Identity, Ontology, and Frege's Problem

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Abstract

It is argued that the semantic difference between sentences of the form "$a = a$" and "$a = b$" reflects a difference in their truth-makers. My account of these truth-makers combines a referential semantics – singular terms are deemed to contribute nothing but their reference to the meaning of sentences in which these occur – with an ontology of logically complex individuals. Against analyses in which Fregean senses are invoked to account for the difference in meaning between (sentences of the form) "$a = a$" and "$a = b$", this ontology invokes an Aristotelian notion of *identity as oneness in substance* as the source of this difference, rendering Fregean senses otiose.

1 Introduction

Call a semantics for singular terms *extensionalist* if it embraces (A), and *classical* if it embraces (B):

(A) The meaning of a singular term is exhausted by its reference.

(B) The reference of a singular term is an entity that is logically simple.

Call such a semantics *inadequate* if it fails to show why assertions of the form "$a = a$" and "$a = b$" differ in meaning.

Fregeans react to the inadequacy of classical extensionalist semantics by rejecting (A). In contrast, I reject (B) – and therewith, the ontology of unanalyzable simples that (B) underwrites. This I replace with an ontology of logically complex individuals – I call these *complexes* – which provide distinct truthmakers for assertions of the form "$a = a$" and "$a = b$". The semantics I urge for singular terms, while

extensionalist in the sense of (A), is thus a non-classical semantics in which variables go proxy for, and singular terms denote, individuals endowed with a logical structure. For such individuals hold formal and material identity apart.

A framework of complexes unifies three types of phenomena. First, it sheds light on the defining properties of identity: symmetry, transitivity, weak reflexivity[1], and indiscernibility for identicals. Usually, such properties are described by axioms not susceptible of proof. However, when the undifferentiated individuals of standard identity theory cede pride of place to complexes, the customary axioms turn out to be consequences of principles more basic [6]. Second, this framework underwrites the difference between formal identity ($a = a$) and material identity ($a = b$), a difference which defies explanation from the standpoint of an ontology of unanalyzable simples. Finally, a framework of complexes shows forth the logical relation between identity and existence.

2   Frege's Problem

A criterion of adequacy for any semantic theory is its ability to settle Frege's problem. Here I explore an ontological dimension to this problem that linguistically-oriented analytic philosophers have failed to address.

Frege's problem is suggested by the commonplace that true identity statements of the form "$a = a$" and "$a = b$" mean different things. Thus, while (1) is purported to be analytic and a truism, (2) is synthetic and extends our knowledge:

(1) Scott = Scott.
(2) Scott = the author of "Waverley".

Yet "Scott" and "the author of 'Waverley'" refer to the same object. Should not (1) and (2) – both of which assert this object is identical with itself – therefore mean the same thing?

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[1]Strong reflexivity – $\forall x(x = x)$ – holds in classical domains only: weak reflexivity – $\forall x(\exists y(x = y) \leftrightarrow x = x)$ – holds in classical and non-classical domains [6].

Taking their cue from turn of the (twentieth) century writings by Gottlob Frege and Bertrand Russell, linguistically-oriented analytical philosophers have sought to discover what sets apart statements like "Scott is Scott" and "Scott is the author of 'Waverley'" without asking, What is it to *be* Scott or the author of "Waverley"? To this question I now turn.

3 F.H. Bradley on Singularity

Concerning what it is to be anything considered *real*, F.H. Bradley writes:

> If we take up anything considered real, no matter what it is, we find in it two aspects. There are always two things we can say about it; and if we cannot say both we have not got reality. There is a 'what' and a 'that', an existence and a content, and the two are inseparable. That anything should be, and should yet be nothing in particular, or that a quality should not qualify and give a character to anything is obviously impossible [4, p. 162].

According to Bradley, to be anything considered real is thus to be both a *that* and a *what*. *A fortiori*, to be Scott or the author of "Waverley" is to be a *that* and a *what*. More generally, to be an individual is to be a *that* and a *what*. How can these marks of individuality be appropriated for logic?

Call the bearer of Scott's properties a *substratum*[2], and identity-with-Scott a *thisness*[3]. To say Scott is both a *that* and a *what* is thus

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[2]**Substratum**. 'Substratum' signifies "something that is underneath (*sub*) a stratum, a layer, a mass, etc. – . The substratum is, then, a support, something that is involved in supporting another thing. Various 'supports' can be considered as forms or varieties of 'substratum': a substance (*sub-stancia*), a subject (*sub-iecto*), a supposition (*sub-positum*). To each of these can be given the name 'substratum', so that this term can be used to designate any of them. 'Substratum' can thus be used as the name in common of anything that is 'underneath'" [5, p. 749].

[3]A *thisness* ... "is the property of being identical with a certain particular individual – not the property that we all share, of being identical with some individual or

to say Scott comprises a substratum and thisness. Of course there is more *to* Scott – and to the author of "Waverley" – than substratum and thisness. However, to settle Frege's problem it suffices to take an individual as the resultant of a substratum and thisness.[4] An individual so constituted I will call a *complex*.

To represent individuals, substrata, thisnesses, and complexes requires four styles of variables and names. For an individual, I will underscore; for a substratum I will use lower-case italics; for a thisness I will use upper-case italics; and for a complex, I will use the operator "·" flanked by its operands. For Scott I will thus write "s̲"; for the substratum and thisness which constitute Scott, I will write "s" and "S"; and for the resultant of these I will write "$s \cdot S$". Similarly, for the author of "Waverley" and its avatars I will write "a̲w̲", "aw", "AW", and "$aw \cdot AW$".

4   Aristotle on Identity

My account of the difference in meaning between "s̲ = s̲" and "s̲ = a̲w̲" – and between assertions of formal and material identity generally – proceeds from a less-than-modern concept of identity and the things it relates. The concept in question: Aristotle's concept of *identity as oneness in substance*. According to Aristotle,

> "Essentially, things are identical in the same way as they are one, whether in matter... or in substance. So it is evident that identity... is oneness, whether of a plurality of things or of a single thing considered as two, as when a thing is said to be identical with itself" (Cited in [1] pp. 640-641)

Concerning Aristotle's view of identity as a kind of oneness, Nicholas White observes:

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other, but my property of being identical with me, your property of being identical with you, etc." [2, p 6].

[4]In contrast, set-theoretic semantics treats such entities as Scott and the author of "Waverley" as "bare individuals" with "no inner structure" [8, p. 186].

"...it is clear that in saying that X and Y are one, Aristotle does not *simply* mean that $[X$ and $Y]$ are parts of the same compound entity; he *also* means that $[X$ and $Y]$ are in some sense the same, *as each other*" [9, p. 187].

Aristotle, White goes on to say, is not "keeping separate the use of 'X and Y are one' to mean that they are in some way identical from its use to say that they make up a unitary entity" (Ibid).

5 Bradley, Aristotle, and Logic

How does the doctrine that an individual by its nature is both a substratum and thisness – and that identity is oneness in substance – bear on what holds apart "$\underline{s} = \underline{s}$" and "$\underline{s} = \underline{aw}$"? Taking $\underline{s}$ as $s \cdot S$ and $\underline{aw}$ as $aw \cdot AW$, (1) comes to (3) and (2) comes to (4):

(3) $s \cdot S = s \cdot S$

(4) $s \cdot S = aw \cdot AW$

But if identity is oneness in substance, (3) is true just in case there is some substance to which s and S both belong; while (4) is true just in case there is some substance to which s, S, aw, and AW all belong. So if $\underline{s}$ and $\underline{aw}$ are $s \cdot S$ and $aw \cdot AW$, and if identity is oneness in substance, an account of Frege's problem is near to hand that requires neither Fregean senses nor a Russellian distinction between grammatical and logical form. For "$\underline{s} = \underline{s}$" and "$\underline{s} = \underline{aw}$" then differ in meaning – as intuitively and pre-theoretically they appear to – because they assert different things.

In Figure 1 there is depicted what I take to be the difference between assertions of formal and material identity. Assertions of identity are true just in case the constituents of the things asserted to be identical are co-implected in substance. Hence "$\underline{s} = \underline{s}$" is true just in case s and S are co-implected in substance; and "$\underline{s} = \underline{aw}$" is true just in case s, S, aw, and AW are co-implected in substance.

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Individuals:   s  =  s      s  =  aw
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               |     |      |      |
Complexes:   s.S  =  s.S   s.S  =  aw.AW
               \    /       \      /
                \  /         \    /
                 \/           \  /
                 V             V
            SUBSTANCE   SUBSTANCE
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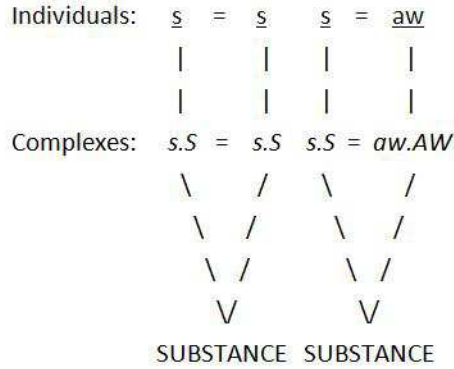
Figure 1. Formal identity vs material identity

6   Identity and Existence

In addition to showing what it is about individuals that sets apart formal and material identity, an ontology of complexes shows forth logical parameters for existence. An individual exists just in case the substratum and thisness which constitute it are co-implected in substance. Otherwise, it does not exist: the marks of an individual's existence are none other than those of its self-identity.[5]

References

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[5]According to Étienne Gilson, to exist was–for Plato–to be one and the same: there was "no difference whatsoever between being and self-identity" [3, p. 24]. "This seems also to be the force of Aristotle's contention that being and unity 'are the same and are one thing in the sense that they are implied in one another' (1003b). The being of a being, here determined as unity, holds elements together and makes a thing one, the same, self-identical, for precisely as long as it lasts. Greek ontology thus thinks of being in terms of identity and identity as self-identity, a unity or unification proper to the thing, its own nature" (Ibid).

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