta\mathbf{w}))$

(Power) $\forall x \exists y \forall \mathbf{z}(\mathbf{z}\Delta y \leftrightarrow \forall \mathbf{w}(\mathbf{w}\Delta\mathbf{z} \rightarrow \mathbf{w}\Delta x))$

(Infinity) $\exists x([y \neq y]_y \Delta x \wedge \forall z(z\Delta x \rightarrow [w\Delta z \vee w = z]^z_w \Delta x))$

(Replacement) $\forall X \forall x(\forall \mathbf{u} \forall \mathbf{v} \forall \mathbf{w}((P_s\mathbf{u} \vee U\mathbf{u}) \wedge (P_s\mathbf{v} \vee U\mathbf{v}) \wedge (P_s\mathbf{w} \vee U\mathbf{w}) \rightarrow (< \mathbf{u}, \mathbf{v} > \Delta X \wedge < \mathbf{u}, \mathbf{w} > \Delta X \rightarrow \mathbf{v} = \mathbf{w})) \rightarrow \exists y \forall \mathbf{z}(\mathbf{z}\Delta y \leftrightarrow \exists \mathbf{w}(\mathbf{w}\Delta x \wedge < \mathbf{w}, \mathbf{z} > \Delta X)))$

(Regularity) $\forall X(\exists \mathbf{y}(\mathbf{y}\Delta X) \rightarrow \exists \mathbf{y}(\mathbf{y}\Delta X \wedge \forall \mathbf{z}(\mathbf{z}\Delta X \rightarrow \mathbf{z} \not\Delta \mathbf{y})))$

(c)′ (Comprehension) $X\Delta[X\Delta Y \wedge A]^Y_X \leftrightarrow X\Delta Y \wedge A$

(Null) $X \not\Delta [X \neq X]_X$

(Pairing) $X\Delta[X = Y \vee X = Z]^{YZ}_X \leftrightarrow X = Y \vee X = Z$

(Union) $X\Delta[\exists Z(X\Delta Z \wedge Z\Delta Y)]^Y_X \leftrightarrow \exists Z(X\Delta Z \wedge Z\Delta Y)$

(Power) $X\Delta[\forall Z(Z\Delta X \rightarrow Z\Delta Y)]^Y_X \leftrightarrow \forall Z(Z\Delta X \rightarrow Z\Delta Y)$

(Infinity) $\exists X([Y \neq Y]_Y \Delta X \wedge \forall Z(Z\Delta X \rightarrow [W\Delta Z \vee W = Z]^Z_W \Delta X))$

(Replacement) $\forall X \forall Y \forall Z((A(X,Y) \wedge A(X,Z)) \rightarrow Y = Z) \rightarrow \forall Y(Y\Delta[\exists X(X\Delta W \wedge A(X,Y))]^W_Y \leftrightarrow \exists X(X\Delta W \wedge A(X,Y)))$

(Regularity) $\forall X(\exists Y(Y\Delta X) \rightarrow \exists Y(Y\Delta X \wedge \forall Z(Z\Delta X \rightarrow Z \not\Delta Y)))$

[8]In T1+ we can prove the following instantiation of the naive Navya-Nyāya property abstraction: "It is a locus of nameabilityness, iff it is nameability." By means of the predicate $N\mathbf{x}$ (for "$\mathbf{x}$ is nameable") "nameability" can be expressed as $[N\mathbf{x}]_{\mathbf{x}}$. The substitution of $[N\mathbf{x}]_{\mathbf{x}}$ for Y and Z in the (c)′-axiom of pairing yields the above-mentioned instantiation of the naive Navya-Nyāya property abstraction: $X\Delta[X = [N\mathbf{x}]_{\mathbf{x}}]_X \leftrightarrow X = [N\mathbf{x}]_{\mathbf{x}}$ The existence of the hyper-class-like property "nameabilityness", i.e., $[X = [N\mathbf{x}]_{\mathbf{x}}]_X$ can be inferred from this as a corollary.

3 Negative properties

We are now prepared to formalize some of the Navya-Nyāya intuitions about negative properties and to prove them in T1+. First of all, we will translate the two types of "absence" (*abhāva*) into the language of T1+.

An absence can be regarded as the result of applying an operation to a property, which Bealer calls "negation". Following his terminology the term $[\neg F\mathbf{x}]_{\mathbf{x}}$ is the result of "negating" the term $[F\mathbf{x}]_{\mathbf{x}}$. Bealer introduces several operations on properties in order to explain how the denotation of a complex term $[A]_\alpha$ can be determined from the denotation(s) of the relevant syntactically simpler term(s) (cf. [2], p. 46f). Interestingly, some of these operations were also taken into account by Navya-Nyāya logicians.

3.1 Mutual absence

The term $[\neg F\mathbf{x}]_{\mathbf{x}}$ may serve as a formal representation of the "mutual absence" (*anyonyābhāva*), i.e., of the "difference" (*bheda*) from an F (more accurately: from anything which is an F). The indefinite article has been added here in front of F in order to facilitate a smooth English translation. In Sanskrit there is no article. A phrase like "the mutual absence of a cloth" is commonly expressed by means of a compound (*paṭānyonyābhāva*) and the literal meaning would be "cloth-mutual-absence". Similarly, "the relational absence of a pot" would be renderd as a compound which literally translates into "pot-relational-absence" (*ghaṭasaṃsargābhāva*). When asked to specify the absentees, the so-called "counterpositives" (*pratiyogin*) of these absences, a Navya-Naiyāyika might say "cloth" (*paṭa*) and "pot" (*ghaṭa*), where "cloth" and "pot" are meant in the sense of expressions which refer to any cloth or any pot, respectively.[9]

[9]In order to emphasize that no reference to one particular cloth or pot is intended here, a Navya-Naiyāyika might say that in these cases the counterpositiveness is "limited" (*avacchinna*) by clothness or potness, respectively. On the other hand, by adding a demonstrative like "this" (*etad*) in front of "cloth" or "pot" he might indicate that these expressions are meant in the sense of singular terms.

We can regard $[\neg F\mathbf{x}]_{\mathbf{x}}$ as a shorthand version of the following formalization which duly mirrors the fact that Navya-Naiyāyikas conceive of a mutual absence, i.e., of a difference, as a denial of an identity between the absentee and the locus of the absence:

(†) $[\neg\exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x} = \mathbf{y})]_{\mathbf{x}}$

3.2 Relational absence

The "relational absence" (*saṃsargābhāva*) of an F (more accurately: of anything which is an F) can be construed as a property which characterizes something as being devoid of (or: no locus of) anything which is an F. In order to formalize this property we will use the predicate $\mathbf{x}L\mathbf{y}$ with the intended meaning "$\mathbf{x}$ is a locus of $\mathbf{y}$". L is supposed to be a more general occurrence relation than the Δ-relation in the sense that $\mathbf{x}L\mathbf{y}$ might also be true if $\mathbf{y}$ is an urelement. Thus, we can infer $\mathbf{x}L\mathbf{y}$ from $\mathbf{x}\Delta\mathbf{y}$, but not vice versa.

Using the L-relation for the purpose of formalizing a relational absence is appropriate in cases where there is no specification of the occurrence relation which fails to subsist between the absentee and the locus of the absence. Thus, an unspecified relational absence can be formalized in the following way:

(‡) $[\neg\exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$

If the relational absence is more specifically meant in the sense of a relational absence via contact or inherence etc., we can replace L by the corresponding symbols for these relations (such as, e.g., C or I).

Since the present formalization of relational absences has basically the same syntactic structure as a mutual absence, namely $[\neg\phi(\mathbf{x})]_{\mathbf{x}}$, where $\phi(\mathbf{x}) :\leftrightarrow \exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$, we can also regard a relational absence as a negation, i.e., as the negation of the property "being a locus of an F" ($[\exists\mathbf{y}(F\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$). Possessing a relational absence of an F means to be different from a locus of an F. So, a relational absence turns out to be a special case of a mutual absence. Both can be regarded as negations.

Navya-Naiyāyikas see the essential difference between the two types of absence in the relation by which the absentee fails to reside in the locus of the absence. In the case of mutual absence this relation is identity. In the case of relational absence it is some kind of occurrence relation. This distinctive feature is duly mirrored in the present formalizations (†) and (‡), because they differ only with respect to the relations (L and =).

3.3 Identities concerning iterated absences

[6] (p. 147f) contains a proof of the following identity concerning iterated absences, which is endorsed by Mathurānātha (cf. [9], p. 71 and [11], p. 152f):

(Id) The relational absence (*saṃsargābhāva*) of the difference (*bheda*) from a pot is identical to potness.

The difference from a pot can obviously be represented as ...

$[\neg P\mathbf{x}]_{\mathbf{x}}$, where $P\mathbf{x}$ translates into "$\mathbf{x}$ is a pot".

Since not only urelements other than pots, but also all class-like individuals are different from pots, one might be inclined to regard the difference from a pot as a property which applies to all class-like individuals including properly class-like properties. However, T1+ does not yield the existence of a hyper-class-like property which applies to every properly class-like property, as there is also no universal set in ZF. Therefore our formalization of the difference from a pot restricts the range of loci to urelements and set-like individuals.

In order to obtain a formal representation of the absence of the difference from a pot one might replace $F\mathbf{y}$ in (‡) by ...

$\mathbf{y} = [\neg P\mathbf{x}]_{\mathbf{x}}$.

Then the relational absence of the difference from a pot (*ghaṭabhedābhāva*) can be expressed as ...

$[\neg\exists\mathbf{y}(\mathbf{y} = [\neg P\mathbf{x}]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{y})]_{\mathbf{x}}$ ("being no locus of anything which is identical to the difference from a pot").

If the difference from a pot is supposed to be a property of all set-like properties and urelements other than pots, one might argue that the relational absence of the difference from a pot applies to all properly class-like properties. However, as noted above, the existence of a hyper-class-like property which applies to every properly class-like property is not warranted by T1+. Therefore our formalization of the relational absence of the difference from a pot restricts the range of loci to urelements and set-like individuals.

Now (Id) can be rendered as a T1+ proposition and we can prove it in T1+:

Theorem (Id):
$[\neg\exists\mathbf{y}(\mathbf{y} = [\neg P\mathbf{x}]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{y})]_{\mathbf{x}} = [P\mathbf{x}]_{\mathbf{x}}$

The proof contains an application of the following instantiation of (C):

$$\forall\mathbf{x}(P_s\mathbf{x} \vee U\mathbf{x} \rightarrow (\mathbf{x}\Delta[\neg P\mathbf{x}]_{\mathbf{x}} \leftrightarrow \neg P\mathbf{x}))$$

Hence, a substitution of equivalents by means of the wff $\mathbf{x}\Delta[\neg P\mathbf{x}]_{\mathbf{x}} \leftrightarrow \neg P\mathbf{x}$ is admissible if we require the variable $\mathbf{x}$ to range over urelements and set-like properties:

Proof of (Id)[10]:

(A1)	$\neg\neg P\mathbf{x} \leftrightarrow P\mathbf{x}$
(C)	$\neg\mathbf{x}\Delta[\neg P\mathbf{x}]_{\mathbf{x}} \leftrightarrow P\mathbf{x}$
(1st-order logic)	$\neg\exists\mathbf{y}(\mathbf{y} = [\neg P\mathbf{x}]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{y}) \leftrightarrow P\mathbf{x}$
(R2, R3)	$\Box\forall\mathbf{x}(\neg\exists\mathbf{y}(\mathbf{y} = [\neg P\mathbf{x}]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{y}) \leftrightarrow P\mathbf{x})$
(A8, R1)	$[\neg\exists\mathbf{y}(\mathbf{y} = [\neg P\mathbf{x}]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{y})]_{\mathbf{x}} = [P\mathbf{x}]_{\mathbf{x}}$ ■

[10] In order to be very precise, one might want to add "$\wedge(P_s\mathbf{x} \vee U\mathbf{x})$" on each side of the equivalences in the first four lines. The last line of the proof would then be $[\neg\exists\mathbf{y}(\mathbf{y} = [\neg P\mathbf{x}]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{y}) \wedge (P_s\mathbf{x} \vee U\mathbf{x})]_{\mathbf{x}} = [P\mathbf{x} \wedge (P_s\mathbf{x} \vee U\mathbf{x})]_{\mathbf{x}}$ and this is, of course, equivalent to (Id).

Maheśa Chandra states two other identities concerning iterated absences, namely the following reduction rules, which are referred to as (Id′) and (Id″) below: *tathāhi dvitīyābhāvaḥ (ghaṭābhāvābhāvaḥ) pratiyogi(ghaṭa)svarūpas tṛtīyābhāvaḥ (ghaṭābhāvābhāvābhāvaḥ) prathamābhāva(ghaṭābhāva)svarūpa iti prathamābhāvasya (ghaṭābhāvasya) ghaṭa iva dvitīyābhāvo 'pi (ghaṭābhāvābhāvo 'pi) pratiyogī.* ([3], p. 15, 27f = [7], p. 81) – "So, the second absence (the absence of the absence of a pot) is essentially identical to the counterpositive (pot). The third absence (the absence of the absence of the absence of a pot) is essentially identical to the first absence (the absence of a pot). So, the second absence (the absence of the absence of a pot) is like 'pot' of the first absence (the absence of a pot) a counterpositive (author's note: The "second absence" *ghaṭābhāvābhāva* is the counterpositive of the "third absence" *ghaṭābhāvābhāvābhāva*.)."

(Id′) The relational absence of the relational absence of a pot is identical to "pot".[11]

(Id″) The relational absence of the relational absence of the relational absence of a pot is identical to the relational absence of a pot.[12]

[11] As noted by Matilal, the Navya-Naiyāyika Raghunātha Śiromaṇi rejected this identity. "Raghunātha, however, in his intensionalist vein, argued against the identification of x with $\sim\sim x$ (author's note: "$\sim$" is Matilal's abbreviation of "relational absence".). For, he thought, the notion of negation conveyed by the second can never be conveyed by the first, and hence it is difficult to think of them as non-distinct." ([10], p. 5) For the same reason Raghunātha would also not have been in favour of (Id), which was like (Id′) generally accepted in Navya-Nyāya (cf. [10], p. 7). Raghunātha's intuitions concerning properties are closer to Bealer's system T2 (cf. [2], p. 64f). In T2 axiom A8 of T1 is replaced by $\mathscr{A}8$: $[A]_\alpha = [B]_\alpha \to (A \leftrightarrow B)$ Moreover, T2 contains an axiom which coincides with Raghunātha's argument against (Id′), namely $\mathscr{A}9$: $t \neq r$ (where t and r are non-elementary complex terms of different syntactic kinds). According to our formalization techniques the left side of (Id′) is the negation of a property, whereas the right side should be interpreted as a non-negated property. Hence, they belong to different syntactic categories and therefore $\mathscr{A}9$ forces us to reject (Id′).

[12] Some other kinds of iterated absences might have been taken into account here, especially those starting with a difference, such as the difference from the absence of a pot or the difference from the difference from a pot. However, as noted by Mati-

In order to explicate the right side of (Id′) in an appropriate way one might substitute "pot" (*ghaṭa*) by "being a locus of a pot" (*ghaṭa-vattva*), since this is common practice in Navya-Nyāya (cf. [11], p. 115). After all, the property "being a locus of a pot" is equi-locatable with every pot. Even though the Navya-Naiyāyikas do regard expressions like *ghaṭa* and *ghaṭavattva* as interchangeable, this is not unproblematic, because a pot possesses potness, whereas the property "being a locus of a pot" does not.

Theorem (Id′):
$$[\neg\exists \mathbf{z}(\mathbf{z} = [\neg\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{z})]_{\mathbf{x}} = [\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$$

Since not only urelements other than loci of pots, but also all class-like individuals are no loci of pots, one might be inclined to regard the relational absence of a pot as a property which applies to all class-like individuals including properly class-like properties. However, as noted above, T1+ does not yield the existence of a hyper-class-like property which applies to every properly class-like property. Therefore our formalization of the relational absence of a pot restricts the range of loci to urelements and set-like individuals.

If the relational absence of a pot is supposed to be a property of all set-like properties and urelements other than loci of pots, one might argue that the relational absence of the relational absence of a pot applies to all properly class-like properties. However, since the existence of a hyper-class-like property which applies to every prop-

lal, these kinds of absences were ignored in Navya-Nyāya (cf. [10], p. 8), because they were regarded as "not very interesting" (ibid.). This is quite obvious from the perspective of our formalization techniques, because any iterated absence starting with a difference can be expressed as $[\mathbf{x} \neq \ldots]_{\mathbf{x}}$, where "..." stands for an absence or a difference. Only the dotted part might be reducible to a less complex term. A "difference" in the beginning of an iterated absence is invariant under the application of any reduction procedure. Hence, there is no coreferential syntactically simpler term which corresponds to "the difference from the absence of a pot" or "the difference from the difference from a pot". Each of these properties resides in everything except for one individual, namely the absence of a pot or the difference from a pot, respectively.

erly class-like property is not warranted by T1+, we have to impose the above-mentioned restriction on the formalization of the relational absence of the relational absence of a pot as well.

The proof of (Id′) contains an application of the following instantiation of (C):

$$\forall \mathbf{x}(P_s\mathbf{x} \lor U\mathbf{x} \to (\mathbf{x}\Delta[\neg\exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \leftrightarrow \neg\exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})))$$

Hence, a substitution of equivalents by means of the wff $\mathbf{x}\Delta[\neg\exists \mathbf{y}(P\mathbf{y}\land \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \leftrightarrow \neg\exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y}))$ is admissible if we require the variable $\mathbf{x}$ to range over urelements and set-like properties:

Proof of (Id′):

(A1)	$\neg\neg\exists \mathbf{y}(P\mathbf{x} \land \mathbf{x}L\mathbf{y}) \leftrightarrow \exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})$
(C)	$\neg\mathbf{x}\Delta[\neg\exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \leftrightarrow \exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})$
(1st-order logic)	$\neg\exists \mathbf{z}(\mathbf{z} = [\neg\exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \land \mathbf{x}\Delta\mathbf{z}) \leftrightarrow \exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})$
(R2, R3)	$\Box\forall \mathbf{x}(\neg\exists \mathbf{z}(\mathbf{z} = [\neg\exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \land \mathbf{x}\Delta\mathbf{z}) \leftrightarrow \exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y}))$
(A8, R1)	$[\neg\exists \mathbf{z}(\mathbf{z} = [\neg\exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \land \mathbf{x}\Delta\mathbf{z})]_{\mathbf{x}} = [\exists \mathbf{y}(P\mathbf{y} \land \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$ ■

In order to prove (Id″) and any other reduction rule which states the identity of an uneven number of such relational absences to a single relational absence, it suffices to prove:

(Id*) The relational absence of the property "being a locus of a pot" is identical to the relational absence of a pot.

By adding one relational absence on both sides of (Id′) we can infer from (Id′) that the relational absence of the relational absence of the relational absence of a pot is identical to the relational absence of <u>pot</u> (where the underlined "pot" is supposed to be explicated in the sense of "the property 'being a locus of a pot'"). On account of (Id*) the relational absence of "pot", i.e., of the property "being a locus of a pot", is identical to the relational absence of a pot, and this proves (Id″).

THEOREM (Id*):
$[\neg\exists \mathbf{z}(\mathbf{z} = [\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{z})]_{\mathbf{x}} = [\neg\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$

The proof contains an application of the following instantiation of (C):

$$\forall \mathbf{x}(P_s\mathbf{x} \vee U\mathbf{x} \rightarrow (\mathbf{x}\Delta[\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \leftrightarrow \exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y}))).$$

It is plausible to asume that neither of the members of the equivalence $\mathbf{x}\Delta[\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \leftrightarrow \exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$ in this formula is true of any $\mathbf{x}$ which fulfills the condition $\neg(P_s\mathbf{x} \vee U\mathbf{x})$, i.e., $\forall \mathbf{x}(\neg(P_s\mathbf{x} \vee U\mathbf{x}) \rightarrow \neg\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y}) \wedge \neg\mathbf{x}\Delta[\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}})$, because individuals which are neither set-like properties nor urelements are class-like properties, i.e., they are different from loci of pots and do not possess the property to be loci of pots. Hence, the equivalence $\mathbf{x}\Delta[\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \leftrightarrow \exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$ can be applied unconditionally in this case.

PROOF of (Id*):

(A1)	$\neg\exists \mathbf{y}(P\mathbf{x} \wedge \mathbf{x}L\mathbf{y}) \leftrightarrow \neg\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$
(C)	$\neg\mathbf{x}\Delta[\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \leftrightarrow \neg\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$
(1st-order logic)	$\neg\exists \mathbf{z}(\mathbf{z} = [\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{z}) \leftrightarrow \neg\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})$
(R2, R3)	$\Box\forall \mathbf{x}(\neg\exists \mathbf{z}(\mathbf{z} = [\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{z}) \leftrightarrow \neg\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y}))$
(A8, R1)	$[\neg\exists \mathbf{z}(\mathbf{z} = [\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}} \wedge \mathbf{x}\Delta\mathbf{z})]_{\mathbf{x}} = [\neg\exists \mathbf{y}(P\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$ ■

3.4 Sheffer stroke applied to properties

In Mathurānātha Tarkavāgīśa's Vyāptipañcakarahasyam (quoted in [9], p. 64f) this operation is named "conjoint absence" (*ubhayābhāva*).

Maheśa Chandra characterizes it as "an absence due to prefixing 'being both' ": *. . . paṭaghaṭobhayatvarūpeṇa vobhayatvapuraskāreṇābhāvo . . .* ([3], p. 14, 5f = [7], p. 76) – ". . . or an absence due to prefixing 'being both' in the form of 'being both, [i.e.] cloth and pot' . . . "

Following Bealer, who names property operators after their corresponding propositional operators, one can regard the Sheffer stroke applied to properties as the negation of the conjunction of properties. An example of such a property is the absence of both, cloth and pot, in a house where there is a cloth, but no pot. *evaṃ gṛhe kevalasya paṭasya sattve 'pi ghaṭasyābhāvena paṭaghaṭobhayasyāpy abhāvo 'sty eva. ekābhāvenobhayābhāvasyāvaśyaṃbhāvitvād . . .* ([3], p. 14, 8f = [7], p. 76) – "So, when there is only a cloth in the house, there is absence of both, a cloth and a pot <collectively>, because of the absence of a pot, because the absence of both <collectively> is necessary on account of the absence of one."

The condition that there is a cloth but no pot in the house can be formalized as . . .

$\exists \mathbf{y}(C\mathbf{y} \wedge hL\mathbf{y}) \wedge \neg\exists \mathbf{z}(P\mathbf{z} \wedge hL\mathbf{z})$ (where $C\mathbf{x}$ is to be read as "$\mathbf{x}$ is a cloth", $P\mathbf{x}$ as "$\mathbf{x}$ is a pot", $\mathbf{x}L\mathbf{y}$ as "$\mathbf{x}$ is a locus of $\mathbf{y}$" and h as "the house").

Now, if there is a cloth but no pot in the house, then it is not the case that there is a cloth and a pot in the house. This can be rendered as an implication with an alternative formalization of the consequent by means of the Sheffer stroke:

$$\exists \mathbf{y}(C\mathbf{y}\wedge hL\mathbf{y}) \wedge\neg\exists \mathbf{z}(P\mathbf{z}\wedge hL\mathbf{z}) \rightarrow \underbrace{\neg(\exists \mathbf{y}(P\mathbf{y}\wedge hL\mathbf{y})\wedge \exists \mathbf{z}(C\mathbf{z}\wedge hL\mathbf{z}))}_{\exists \mathbf{y}(P\mathbf{y}\wedge hL\mathbf{y})\uparrow \exists \mathbf{z}(C\mathbf{z}\wedge hL\mathbf{z})}$$

Since h denotes an urelement, we can apply (C) and substitute the consequent by an equivalent formula which expresses the fact that the house is a locus of the negation of the conjunction of the properties $[\exists \mathbf{y}(C\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$ ("being a locus of a cloth") and $[\exists \mathbf{z}(P\mathbf{z} \wedge \mathbf{x}L\mathbf{z})]_{\mathbf{x}}$

("being a locus of a pot"):

$h\Delta[\exists\mathbf{y}(C\mathbf{y} \wedge \mathbf{x}L\mathbf{y}) \uparrow \exists\mathbf{z}(P\mathbf{z} \wedge \mathbf{x}L\mathbf{z})]_{\mathbf{x}}$

The negation of each of the properties $[\exists\mathbf{y}(C\mathbf{y} \wedge \mathbf{x}L\mathbf{y})]_{\mathbf{x}}$ and $[\exists\mathbf{z}(P\mathbf{z} \wedge \mathbf{x}L\mathbf{z})]_{\mathbf{x}}$ yields the term for the corresponding absence, i.e., the absence of a cloth and the absence of a pot, respectively. Therefore it makes sense to regard the negation of their conjunction, i.e., $[\exists\mathbf{y}(C\mathbf{y} \wedge \mathbf{x}L\mathbf{y}) \uparrow \exists\mathbf{z}(P\mathbf{z} \wedge \mathbf{x}L\mathbf{z})]_{\mathbf{x}}$, as a formal equivalent of the "conjoint absence" (*ubhayābhāva*) of a cloth and a pot.

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Eberhard Guhe

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Fudan University, Shanghai
E–mail: eberhard@guhe.de