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A New Full-Newton Step $O(n)$ Infeasible Interior-Point Algorithm for $P_*(\kappa)$-horizontal Linear Complementarity Problems

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Abstract

In this paper, we first present a brief review about the feasible interior-point algorithm for $P_*(\kappa)$-horizontal linear complementarity problems (HLCPs) based on new directions. Then we present a new infeasible interior-point algorithm for these problems. The algorithm uses two types of full-Newton steps which are called feasibility steps and centering steps. The algorithm starts from strictly feasible iterations of a perturbed problem, and feasibility steps find strictly feasible iterations for the next perturbed problem. By accomplishing a few centering steps for the new perturbed problem, we obtain strictly feasible iterations close enough to the central path of the new perturbed problem and prove that the same result on the order of iteration complexity can be obtained.

Keywords: Horizontal linear complementarity problem, infeasible interior-point method, central path.

1 Introduction

Interior-point methods (IPMs) have been studied for decades by many researchers. After Karmarkar's pioneer work on interior-point polynomial algorithm for linear programming (LP) [1], interior-point polynomial algorithms have been investigated by many researchers. For example, Ye and Tse [2] extended Karmarkar's algorithm for convex

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quadratic programming (CQP) and proved that it has polynomial complexity bound $O\left(n\log\left(\frac{1}{\varepsilon}\right)\right)$. IPMs also are the powerful tools to solve some widely used mathematical problems such as, semidefinite optimization (SDO) [3, 4] and linear complementarity problem (LCP) [5, 6, 7, 8]. These methods are so-called feasible IPMs. Feasible IPMs start with a strictly feasible interior point and keep feasibility during the solution process. Infeasible IPMs (IIPMs) start with an arbitrary positive point and feasibility is reached as optimality is approached. The choice of the starting point in IIPMs is crucial for the performance. Very recently, in [9, 10], Mansouri et al. presented the first full-Newton step IIPM for LCPs, which is an extension of the work for LP [11, 12, 13]. These algorithms use an intermediate problem which is a suitable perturbation of the given original problem so that at any stage the iterations are strictly feasible for the current perturbed problem. In each iteration the size of the perturbation decreases at the same speed as the barrier parameter μ. When μ changes to a smaller value, the perturbed problem also changes, and hence also the current central path. The iterations are kept feasible for the new perturbed problem and close to its central path. To achieve this, the algorithm uses a so-called feasibility step. This step serves to get iterations that are strictly feasible for the new perturbed problem and belong to the region of quadratic convergence of its μ^+-center, where μ^+ is the barrier parameter after updating. Now the algorithm can start from the point obtained in the feasibility step and perform few centering steps to obtain iterations that are close enough to the μ^+-center of the new perturbed problem. This process continues until the algorithm finds an ε-solution or detects that the problem has no optimal solution with zero duality gap. In this paper, we discuss an extension to HLCP of the just described algorithm, using Darvay's method [14]. We show that whose complexity is at least as good as the best known complexity of IIPMs. We use the results of analysis of the centering full Newton steps in [15].

The paper is organized as follows: in Section 2 we first recall some tools in the analysis of a feasible IPM that we use also in the analysis of IIPMs proposed in this paper. In Section 3 we describe an IIPM for HLCP. The analysis of the feasibility step of our method, the most tedious part of the analysis, is carried out in Section 4. In Section 5 we will derive a complexity bound for our algorithm. In section 6 some numerical results are presented. Finally, some concluding remarks follow in Section 7.

Some notations used throughout the paper are as follows. Vectors are denoted by lower-case Latin letters and matrices by capital Latin letters. $\mathbb{R}^n_+ (\mathbb{R}^n_{++})$ is the nonnegative (positive) orthant of $\mathbb{R}^n$. Further, X is the diagonal matrix whose diagonal elements are the coordinates of the vector x, i.e., $X = \operatorname{diag}(x)$, and I denotes the identity matrix of appropriate dimension. The vector $xs = Xs$ is the componentwise product (Hadamard product) of the vectors x and s, and for $\alpha \in \mathbb{R}$ the vector x^α denotes the vector whose i-th component is x_i^α . We denote the vector of ones by e. As usual, $\|\cdot\|$ denotes the 2-norm for vectors and matrices. $\min(x)$ (or $\max(x)$) denotes the smallest (or largest) value of the components of x. C^1 is the set of all continuously differentiable functions in $\mathbb{R}$. Finally, if $z \in \mathbb{R}^n_+$ and $f : \mathbb{R}_+ \to \mathbb{R}_+$, then $f(z)$ denotes the vector in $\mathbb{R}^n_+$ whose i-th component is $f(z_i)$, with $1 \leq i \leq n$. We write $f(x) = O(g(x))$ if $f(x) \leq cg(x)$ for some positive constant c.

2 Preliminaries

2.1 The HLCP problem

In the HLCP, we seek a vector pair $(x, s) \in \mathbb{R}^{2n}$ that satisfies the conditions

$$Qx + Rs = b, \quad (x, s) \geq 0, \quad x^T s = 0, \qquad (P)$$

where $b \in \mathbb{R}^n$, and $Q, R \in \mathbb{R}^{n\times n}$. The standard (monotone) linear complementarity problem (SLCP) is obtained by taking $R = -I$, and Q positive semidefinite matrix. The class P_* matrices are introduced by Kojima et al. [16]. The matrix pair (Q, R) belongs to P_* if there exists a constant $\kappa \geq 0$ such that

$$Qu + Rv = 0 \;\Rightarrow\; (1 + 4\kappa) \sum_{i\in\mathcal{I}^+} u_i v_i + \sum_{i\in\mathcal{I}^-} u_i v_i \geq 0 \quad \forall u, v \in \mathbb{R}^n,$$

where $\mathcal{I}^+ = \{i : u_i v_i > 0\}$ and $\mathcal{I}^- = \{i : u_i v_i < 0\}$. Then we say that the pair (Q, R) is a $P_*(\kappa)$-pair or equivalently the HLCP is called a $P_*(\kappa)$-HLCP. For $\kappa = 0$, $P_*(0)-$HLCP is called the monotone HLCP.

2.2 Central path for the $P_*(\kappa)$-HLCP

Because of nonnegativity of x and s in (P), solving HLCP is equivalent to finding a solution of the following system of equations:

$$\begin{array}{rcll} Qx + Rs & = & b, & x \geq 0, \\ xs & = & 0, & s \geq 0. \end{array} \tag{1}$$

The classical path-following IPMs consist in introducing a positive parameter μ. One considers the nonlinear system parameterized by μ:

$$\begin{array}{rcll} Qx + Rs & = & b, & x \geq 0, \\ xs & = & \mu e, & s \geq 0, \end{array} \tag{2}$$

where e denotes the all-one vector. It is shown in [17] that under interior-point condition (IPC), i.e., the existence of a vector pair $(x, s) > 0$ with $Qx + Rs = b$, there exists one unique solution $(x(\mu), (s(\mu))$. The path $\mu \to (x(\mu), (s(\mu))$ is called the central path. It is known that when $\mu \to 0$, $(x(\mu), (s(\mu))$ goes to a solution of (P).

2.3 Feasible full-Newton step

In this section we briefly present the feasible IPM in [15]. Consider the function

$$\varphi \in C^1, \, \varphi : \mathbb{R}_+ \to \mathbb{R}_+,$$

and suppose that the inverse function φ^{-1} exists. The system of equations in (2) can be written equivalently in the following form:

$$\begin{array}{rcll} Qx + Rs & = & b, & x \geq 0, \\ \varphi\left(\frac{xs}{\mu}\right) & = & \varphi(e), & s \geq 0. \end{array} \tag{3}$$

If we use Newton's method for linearizing the system (3), we get the following system for the search directions Δx and Δs:

$$\begin{array}{rcll} Q\Delta x + R\Delta s &=& 0, & x \geq 0, \\ \frac{s}{\mu}\varphi'\left(\frac{xs}{\mu}\right)\Delta x + \frac{x}{\mu}\varphi'\left(\frac{xs}{\mu}\right)\Delta s &=& \varphi(e) - \varphi\left(\frac{xs}{\mu}\right), & s \geq 0, \end{array}$$

which is equivalent to the following system:

$$\begin{array}{l} Q\Delta x + R\Delta s = 0, \qquad x \geq 0, \\ s\Delta x + x\Delta s = \mu\left(\varphi'\left(\frac{xs}{\mu}\right)\right)^{-1}\left(\varphi(e) - \varphi\left(\frac{xs}{\mu}\right)\right), \quad s \geq 0. \end{array} \tag{4}$$

We define the following notations:

$$v = \sqrt{\frac{xs}{\mu}}, \quad d_x = \frac{v\Delta x}{x}, \quad d_s = \frac{v\Delta s}{s}. \tag{5}$$

Then we have

$$\mu v\left(d_x + d_s\right) = x\Delta s + s\Delta x, \tag{6}$$

and

$$d_x d_s = \frac{\Delta s\Delta x}{\mu}. \tag{7}$$

Consequently we have the scaled Newton-system as follows:

$$\begin{array}{rcl} \bar{Q}d_x + \bar{R}d_s &=& 0, \\ d_x + d_s &=& p_v, \end{array} \tag{8}$$

where

$$\bar{Q} = QXV^{-1}, \quad \bar{R} = RSV^{-1}, \quad p_v = \frac{\varphi(e) - \varphi\left(v^2\right)}{v\varphi'\left(v^2\right)}.$$

If $\varphi(t) = t$, then $p_v = v - v^{-1}$, and we obtain the standard algorithm. Now we take $\varphi(t) = \sqrt{t}$ based on Darvay's technique for LP [14]. So we have

$$p_v = 2(e - v). \tag{9}$$

Then the systems (4) and (8) are equivalent to the following systems, respectively:

$$\begin{array}{rcll} Q\Delta x + R\Delta s & = & 0, & x \geq 0, \\ s\Delta x + x\Delta s & = & 2\mu v(e-v), & s \geq 0, \end{array} \tag{10}$$

and

$$\begin{array}{rcl} \bar{Q}d_x + \bar{R}d_s & = & 0, \\ d_x + d_s & = & 2\,(e-v)\,. \end{array} \tag{11}$$

We derive the new search directions d_x and d_s by solving (11) and then we compute Δx and Δs via (5). The new iterations are given by

$$\begin{array}{rcl} x^+ & = & x + \Delta x, \\ s^+ & = & s + \Delta s. \end{array} \tag{12}$$

In the analysis of the algorithm we use the norm-based proximity measure to the central path as follows:

$$\delta(v) := \delta(x, s; \mu) = \frac{\|p_v\|}{2} = \|e - v\|\,. \tag{13}$$

The algorithm starts with (x^0, s^0) such that $\delta(x^0, s^0; \mu^0) < \tau := \frac{1}{2(1+4\kappa)}$. In each iteration the search directions at the current iterations with respect to the current value of μ be computed and then (12) be applied to get new iterations. The algorithm terminates with a point in τ-neighborhood of the central path that satisfies $n\mu \leq \varepsilon$. The following lemmas are crucial in the analysis of the algorithm. We recall them without proof.

Lemma 2.1 (Lemma 7 in [15]). *Let* $\delta := \delta(x, s; \mu) \leq \frac{1}{\sqrt{1+4\kappa}}$ *and* $\mu^+ = (1-\theta)\mu$, *where* $0 < \theta < 1$. *Then*

$$\delta(x, s; \mu^+) \leq \frac{\theta\sqrt{n} + (1+4\kappa)\,\delta^2}{1-\theta \; + \sqrt{(1-\theta)\,(1-(1+4\kappa)\,\delta^2)}}.$$

Algorithm 1. Feasible IPM for $P_*(\kappa)$-HLCPs

Input:
Accuracy parameter $\varepsilon > 0$;
threshold parameter $\tau < 1$;
barrier update parameter θ, $0 < \theta < 1$;
feasible pair (x^0, s^0) with $(x^0)^T s^0 = n\mu^0$ and $\mu^0 > 0$ such that $\delta(x^0, s^0; \mu^0) \le \tau$.
begin
$x := x^0;\ s := s^0;\ \mu := \mu^0;$
while $n\mu \ge \varepsilon$ **do**
begin
$(x, s) := (x, s) + (\Delta x, \Delta s);$
$\mu := (1-\theta)\mu;$
end
end

Lemma 2.2 (Lemma 6 in [15]). *After a full Newton-step, one has*

$$(x^+)^T s^+ \le n\mu.$$

Lemma 2.3 (Lemma 5 in [15]). *Let* $\delta := \delta(x, s; \mu) \le \frac{1}{\sqrt{1+4\kappa}}$. *Then*

$$\delta(x^+, s^+; \mu) < \frac{(1+4\kappa)\delta^2}{1+\sqrt{1-(1+4\kappa)\delta^2}}.$$

Thus $\delta(x^+, s^+; \mu) \le \left(\sqrt{1+4\kappa}\ \delta\right)^2$, *which shows the quadratic convergence of the algorithm.*

The following result establishes a polynomial iteration bound of the above described algorithm; it easily follows from the above lemmas.

Theorem 1 (Theorem 1 in [15]). *If $\theta = \frac{1}{2(1+4\kappa)\sqrt{n}}$, then the algorithm requires at most*

$$4(1+4\kappa)\sqrt{n}\log\frac{n\mu^0}{\varepsilon}$$

iterations.

3 Infeasible full-Newton step IPM

In this section we present an infeasible interior-point algorithm that generates an ε-solution of $P_*(\kappa)$-HLCPs.

3.1 The perturbed problem and its central path

We use a triple $\left(x^0, s^0, \mu^0\right) > 0$ with $x^0 s^0 = \mu^0 e$ for some (positive) number μ^0 to start our IIPM. We denote the value of the residual at these initial points as r^0, as

$$r^0 = b - Qx^0 - Rs^0. \tag{14}$$

Now for any ν with $0 < \nu \leq 1$, we consider the perturbed problem (P_ν), defined by

$$b - Qx - Rs \;=\; \nu r^0, \qquad (x,\, s) \geq 0. \tag{P_ν}$$

Note that if $\nu = 1$, then $(x,\, s) = \left(x^0,\, s^0\right)$ yields a strictly feasible solution of (P_ν). We conclude that if $\nu = 1$ then (P_ν) satisfies the IPC.

Lemma 3.1. *Let the original problem (P) be feasible, then the perturbed problem (P_ν) satisfies IPC.*

Proof. The proof is similar to the proof of Lemma 4.1 in [9]. □

We assume that (P) is feasible. Letting $0 < \nu \leq 1$, Lemma 3.1 implies that the problem (P_ν) satisfies the IPC, and hence its central path exists. This means that the system

$$\begin{array}{rcl} b - Qx - Rs &=& \nu r^0, \quad x \geq 0, \quad s \geq 0, \\ xs &=& \mu e, \end{array} \tag{15}$$

has a unique solution, for every $\mu > 0$. In the sequel this unique solution is denoted by $(x(\mu,\nu),\, s(\mu,\nu))$. It is the μ-center of the perturbed problem (P_ν). Note that since $x^0 s^0 = \mu^0 e$, $(x^0,\, s^0)$ is the μ^0-center of the perturbed problem (P_1). In other words, $\left(x\left(\mu^0,1\right),\, s\left(\mu^0,1\right)\right) = \left(x^0, s^0\right)$. In the sequel the parameters μ and ν always satisfy the relation $\mu = \nu\mu^0$.

3.2 New feasibility search directions

For the search directions in the feasibility step we use the pair $\left(\Delta^f x, \Delta^f s\right)$ such that the new iterations

$$\begin{array}{rcl} x^f & = & x + \Delta^f x, \\ s^f & = & s + \Delta^f s. \end{array} \tag{16}$$

be feasible for (P_{ν^+}), where $\nu^+ = (1-\theta)\nu$. According to the definition of (P_ν), we have

$$b - Q(x + \Delta^f x) - R(s + \Delta^f s) = \nu^+ r^0, \quad \left(x + \Delta^f x,\, s + \Delta^f s\right) > 0.$$

Since $(x,\, s)$ is feasible for (P_ν), it follows that $\Delta^f x$ and $\Delta^f s$ should satisfy

$$Q\Delta^f x + R\Delta^f s = \theta\nu r^0.$$

Now we propose new feasibility search directions for HLCPs based on Darvay's method in LP [14]. Consider the function

$$\varphi \in C^1 \ , \qquad \varphi : \mathbb{R}_+ \to \mathbb{R}_+,$$

and suppose that the inverse function φ^{-1} exists. The system of equations in (15) can be rewritten equivalently in the following form:

$$\begin{array}{rcll} b - Qx - Rs & = & \nu r^0, & \qquad x \geq 0, \\ \varphi\left(\frac{xs}{\mu}\right) & = & \varphi(e), & \qquad s \geq 0. \end{array} \tag{17}$$

We define:

$$d_x^f = \frac{v\Delta^f x}{x}, \qquad d_s^f = \frac{v\Delta^f s}{s}, \tag{18}$$

where v is defined in (5). Let $\varphi(t) = \sqrt{t}$, so after using Newton's method and linearizing the system (17), we get the following system for the feasibility search directions $\Delta^f x$ and $\Delta^f s$:

$$\begin{array}{rcll} Q\Delta^f x + R\Delta^f s & = & \theta\nu r^0, & x \geq 0, \\ s\Delta^f x + x\Delta^f s & = & 2\mu v(e-v), & s \geq 0, \end{array} \tag{19}$$

and the following system for scaled feasibility search directions d_x^f and d_s^f:

$$\begin{array}{rcl} \bar{Q}d_x^f + \bar{R}d_s^f & = & \theta\nu r^0, \\ d_x^f + d_s^f & = & 2\left(e-v\right), \end{array} \tag{20}$$

where

$$\bar{Q} = QXV^{-1}, \quad \bar{R} = RSV^{-1}, \quad X = \mathrm{diag}\,(x), \quad S = \mathrm{diag}\,(s).$$

We derive the new search directions d_x^f and d_s^f by solving (20) and then we compute $\Delta^f x$ and $\Delta^f s$ via (18).

3.3 Description of the algorithm

Suppose that $\nu = 1$ and $\mu = \mu^0$. Then $(x,\, s) = \left(x^0,\, s^0\right)$ is the μ-center of the perturbed problem (P_ν). The algorithm is started by these iterations. We measure proximity to the μ-center of the perturbed problem (P_ν) by the quantity $\delta(v)$ as defined in (13). We assume that at the start of each iteration, just before the μ-update, $\delta(v)$ is smaller than or equal to a (small) threshold value $\tau > 0$. So this is certainly true at the start of the first iteration.

One (main) iteration of the algorithm works as follows. Suppose that for some $\mu \in \left(0,\, \mu^0\right)$ we have (x, s) satisfying the feasibility condition (15) for $\nu = \frac{\mu}{\mu^0}$, and $\delta(x, s, \mu) \leq \tau$. We reduce ν to $\nu^+ = (1-\theta)\,\nu$ and μ to $\mu^+ = (1-\theta)\,\mu$, with $\theta \in (0,1)$, and find new iterations $(x^+,\, s^+)$ that satisfy (15), with μ replaced by μ^+ and ν by ν^+, and such that $x^T s \leq n\mu^+$ and $\delta(x^+, s^+; \mu^+) \leq \tau$. In each main iteration first we accomplish one feasibility step to get iterations $\left(x^f,\, s^f\right)$

that are strictly feasible for (P_{ν^+}) and close enough to its central path, and then we perform a few centering steps to improve the closeness of $\left(x^f,\, s^f\right)$ to the central path of (P_{ν^+}) and obtain a pair $(x,\, s)$ that satisfies the condition $\delta\,(x,\, s;\; \mu) \leq \tau$. Since during the centering steps the iterations stay feasible for P_{ν^+}, it follows that for the analysis of the centering steps we can use the analysis for the feasible IPM, presented in Section 2.

Algorithm 2. Infeasible IPM for $P_*(\kappa)$-HLCP

Input:
  Accuracy parameter $\varepsilon > 0$;
  threshold parameter $\tau < 1$;
  barrier update parameter θ, $0 < \theta < 1$;
  feasible pair $\left(x^0,\, s^0\right)$ with $(x^0)^T s^0 = n\mu^0$ and $\mu^0 > 0$ such that $\delta\left(x^0,\, s^0,\, \mu^0\right) \leq \tau$.
begin
  $x := x^0;\; s := s^0;\; \mu := \mu^0;$
  while $\max(n\mu, ||r||) \geq \varepsilon$ **do**
  begin
    feasibility step:
      $(x,\, s) := (x,\, s) + (\Delta^f x,\, \Delta^f s);$
    μ-update:
      $\mu := (1-\theta)\mu;$
    centering steps:
    while $\delta(v) \geq \tau$ **do**
    begin
      $(x,\, s) := (x,\, s) + (\Delta x,\, \Delta s)\,;$
    end
  end
end

3.4 Some useful tools

In this subsection we present some technical lemmas which we need in the rest of the paper.

Lemma 3.2. *One has*

$$\min(v) \geq 1 - \delta(v).$$

Proof. Using (13), one has

$$\delta(v) = \|e - v\| \geq |1 - \min(v)| \geq 1 - \min(v).$$

which results the lemma. □

Lemma 3.3. *One has*

$$\|v\| \leq \sqrt{n} + \delta(v).$$

Proof. Due to Cauchy-Schwartz inequality, one has

$$e^T v \leq |e^T v| \leq \|e\| \, \|v\| \, .$$

Using the above inequality and (13), one has

$$\delta^2(v) = \sum_{i=1}^{n} (1 - v_i)^2 = \|v\|^2 - 2e^T v + n \geq (\|v\| - \|e\|)^2 \, ,$$

which completes the proof. □

4 Analysis of the feasibility step

In this section we present some conditions for strict feasibility of the feasibility step and an upper bound for the proximity function after a feasibility step.

Lemma 4.1. *The new iterations* $\left(x^f, s^f\right)$ *are strictly feasible if*

$$\left\| d_x^f d_s^f \right\|_\infty < 1 - \delta^2(v).$$

Proof. The proof of this Lemma is similar to the proof of Lemma 10 in [18] and therefore is omitted. □

To simplify the notation in the sequel we introduce

$$\omega(v) := \frac{1}{2}\left(\left\|d_x^f\right\|^2 + \left\|d_s^f\right\|^2\right).$$

Lemma 4.2. *If $\omega(v) < 1 - \delta^2(v)$, then the iterations $\left(x^f, s^f\right)$ are strictly feasible.*

Proof. The proof is similar to the proof of lemma 11 in [18]. □

Assuming $\omega(v) < 1 - \delta^2(v)$, which guarantees the strict feasibility of the iterations $\left(x^f, s^f\right)$. The next lemma gives an upper bound for $\delta\left(x^f, s^f; \mu\right)$.

Theorem 4.3. *If the new iterations $\left(x^f, s^f\right)$ are strictly feasible, then*

$$\delta\left(x^f, s^f; \mu^+\right) \leq \frac{\theta\sqrt{n} + \delta^2(v) + \omega(v)}{\sqrt{1-\theta}\left(\sqrt{1-\theta} + \sqrt{1-\delta^2(v) - \omega(v)}\right)}. \tag{21}$$

Proof. The proof is similar to the proof of Theorem 2 in [18] and therefore is omitted. □

Note that in the algorithm for using the quadratically convergence of the centering steps after doing the feasibility step we need that the following condition is satisfied

$$\delta\left(x^f, s^f; \mu^+\right) < \frac{1}{\sqrt{1+4\kappa}}. \tag{22}$$

Using Theorem 4.3, this is certainly true, if

$$\frac{\theta\sqrt{n} + \delta^2(v) + \omega(v)}{\sqrt{1-\theta}\left(\sqrt{1-\theta} + \sqrt{1-\delta^2(v) - \omega(v)}\right)} < \frac{1}{\sqrt{1+4\kappa}}. \tag{23}$$

By some elementary calculations, we obtain that if

$$\begin{aligned} \omega(v) &< \frac{1}{4\sqrt{1+4\kappa}}, &(24)\\ \delta(v) &< \tau < \frac{1}{4\sqrt{1+4\kappa}}, &(25)\\ 0 \le \theta &< \frac{1}{2n(1+4\kappa)}, &(26)\end{aligned}$$

then the inequality (23) is satisfied. In other words, the inequalities (24), (25) and (26) imply that the iterations $\left(x^f,\, s^f\right)$ are strictly feasible and lie in the quadratic convergence neighborhood with respect to the μ^+-center of (P_{ν^+}). We proceed by considering the value $\omega(v)$ in more detail.

4.1 An upper bound for $\omega(v)$

We start by finding some bounds for the solution of a linear system of the form

$$\begin{aligned} su + xv &= a,\\ Qu + Rv &= \tilde{b}. \end{aligned} \qquad (27)$$

Lemma 4.4 (Lemma 3.3 in [19])**.** *If HLCP be $P_*(\kappa)$, then for any $z = (x,s) \in \mathbb{R}^{2n}_{++}$ and any $a, \tilde{b} \in \mathbb{R}^n$ the linear system (27) has a unique solution $w = (u,v)$ and the following inequality is satisfied:*

$$\|w\|_z \le \sqrt{1+2\kappa}\,\|\tilde{a}\| + \left(1+\sqrt{2+4\kappa}\right)\xi(z,\tilde{b}),$$

where

$$\tilde{a} = (xs)^{-\frac{1}{2}}\ a, \quad \|w\|_z^2 = \|(u,v)\|_z^2 = \|Du\|^2 + \left\|D^{-1}v\right\|^2, \quad D = X^{-\frac{1}{2}}S^{\frac{1}{2}},$$

and

$$\xi(z,\tilde{b})^2 = \min\{\|(\tilde{u},\tilde{v})\|_z^2 : Q\tilde{u} + R\tilde{v} = \tilde{b}\}.$$

Comparing system (27) with (19) and considering $w = (u, v) = (\Delta^f x, \Delta^f s)$, $a = 2\mu v(e - v)$, $\tilde{b} = \theta\nu r^0$, $z = (x, s)$ in the system (27), we have

$$\left\| D\Delta^f x \right\|^2 + \left\| D^{-1}\Delta^f s \right\|^2 \leq \tag{28}$$
$$\leq \left(2\sqrt{1+2\kappa} \left\| (xs)^{-1/2} \left(\mu v(e-v) \right) \right\| + (1+\sqrt{2+4\kappa})\xi(z, \theta\nu r^0) \right)^2.$$

Note that

$$\left\| (xs)^{-\frac{1}{2}} (\mu v(e-v)) \right\| = \sqrt{\mu} \left\| e - v \right\| = \sqrt{\mu}\delta(v).$$

Also by definition of $\xi(z, \tilde{b})$, we have

$$\xi(z, \theta\nu r^0) = \theta\nu\xi(z, r^0).$$

By definitions of d_x^f and d_s^f, we obtain $D\Delta^f x = \sqrt{\mu}\ d_x^f$ and $D^{-1}\Delta^f s = \sqrt{\mu}\ d_s^f$. Substituting the above equations in (28), we have

$$\omega(v) \leq \frac{1}{\mu} \left(\sqrt{2\mu(1+2\kappa)}\delta(v) + \frac{1}{\sqrt{2}} \left(1 + \sqrt{2+4\kappa} \right) \theta\nu\xi(z, r^0) \right)^2. \tag{29}$$

To proceed, we have to specify our initial iterates $\left(x^0,\, s^0\right)$. We assume that ρ_p and ρ_d are such that

$$\|x^*\| \leq \rho_p, \quad \|s^*\| \leq \rho_d, \tag{30}$$

for some optimal solutions $(x^*,\, s^*)$ of (P), and as usual we start the algorithm with

$$x^0 = \rho_p\, e, \quad s^0 = \rho_d\, e, \quad \mu^0 = \rho_p\, \rho_d. \tag{31}$$

Note that for such starting points we have clearly

$$x^* - x^0 \leq \rho_p e, \tag{32}$$

$$s^* - s^0 \leq \rho_d e. \tag{33}$$

Now we find an upper bound for $\xi(z, r^0)$.

Lemma 4.5. *Let $\xi(\cdot,\cdot)$ be as defined in Lemma 4.4. Then we have*

$$\xi(z, r^0) \leq \sqrt{\frac{\rho_p^2}{\mu v_{\min}^2}\,\|s\|_1^2 + \frac{\rho_d^2}{\mu v_{\min}^2}\,\|x\|_1^2}\;.$$

Proof. By definition of $\xi(z, \tilde{b})$, we have

$$\begin{aligned} \xi(z, r^0)^2 &= \min\{\|(\tilde{u}, \tilde{v})\|_z^2 : Q\tilde{u} + R\tilde{v} = r^0\} \\ &= \min\{\|D\tilde{u}\|^2 + \left\|(D)^{-1}\tilde{v}\right\|^2 : Q\tilde{u} + R\tilde{v} = r^0\}. \end{aligned}$$

We also have

$$\begin{aligned} r^0 = b - Qx^0 - Rs^0 &= Qx^* + Rs^* - Qx^0 - Rs^0 \\ &= Q(x^* - x^0) + R(s^* - s^0), \end{aligned}$$

thus by applying (32) and (33) the following inequalities are satisfied

$$\begin{aligned} \xi(z, r^0)^2 &\leq \left\|D(x^* - x^0)\right\|^2 + \left\|D^{-1}(s^* - s^0)\right\|^2 \leq \\ &\leq \|\rho_p De\|^2 + \left\|\rho_d D^{-1} e\right\|^2 = \\ &= \rho_p^2 \left\|\sqrt{\frac{s}{x}}\right\|^2 + \rho_d^2 \left\|\sqrt{\frac{x}{s}}\right\|^2 \leq \rho_p^2 \left\|\sqrt{\frac{s}{x}}\right\|_1^2 + \rho_d^2 \left\|\sqrt{\frac{x}{s}}\right\|_1^2 = \\ &= \frac{\rho_p^2}{\mu} \left\|\sqrt{\frac{\mu}{xs}}\; s\right\|_1^2 + \frac{\rho_d^2}{\mu} \left\|\sqrt{\frac{\mu}{xs}}\; x\right\|_1^2 = \frac{\rho_p^2}{\mu} \left\|\frac{s}{v}\right\|_1^2 + \frac{\rho_d^2}{\mu} \left\|\frac{x}{v}\right\|_1^2 \leq \\ &\leq \frac{\rho_p^2}{\mu v_{\min}^2}\,\|s\|_1^2 + \frac{\rho_d^2}{\mu v_{min}^2}\,\|x\|_1^2\;. \end{aligned}$$

The proof is completed. □

In what follows we obtain some upper bounds for $\|x\|_1$ and $\|s\|_1$.

Lemma 4.6. *Let $(x,\, s)$ be feasible for the perturbed problem (P_ν) and $\left(x^0,\, s^0\right)$ as defined in (31). Then for any optimal solution $(x^*,\, s^*)$, we have*

$$\begin{aligned} &\nu \left(x^T s^0 + s^T x^0\right) \leq \\ &\quad \leq (1 + 4\kappa)\big(\nu^2 n\mu^0 + \nu(1 - \nu)\big((x^*)^T s^0 + (x^0)^T s^*\big) + x^T s\big)\,. \end{aligned}$$

Proof. Since $r^0 = b - Qx^0 - Rs^0$ and $b - Qx - Rs = \nu r^0$, by definition of perturbed problem, we have

$$\begin{aligned} & Q\,(\nu x^0 + (1-\nu)x^* - x) + R(\nu s^0 + (1-\nu)s^* - s) = \\ = \;& \nu(Qx^0 + Rs^0) + (1-\nu)(Qx^* + Rs^*) - (Qx + Rs) = \\ = \;& \nu(b - r^0) + (1-\nu)b - (b - \nu r^0) = 0. \end{aligned}$$

Thus if

$$\mathcal{I}^+ = \{i : \left(\nu x^0 + (1-\nu)x^* - x\right)_i \left(\nu s^0 + (1-\nu)s^* - s\right)_i > 0\},$$

and

$$\mathcal{I}^- = \{i : \left(\nu x^0 + (1-\nu)x^* - x\right)_i \left(\nu s^0 + (1-\nu)s^* - s\right)_i < 0\},$$

then the $P_*(\kappa)$ property implies that

$$\begin{aligned} &(1+4\kappa)\sum_{\mathcal{I}^+}(\nu x^0 + (1-\nu)x^* - x)_i(\nu s^0 + (1-\nu)s^* - s)_i + \\ &\quad + \sum_{\mathcal{I}^-}(\nu x^0 + (1-\nu)x^* - x)_i(\nu s^0 + (1-\nu)s^* - s)_i \geq 0. \end{aligned}$$

So we have

$$\begin{aligned} &\sum_{\mathcal{I}^+}(\nu x^0 + (1-\nu)x^* - x)_i(\nu s^0 + (1-\nu)s^* - s)_i + \\ &\quad + \sum_{\mathcal{I}^-}(\nu x^0 + (1-\nu)x^* - x)_i(\nu s^0 + (1-\nu)s^* - s)_i \geq \\ &\quad \geq -4\kappa \sum_{\mathcal{I}^+}(\nu x^0 + (1-\nu)x^* - x)_i(\nu s^0 + (1-\nu)s^* - s)_i. \end{aligned}$$

Thus we obtain

$$\begin{aligned} &[\nu x^0 + (1-\nu)x^* - x]^T[\nu s^0 + (1-\nu)s^* - s] \geq \\ &\quad \geq -4\kappa \sum_{\mathcal{I}^+}(\nu x^0 + (1-\nu)x^* - x)_i(\nu s^0 + (1-\nu)s^* - s)_i \geq \\ &\quad \geq \sum_{\mathcal{I}^+}\left(\nu^2 x_i^0 s_i^0 + \nu(1-\nu)(x_i^* s_i^0 + x_i^0 s_i^*) + x_i s_i\right) \geq \\ &\quad \geq -4\kappa\left(\nu^2 (x^0)^T(s^0) + \nu(1-\nu)((x^*)^T s^0 + (x^0)^T s^*) + x^T s\right). \end{aligned}$$

Since $(x^*)^T s^* = 0$, $s^T x^* + x^T s^* \geq 0$ and $s^T x^0 + x^T s^0 \geq 0$, we deduce that

$$\begin{aligned}
&-4\kappa\left(\nu^2 (x^0)^T (s^0) + \nu(1-\nu)((x^*)^T s^0 + (x^0)^T s^*) + x^T s\right) \leq \\
&\quad \leq [\nu x^0 + (1-\nu)x^* - x]^T [\nu s^0 + (1-\nu)s^* - s] = \\
&\quad = \nu^2 n\mu^0 + \nu(1-\nu)((x^*)^T s^0 + (x^0)^T s^*) - \nu(s^T x^0 + x^T s^0) + \\
&\quad\; + x^T s - (1-\nu)(s^T x^* + x^T s^*) + (1-\nu)(x^*)^T s^* \leq \\
&\quad\; \leq \nu^2 n\mu^0 + \nu(1-\nu)((x^*)^T s^0 + (x^0)^T s^*) - \nu(s^T x^0 + x^T s^0) + x^T s.
\end{aligned}$$

Therefore we have

$$\begin{aligned}
&\nu(x^T s^0 + s^T x^0) \leq \\
&\quad \leq (1+4\kappa)\big(\nu^2 n\mu^0 + \nu(1-\nu)\big((x^*)^T s^0 + (x^0)^T s^*\big) + x^T s\big).
\end{aligned}$$

The proof is completed. □

Lemma 4.7. *Let (x, s) be feasible for the perturbed problem (P_ν) and $\left(x^0, s^0\right)$ as defined in (31). Then we have*

$$\|x\|_1 \leq (1+4\kappa)\left(2n + \left(\sqrt{n} + \delta(v)\right)^2\right)\rho_p, \tag{34}$$

$$\|s\|_1 \leq (1+4\kappa)\left(2n + \left(\sqrt{n} + \delta(v)\right)^2\right)\rho_d. \tag{35}$$

Proof. Using Lemma 4.6 and Lemma 3.3, this lemma may be proved in the same way as the proof of Lemma 16 in [18]. □

Substituting (34) and (35) in Lemma 4.5 and noting Lemma 3.2 gives

$$\begin{aligned}
\xi(z, r^0) &\leq \sqrt{\frac{2\rho_p^2\rho_d^2}{\mu\left(1-\delta(v)\right)^2}(1+4\kappa)^2\left(2n + \left(\sqrt{n}+\delta(v)\right)^2\right)^2} = \\
&= \sqrt{\frac{2}{\mu}}\;\frac{(1+4\kappa)\rho_p\rho_d\left(2n + (\sqrt{n}+\delta(v))^2\right)}{(1-\delta(v))}.
\end{aligned} \tag{36}$$

Now by substituting (36) in (29), we have

$$\begin{aligned}\omega(v) \ &\leq \ \frac{1}{\mu}\Bigg(\sqrt{2\mu(1+2\kappa)}\delta(v) + \\ &+ \ \frac{\theta\nu\left(1+\sqrt{2+4\kappa}\right)(1+4\kappa)\rho_p\rho_d\left(2n+\left(\sqrt{n}+\delta(v)\right)^2\right)}{\sqrt{\mu}\left(1-\delta(v)\right)}\Bigg)^2 = \\ &= \ \Bigg(\sqrt{2(1+2\kappa)}\delta(v) + \\ &+ \ \frac{\theta\left(1+\sqrt{2+4\kappa}\right)(1+4\kappa)\left(2n+\left(\sqrt{n}+\delta(v)\right)^2\right)}{1-\delta(v)}\Bigg)^2. \end{aligned} \tag{37}$$

4.2 Value for θ

We have found that $\delta(v) < \frac{1}{\sqrt{1+4\kappa}}$ holds if the inequalities (24) and (25) and (26) are satisfied. Then by (37), inequality (24) holds if

$$\sqrt{2(1+2\kappa)}\delta(v) + \frac{\theta\left(1+\sqrt{2+4\kappa}\right)(1+4\kappa)\left(2n+\left(\sqrt{n}+\delta(v)\right)^2\right)}{1-\delta(v)} < \\ < \frac{1}{2\sqrt[4]{1+4\kappa}}.$$

Obviously, the left-hand side of the above inequality is increasing in $\delta(v)$. Using this, one may easily verify that the above inequality is satisfied if

$$\tau = \frac{1}{32(1+4\kappa)}, \quad \theta = \frac{1}{50n(1+4\kappa)^2}, \tag{38}$$

which is in agreement with (24) and (25).

Note that by Lemma 4.2, to keep the iterates (x^f, s^f) be feasible, the following condition must be satisfied

$$\omega(v) < 1 - \delta^2(v).$$

It follows from (37) that the above inequality certainly holds if

$$\sqrt{2(1+2\kappa)}\delta(v) + \frac{\theta\left(1+\sqrt{2+4\kappa}\right)(1+4\kappa)\left(2n+\left(\sqrt{n}+\delta(v)\right)^2\right)}{1-\delta(v)} < \\ < \sqrt{1-\delta^2(v)}. \quad (39)$$

It is easy to verify that, for the above inequality, the left side is monotone increasing according to $\delta(v)$, while the right hand side is monotone decreasing according to $\delta(v)$. Using (38), an upper bound for the left hand side of inequality (39) is 0.1969, while a lower bound for the right hand side of inequality (39) is 0.9995. In this case, we conclude that the iterate $(x^f,\, s^f)$ is strictly feasible.

5 Complexity analysis

Let $\delta\left(x^f,\, s^f;\, \mu^+\right) \le \frac{1}{2(1+4\kappa)}$, which is in agreement with (22). Starting at (x^f, s^f) we repeatedly apply full Newton steps until the $k-$iterate, denoted as $(x^+,\, s^+) := (x^k, s^k)$, satisfies $\delta\left(x^k,\, s^k;\, \mu^+\right) \le \tau$. We can estimate the required number of (centering) Newton steps by using Lemma 2.3. To simplify notations we define for the moment $\delta(v^k) = \delta\left(x^k, s^k; \mu^+\right)$, $\delta(v^0) = \delta\left(x^f, s^f; \mu^+\right)$ and $\gamma = \sqrt{1+4\kappa}$. Note that $\gamma \ge 1$. It then follows that

$$\delta\left(v^k\right) \le \left(\gamma\delta\left(v^{k-1}\right)\right)^2 \le \left(\gamma\left(\gamma\delta\left(v^{k-2}\right)\right)^2\right)^2 \le \\ \le \ldots \le \gamma^{2+4+\ldots+2^k}\left(\delta\left(v^0\right)\right)^{2^k}.$$

This gives

$$\delta\left(v^k\right) \le \gamma^{2^{k+1}-2}\left(\delta\left(v^0\right)\right)^{2^k} = \gamma^{-2}\left(\gamma^2\delta\left(v^0\right)\right)^{2^k} \le \left(\gamma^2\delta\left(v^0\right)\right)^{2^k}.$$

Using the definition of γ and $\delta\left(v^0\right) \le \frac{1}{2(1+4\kappa)}$ we obtain

$$\gamma^2\delta\left(v^0\right) \le \left(\sqrt{1+4\kappa}\right)^2 \frac{1}{2(1+4\kappa)} = \frac{1}{2}.$$

Hence we certainly have $\delta\left(x^+, s^+; \mu^+\right) \leq \tau$ if $\left(\frac{1}{2}\right)^{2^k} \leq \tau$. Taking logarithms at both sides, this reduces to

$$2^k \log_2 \frac{1}{2} \leq \log_2 \tau.$$

Thus we find that after no more than

$$\left\lceil \log_2 \left(\log_2 \frac{1}{\tau} \right) \right\rceil \tag{40}$$

centering steps we have iterations (x^+, s^+) that satisfy $\delta\left(x^+,\ s^+;\ \mu^+\right) \leq$ $\leq \tau$. Substituting the value of τ from (38) in (40) gives that in the algorithm, at most $\log_2\left(\log_2\left(32(1+4\kappa)\right)\right)$ centering steps are needed. Note that in each main iteration both the value of $x^T s$ and the norm of the residual are reduced by the factor $1-\theta$. Hence, the total number of main iterations is bounded above by

$$\frac{1}{\theta} \log \frac{\max\left\{\left(x^0\right)^T s^0,\ \left\|r^0\right\|\right\}}{\varepsilon}.$$

Due to (38) we may take

$$\theta = \frac{1}{50n(1+4\kappa)^2}.$$

Hence the total number of inner iterations is bounded above by

$$50n\left(1+4\kappa\right)^2 \log_2\left(\log_2\left(32(1+4\kappa)\right)\right) \log \frac{\max\left\{\left(x^0\right)^T s^0,\ \left\|r^0\right\|\right\}}{\varepsilon}.$$

Thus we may state without further proof the main result of the paper.

Theorem 5.1. *If (P) has an optimal solution $(x^*,\, s^*)$ such that $\|x^*\|_\infty \leq \rho_p$ and $\|s^*\|_\infty \leq \rho_d$, then after at most*

$$50n\left(1+4\kappa\right)^2 \log_2\left(\log_2\left(32(1+4\kappa)\right)\right) \log \frac{\max\left\{\left(x^0\right)^T s^0,\ \left\|r^0\right\|\right\}}{\varepsilon}$$

iterations, the algorithm finds an ε-solution of HLCP. □

Table 1. The number of iterations for the examples.

Examples	The number of iterations
6.1	107
6.2 with n=10	111
6.2 with n=20	117
6.2 with n=30	121

6 Numerical results

The algorithm was tested on some $P_*(0)$ (monotone) linear complementarity problems. So $R = -I$. For the algorithm, the initialization parameters ρ_p and ρ_d are assumed as described in Subsection 4.1, and $\tau = 0.031$, $\varepsilon = 10^{-4}$ and $\theta = 0.1$. Table 1 shows the number of iterations to obtain $\varepsilon-$solutions of the following examples with Algorithm 2.

Example 6.1.

$$Q = \begin{bmatrix} 1 & 0 & -0.5 & 0 & 1 & 3 & 0 \\ 0 & 0.5 & 0 & 0 & 2 & 1 & -1 \\ -0.5 & 0 & 1 & 0.5 & 1 & 2 & -4 \\ 0 & 0 & 0.5 & 0.5 & 1 & -1 & 0 \\ -1 & -2 & -1 & -1 & 0 & 0 & 0 \\ -3 & -1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \\ -1 \\ 1 \\ -5 \\ -4 \\ 1.5 \end{bmatrix}.$$

Example 6.2.

$$Q = \begin{bmatrix} 1 & 2 & 2 & \dots & 2 \\ 0 & 1 & 2 & \dots & 2 \\ 0 & 0 & 1 & \dots & 2 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}.$$

7 Concluding remarks and further research

In this paper, an infeasible full-Newton step method for solving $P_*(\kappa)$-HLCP is presented. Based on new search directions, we established that the complexity of the algorithm is at least as good as the currently best known iteration bound for infeasible methods. Future research could be done on representing the other type of infeasible interior-point algorithms by analyzing the algorithm with another function $\varphi(t) \in C^1$.

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