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Solving Problems in Various Domains by Hybrid Models of High Performance Computations

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Abstract

This work presents a hybrid model of high performance computations. The model is based on membrane system (P system) where some membranes may contain quantum device that is triggered by the data entering the membrane. This model is supposed to take advantages of both biomolecular and quantum paradigms and to overcome some of their inherent limitations. The proposed approach is demonstrated through two selected problems: SAT, and image retrieving.

1 Introduction

The present paper concerns definition and investigation of new computational models based on combination of biomolecular and quantum approaches. This new approach springs from practical needs of several disciplines delivering the hard tasks. Both existing quantum- and bio- models of calculation are widely used to solve hard tasks being not always satisfactory. Each paradigm has its own advantages and disadvantages. The proposed hybrid model supposes to compensate restrictions of existing models and to expose their benefits.

We will use membrane computing, or P systems [1] that was motivated by the structure and functioning of a living cell. The model is based on a cell-like hierarchical arrangement of membranes. (There is a variation named the tissue model that uses a non-hierarchical arrangement.) Membranes delimit compartments where objects presented by multisets, numbers, or strings evolve according to the given evolution rules. Many variants dependent on permitted rules and operations exist; we will not restrict ourselves by a particular type.

Quantum computing uses quantum properties to represent data and perform operations over them [2]. Each quantum computation inevitably includes non-quantum steps. Quantum algorithms always begin with the (non-quantum) preparation of the initial observable (classical) state. Then a sequence of quantum operations is applied to the system whose states are unobservable during the process. At the end, a (non-quantum) measurement is performed, and the quantum system collapses to its final observable state.

One of the first hybrid computational models was proposed by A. Leporati [3]. His hybrid of membrane and quantum systems develops previously introduced UREM (Unit Rules and Energy assigned to Membranes) P systems. The former adds "energy" to the objects in a membrane system, and rules can be applied to objects inside a membrane only if there is enough energy to do so. Quantum UREM P system changes objects and rules: objects are represented as pure states of a quantum system, and rules are quantum operators. The result is a hybrid computation device, a membrane system with quantum operations.

We propose a different variant of hybrid model that keeps the entire power of P systems. Our hybrid model is the classical P system framework in which two types of membranes coexist: classical membranes, and quantum membranes, the latter containing a quantum device inside. Entrance of an object into a quantum membrane triggers the quantum computation, while the entered object is available as data in the initial state of the quantum device.

The process of hybrid calculation is illustrated by solutions of two problems. These problems represent the opposite sides of problems range: from theoretical computing (SAT problem) to everyday practical application (medical image retrieval). Presentation of such different problems intends to demonstrate general potential of the proposed hybrid model.

This is an introductory paper dedicated to the proposed hybrid model of calculations. The article demonstrates the applicability and operability of the model on two selected examples. More applications and other aspects like estimations of efficiency and consumed resources, synchronization, hybrid simulator, etc., are subjects of further works.

2 P Systems and Hybrid Model

The first problem we selected to illustrate our construction is the SAT problem. The second selected problem is the image retrieval problem.

Transitional P systems with inhibitors. For SAT problem, we use non-cooperative transitional P systems with atomic inhibitors [4]. Although this class of systems is not even computationally complete [5], it fits well to illustrate the power of the hybrid model. We introduce here only the necessary definitions.

A non-cooperative transitional P system with atomic inhibitors with input is defined as a tuple

$$\Pi = (O, \Sigma, \mu, w_1, \ldots, w_m, R_1, \ldots, R_m, i_0),$$

where: O is a finite alphabet; $\Sigma \subseteq O$ is the input subalphabet; μ is a membrane structure (a rooted tree, traditionally represented by bracketed expression, e.g., $[\ [\ \ ]_2\ ]_1$ denotes membrane 2 in membrane 1, and the set of labels of membranes from μ is $H = 1, \ldots, m$); w_i, $i \in H$, are the initial multisets associated to regions i (directly inside the corresponding membrane), traditionally represented by strings over O (only the multiplicities of symbols being relevant, not their order); R_i, $i \in H$, are the sets of rules associated to regions i; and i_0 is the label of input membrane (an input multiset over Σ is added to the initial multiset in region i_0 into the starting configuration). In this paper, multisets w_i and sets R_i are omitted if i is a label of a quantum membrane.

The rules of the corresponding model are of the forms $a \to u$ or $a \to u|_{\neg b}$, where $a, b \in O$, $u \in (O \times Tar)^*$, $Tar = \{here, out\} \cup \{in_j \mid j\}$, and in this case j denotes a label of immediately inner membrane. The effect of a rule is replacing object a with a multiset of objects specified in the right side, in the regions specified by target indications (*here* may be omitted). A rule with inhibitor b is applicable whenever b absent. A transition step consists in parallel application of applicable rules to all possible objects (non-deterministically if there is a choice). The computation stops when no rules are applicable.

Symport/antiport tissue P systems. For the image retrieval problem, we use tissue P systems with symport/antiport rules. A tissue P system with symport/antiport rules with input is defined by a tuple

$$\Pi = (O, \Sigma, E, d, w_1, \cdots, w_d, R, i_0, o_0),$$

where: O is the alphabet; Σ is the input subalphabet; E is the set of objects occurring in the environment in infinitely many copies; d is the degree of the system ($H = \{1, \ldots, d\}$ is the set of labels of regions called cells, and the environment region is labeled by 0), w_i, $i \in H$, is the initial content of cell i; R is the set of rules; i_0 is the label of the input region (where a multiset over Σ is additionally placed in the beginning of the computation); o_0 is the label of the output region.

The rules have the form $(i, u/v, j)$, meaning that a multiset u may move from region i to region j, coupled with moving of a multiset v from region j to region i. The rules are applied non-deterministically, in the maximally parallel mode (i.e., no further rules can be applicable to the idle objects).

The structure of the tissue is deduced from rules. A rule $(i, u/v, j)$ means that the i-th and the j-th cells are neighboring.

Hybrid model. We now present the formal description of hybrid model

$$\beta = (\mathit{\Pi}, H_Q, N_Q, Inp, Outp, Q_1, \ldots, Q_m).$$

A hybrid system β is defined by a membrane system $\mathit{\Pi}$ (let us denote its membrane/cell label set as H and assume in the membrane case that membranes with labels in $H_E \subseteq H$ are elementary), where quantum devices Q_j are associated to the elements j of a subset of elementary membranes/cells $H_Q \subseteq H_E$. No membrane rules are associated to the objects inside the quantum membranes. N_Q is the maximum number of qubits in the quantum part of the model.

Special objects are used to transfer data in the quantum membrane and to obtain result of quantum calculation. They are collected in alphabets *Inp* and *Outp*. See Sec. 3 for details of this interaction.

3 Quantum Membranes for the Hybrid Computational Model

Our model is characterized by the existence of quantum membranes. We describe below their internal construction.

Notations. Our quantum device explores function $y = f(x)$. Suppose that x is an integer, $0 \leq x < 2^N$, that is, the argument of the function takes N qubits.

M is the size of the result: y is an integer, $0 \leq y < 2^M$.

We are provided with the initial data. Suppose that the membrane is entered by a K-bit integer z_0 $(0 \leq z_0 < 2^K)$. The quantum device begins to work as z_0 enters the membrane.

Suppose that intermediate data takes R qubits.

Quantum registers. Quantum calculation is to be reversible while function f is not always bijective. Therefore, we need quantum registers both for argument and for result, and we demand that the argument was restored by quantum implementation F of function f. The standard demand is

$$\mathsf{F} \left|x\right\rangle \left|y\right\rangle = \left|x\right\rangle \left|y \oplus f(x)\right\rangle , \tag{1}$$

where $\left|x\right\rangle_N$ is the argument register (the index denotes the size of N qubits), $\left|y\right\rangle_M$ is the result, $\oplus$ is the modulo 2 bitwise addition (bitwise **xor**).

Condition (1) implies the reversibility of transformation F. Moreover, F is its own inverse because:

$$\mathsf{FF} \left|x\right\rangle \left|y\right\rangle = \left|x\right\rangle \left|y \oplus f(x) \oplus f(x)\right\rangle = \left|x\right\rangle \left|y\right\rangle . \tag{2}$$

The quantum register $|z\rangle_K$ keeps initial data, and $|w\rangle_R$ keeps ancillary (intermediate) data. Because of inevitable entanglement, we can regard the quantum memory as one register of $N+M+K+R$ qubits.

The linearity and the reversibility of quantum transformations permit to organize the calculation in such a manner that we can ignore ancillary data $|w\rangle$ in our deductions, as we will see below.

Initialization. Before the quantum calculation starts, each qubit is to be set in one of basis states $|0\rangle_1$, or $|1\rangle_1$. The measurement operation is already embedded to be used after calculation, so we can apply it to our qubits.

As we use the measurement before the calculation, the qubits collapse in the basis states. Measurement is irreversible, therefore it is a classical operation.

Now we are to set qubits in the initial states. For example, if we want to set them in the state $|0\rangle$, we are to check the state of each qubit and invert $|1\rangle$. This is not a quantum transformation because it glues two orthogonal vectors together. (In other words: because of linearity, $\alpha|0\rangle+\beta|1\rangle$ would be transformed to $(\alpha+\beta)|0\rangle$ that implies $|\alpha+\beta|^2=1$, and $\alpha\beta=0$. The operation is not linear, or it cannot be applied to non-basis states.)

Usually, all qubits are initially set to $|0\rangle_1$. (Several quantum algorithms use different initial values though.) In our case, we will use non-quantum tools to prepare the state $|x\rangle|y\rangle|z\rangle|w\rangle = |0\rangle_N|0\rangle_M|z_0\rangle_K|0\rangle_R$. Here z_0 is the number that entered the quantum membrane and initiated the process.

As the last step of the initialization, all qubits from register $|x\rangle$ are at once transformed by one-qubit Hadamard transformation

$$\mathsf{H}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}. \tag{3}$$

As the result, register $|x\rangle$ gets the state

$$|x\rangle=\frac{1}{2^{N/2}}\sum_{0\le x<2^N}|x\rangle, \tag{4}$$

that corresponds to the equal probability of all possible values of argument x. This ends the initialization.

Interaction with membrane environment. The quantum membrane needs a binary number to be initialized. P systems are mostly supposed to work over multisets of objects. If this is the case we can use in the membrane part objects $Z_{i,b} \in Inp$, where $0 \leq i < K$ and $b = 0, 1$, where K is the size of the initial data $|z\rangle_K$ for the quantum device. If the quantum membrane is entered by $Z_{i,0}$ ($Z_{i,1}$), then the i-th qubit ot the quantum register $|z_i\rangle$ should be initialized to $|0\rangle$ ($|1\rangle$). We have several possible techniques:

1. We can demand that exactly K objects $Z_{i,b}$ with all $0 \leq i < K$ enter the quantum membrane simultaneously as we do it in this paper.
2. We can use $|0\rangle$ as the default initialization of $|z\rangle$ in the quantum device and require only $Z_{i,1}$ for selected values of i to enter simultaneously the quantum membrane.
3. We can use an additional object *Qtrigger* whose only mission is to start quantum calculation. In this case the simultaneous input of all $Z_{i,b}$ into the quantum membrane becomes not necessary.
4. We should not demand that $Z_{i,0}$ and $Z_{i,1}$ were mutually exclusive. For example, if the quantum membrane was entered by three instances of $Z_{2,0}$ and one instance of $Z_{2,1}$, then the second qubit of $|z\rangle$ is to be initialized as $|z_2\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$.
5. We can introduce even more objects to represent initial values of all qubits in the quantum device. For example, standard initialization of $|x\rangle$ would correspond to the entrance of both $X_{i,0}$ and $X_{i,1}$ for all $0 \leq i < N$, with obvious notations.

The output of quantum result y outside the quantum membrane is performed classically producing objects from *Outp* after the final measurement.

Quantum calculation and result. Applying F, we get into $|y\rangle$ superposition of values of function $f(x)$ at all possible values of the argument:

$$\mathsf{F}|x\rangle|0\rangle = \frac{1}{2^{N/2}} \sum_{0 \leq x < 2^N} |x\rangle |f(x)\rangle. \quad (5)$$

This is the quantum parallelism.

If this finishes the calculation, the following measurement reduces qubit states to basis and we get the result $|x_1\rangle\,|f(x_1)\rangle$. Here x_1 is a random integer between 0 and $2^N - 1$.

Nobody would mess with quantum calculations were they to produce only a value of the function at a random point. However, we could not stop after transformation F. Let us use simultaneously available values of function f at all values of its argument, and perform over $|f(x)\rangle$ another transformation G that produces some important information on all these values at once: $\mathsf{G}\,|x\rangle\,|f(x)\rangle = |x\rangle\,|\text{something important}\rangle$.

The quantum programmer should elaborately select unitary linear transformations F and G. They are applied using entanglement and quantum parallelism.

During the quantum calculation we got all values of function f but they are unobservable. We can then stop and get no more than one value $f(x_1)$, or we can continue losing the information on particular values of function f but obtaining some data on its more general properties. This is the uncertainty principle in quantum calculations.

Independence of ancillary qubits. Register $|z\rangle$ of initial data and the ancillary register $|w\rangle$ could be ignored as speaking on result of the calculation. This was done, for example, in equation (5). The necessary conditions are:

- after the calculation the qubits of $|z\rangle$ and $|w\rangle$ were not entangled with qubits of $|x\rangle$ and $|y\rangle$;
- resulting values of $|z\rangle$ and $|w\rangle$ do not depend on the initial values of $|x\rangle$ and $|y\rangle$.

The entanglement of all qubits during the calculation should take place because in the opposite case the unused qubits could be deleted from the construction.

The construction of a quantum computer shown in Fig. 1 guarantees this.

$\mathsf{V}_f^\dagger = \mathsf{V}_f^{-1}$ is the inverse transformation to V_f. C_M are M standard "controlled NOT" gates, where qubits from $|f(x)\rangle_M$ are control qubits.

An additional transformation is introduced as follows: V_f is re-

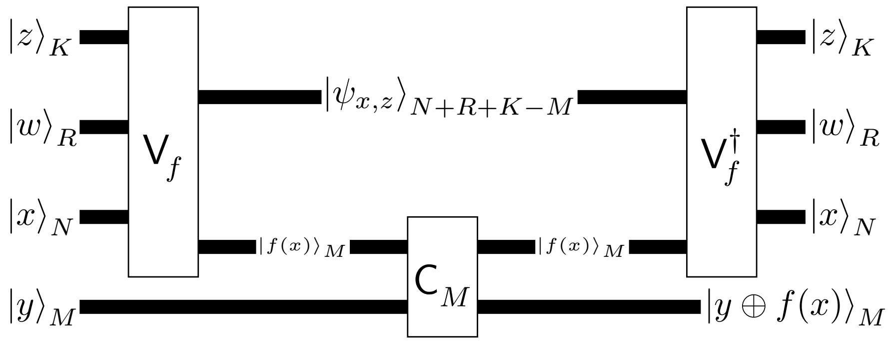

Figure 1. Quantum calculation into a membrane; initialization is not shown

placed by $\mathsf{V}_g\mathsf{V}_f$. The inverse transformations are applied in the reverse order: $\mathsf{V}_f^\dagger\mathsf{V}_g^\dagger$ (Fig. 2).

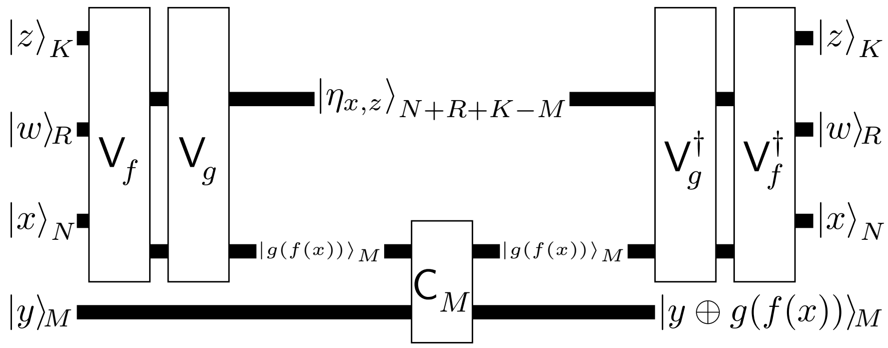

Figure 2. Quantum calculation with an additional transformation

Let us note that, in Fig. 2, transformation V_g can access only entangled qubits $|\psi\rangle$ and $|f(x)\rangle$ while the original $|x\rangle$ and $|z\rangle$ are unavailable. To correct this, we are to apply V_g exactly alike V_f after the termination of all calculations as in Fig. 1, so to speak, "as the second cascade". We need another ancillary register made from M qubits initialized by $|0\rangle$ (Fig. 3; the gray color emphasizes the part corresponding to Fig. 1).

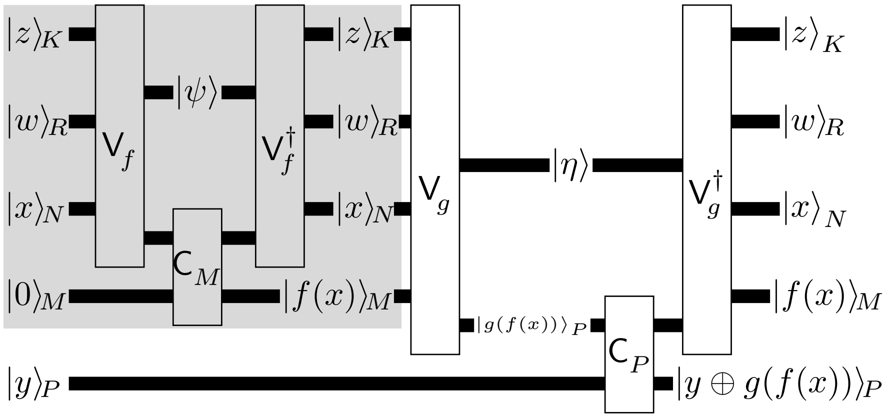

Figure 3. Additional transformation with access to the initial data

The size R of an ancillary register may grow at necessity. The size P of the result should not be equal to M. We get as the result not only $|g(f(x))\rangle$, but $|f(x)\rangle$ either. We can construct a chain of more than two transformations in this manner.

4 Satisfiability

SAT problem. A boolean formula in conjunctive normal form is an expression $\gamma = \bigvee_{1\leq j\leq m} C_j$, where $C_j = \bigwedge_{1\leq l\leq k_j} z_{l,j}$, and $z_{l,j} \in \{x_i, \neg x_i \mid 1 \leq i \leq n\}$, $1 \leq j \leq m$, $1 \leq l \leq k_j$. We assume the input is given by a set of objects from $\Sigma = \{x_{i,j}, \overline{x_{i,j}} \mid 1 \leq i \leq n, 1 \leq j \leq m\}$, each object representing appearance of variable x_i in clause C_j without negation in case of $x_{i,j}$ and with negation in case of $\overline{x_{i,j}}$.

Remark: if x_i does not appear in clause C_j in either form, this fact also needs to be explicitly present as an input to the quantum system we consider. However, it will be the job of membrane subsystem to detect this case and produce the corresponding object.

Membrane system. For the construction we need a bijection l from pairs $(i,j) \mid 1 \leq i \leq n,\ 1 \leq j \leq m$ onto numbers $\{k \mid 1 \leq k \leq mn\}$. We define it by $l(i,j) = (j-1)m + i$. We construct the following

P system (where membrane 2 is a quantum membrane):

$$\begin{aligned}
\Pi &= (O, \Sigma, \mu = [\ [\ \ ]_2\ ]_1, w_1 = s, R_1, i_0 = 1),\\
O &= \Sigma \cup \{I_{k,b}, I'_{k,b} \mid 1 \le k \le 2mn,\ 0 \le b \le 1\}\\
&\cup\ \{s, s', \texttt{yes}, \texttt{no}\} \cup \{y_{i,j} \mid 1 \le i \le n,\ 1 \le j \le m\},\\
R_1 &= \{x_{i,j} \to I'_{2l(i,j)-1,1} I'_{2l(i,j),0}\}\\
&\cup\ \{\overline{x_{i,j}} \to I'_{2l(i,j)-1,1} I'_{2l(i,j),1}\}\\
&\cup\ \{s \to y_{1,1} \cdots y_{n,m} s'\}\\
&\cup\ \{y_{i,j} \to I'_{2l(i,j)-1,0} I'_{2l(i,j),0}|_{\neg I'_{2l(i,j)-1,1}}\\
&\quad \mid 1 \le i \le n,\ 1 \le j \le m\} \cup \{s' \to \lambda\}\\
&\cup\ \{y_{i,j} \to \lambda|_{\neg s'} \mid 1 \le i \le n,\ 1 \le j \le m\}\\
&\cup\ \{I'_{k,b} \to (I_{k,b}, in_2)|_{\neg s'} \mid 1 \le k \le 2mn,\ 0 \le b \le 1\}.
\end{aligned}$$

The P system above takes three steps to prepare the input for the quantum system. The first step generates the input symbols corresponding to variables appearing in some clauses. The second step generates the input symbols corresponding to variables not appearing in some clauses. The third step erases intermediate objects and sends the input into the quantum membrane. The input z_0 consists of $K = 2mn$ bits, that are grouped in m groups of $2n$ bits each. Each of m groups corresponds to a clause, and each two bytes in these groups correspond to a variable. From these bytes, the first one is 0 if the variable is absent in the clause, and it is 1 if the variable is present in the clause. The second byte is 0 for variable x_i and 1 for its negation $\bar{x_i}$. It is obvious that combination 01 is impossible for these bytes. We will denote these bytes by z_p (**p**resence) and z_g (ne**g**ation).

Quantum system. The quantum calculation is quite straightforward and implements formula:

$$C = C \vee (z_p \wedge (z_g \oplus x))$$

to calculate clause C of the propositional form γ. Here C means any of C_j, x means any of x_i, and z_p and z_g mean the corresponding pairs of bits from z_0. γ is calculated as $\gamma = \gamma \wedge C$ for each clause C.

The necessary quantum circuits are shown in Fig. 4. They use standard cNOT and Toffoli gates, and inversion (**not**). We need ancillary qubits to make **and** and **or** reversible.

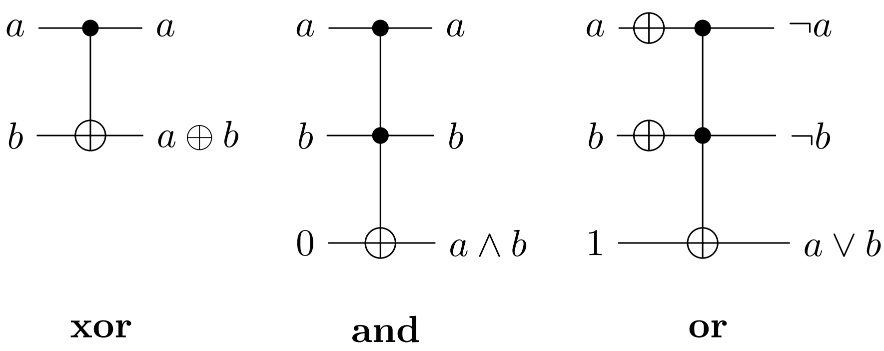

Figure 4. Quantum implementation of Boolean operations

Then Grover's algorithm is applied to search for 1 between possible values of γ. See a detailed step-by-step description in [7].

5 Image Retrieval

Many tasks of medical imaging are affected by image database size. Retrieval of images, similar to a given one, can be considered the most important one among these tasks. During the retrieval, extraction by attributes can be implemented mostly in parallel. The attribute vectors comparison can be also made in parallel for each image as we check the similarity over the whole database. Therefore, images retrieval problem is favorable for massive parallelism provided by unconventional computation, and its binary outputs are suitable for being given by quantum oracle. Basing on this, we chose image retrieval problem as relevant test example for hybrid computational model. It is obvious, however, that in general image retrieval problem is the monstrous task. In current work we only demonstrate how hybrid model can solve the essence subtask of retrieval – estimation of two images similarity.

Definitions. Let Img_{db} is one image from database of grayscale images. Img_u is the pattern image supplied by user. Both images have the size $N_w \times N_h$ pixels. The problem is to learn range of similarity of two images according to given similarity criterion $C_s im$. Let us assume for test purpose that $C_{sim} = N_{sim}/N_{ucont}$, where N_{sim} is the number of matching contour points, N_{ucont} is the total number of points in the contour of the user image.

Basing on general definition of hybrid model given in Sec. 2, the solution of images retrieval problem is represented by the following hybrid model:

$$\beta = (\Pi, H_Q, N_Q, Inp, Outp, Q_1, \ldots, Q_m),$$

where m is the database size as we will need m identical quantum devices for m database images.

For this task P system based calculation implements algorithm of grayscale image region-based segmentation proposed by the Spanish P system research group [6]. Thus, Π is a tissue-like P system. On account of calculation details do not concern the hybrid model functioning, let us give only brief scheme of algorithm. Each pixel is coded by integer representation of its associated grayscale value and mapped to the corresponding multiset object a_{ij} (b_{ij} – for the second image). Graphical-related basis of algorithm is the edge-based segmentation using the cross-like 4-adjacency.

We need two membranes to perform algorithm for each of two images and one H_Q membrane to proceed to retrieval, one more membrane is added to keep the answer.

$$\Pi(n, m) = (\mu, \Sigma, \varepsilon, w_{1u}, w_{2u}, w_{1db}, w_{2db}, q_1, \mathcal{R}, i_\Pi, o_\Pi),$$

where set of membranes is $\mu = [\ ]_1[\ ]_2[\ ]_3[\ ]_4[\ ]_5[\ ]_{collect}$; input alphabet is $\Sigma = \{a_{ij} : a \in \mathcal{C}, 1 \leq i \leq n;\ 1 \leq j \leq m\} \cup \{b_{ij} : a \in \mathcal{C}, 1 \leq i \leq n;\ 1 \leq j \leq m\}$; environment alphabet is $\varepsilon = \{\bar{a}_{ij} : a \in \mathcal{C}, 1 \leq i \leq n, 1 \leq j \leq m, a \in \mathcal{C}\} \cup \{A_{ij} : a \in \mathcal{C},\ 1 \leq i \leq n; 1 \leq j \leq m\, a \in \mathcal{C}\} \cup \{\bar{b}_{ij} : a \in \mathcal{C}, 1 \leq i \leq n,\ 1 \leq j \leq m, b \in \mathcal{C}\} \cup \{B_{ij} : b \in \mathcal{C}, 1 \leq i \leq n; 1 \leq j \leq m\, b \in \mathcal{C}\}$; $w_{1u}, w_{2u}, w_{1db}, w_{2db} = \emptyset$; $i_\Pi = 1$; $o_\Pi = collect$.

The only quantum system for one database picture is Q_1 that will be described below.

Now let us present set $\mathcal{R}$ of communication rules.

Firstly, the segmentation is implemented by P system based calculation using subset $\mathcal{R}_{psyst}$ of $\mathcal{R}$. The $\mathcal{R}_{psyst}$ is presented only for one image, because rules are identical, excepting replacement a by b. Communication rules of $\mathcal{R}_{psyst}$ are divided into types according segmentation steps.

Rules of type 1 look like $(1, a_{ij}b_{kl}/\bar{a}_{ij}A_{ij}b_{kl}, 0)$, where $a, b \in \mathcal{C}, 1 \leq i, k \leq n; 1 \leq j, l \leq m\}$. These rules identify the contour pixels by adjacency of different colors and produce the marks of the edge pixels.

After these marks appear, the rules of type 2 start to be applied in parallel with type 1 rules. Rules of type 2 are as follows:
$(1, \bar{a}_{ij}a_{ij+1}\bar{a}_{i+1j+1}b_{i+1j}/\bar{a}_{ij}\bar{a}_{ij+1}A_{ij+1}\bar{a}_{i+1j+1}b_{i+1j}, 0)$ $a, b \in \mathcal{C}, a < b, 1 \leq i \leq n-1, 1 \leq j \leq m-1$
$(1, \bar{a}_{ij}a_{i-1j+1}\bar{a}_{ij+1}b_{ij+1}/\bar{a}_{ij}\bar{a}_{i-1j}A_{i-1j}\bar{a}_{i-1j+1}b_{ij+1}, 0)$ $a, b \in \mathcal{C}, a < b, 1 \leq i \leq n; 1 \leq j \leq m-1$
$(1, \bar{a}_{ij}a_{ij+1}\bar{a}_{i-1j+1}b_{i-1j}/\bar{a}_{ij}\bar{a}_{ij+1}A_{ij+1}\bar{a}_{i-1j+1}b_{i-1j}, 0)$ $a, b \in \mathcal{C}, 1 \leq i, k \leq n; 1 \leq j, l \leq m-1$
$(1, \bar{a}_{ij}a_{i+1j}\bar{a}_{i+1j+1}b_{ij+1}/\bar{a}_{ij}\bar{a}_{i+1j}A_{i+1j}\bar{a}_{i+1j+1}b_{ij+1}, 0)$ $a, b \in \mathcal{C}, 1 \leq i, k \leq n-1; 1 \leq j, l \leq m-1$

These rules mark with the pixels that are adjacent to two pixels of the same color, which were marked by rules of type 1 but with the condition that the marked objects are adjacent to another pixel with a different color. Together with these operations the object representing the final border pixel is brought from the environment.

Finally, the rules of type 3 $(1, A_{ij}/\lambda, 2)$, for $1 \leq i \leq n, 1 \leq j \leq m$ are applied putting all the edge pixels A_{ij} in the output cells.

The only change, made in the algorithm from [6] to adopt it for retrieval, is the absence of final stage in which segmented image is restored from tissue P system. To prepare the enter in membrane of H_Q we only need the points of contours that divide segmented areas. They are stored in the resulting multiset A_{ij} (B_{ij}). Actually we have the set of contour points that are now independent from color. To solve retrieval problem, points B_{ij} obtained from Img_u have to be checked (in fully parallel mode) only on existence of contour point (points) A_{ij} in correspondent neighborhood for Img_{db}. The question of similarity is mapped to matching of criterion C_{sim} to threshold. This fact makes the problem similar to graph isomorphism one (for case of reduced graph), that has a number of quantum-based solutions in majority applying the adaptation of classical Grover search [7].

To manage H_Q calculation of this problem, we have to apply two extensions of classical Grover search algorithm. Firstly, we need the possibility of starting from an arbitrary state. The convergence of Grover search in this case is proved in [8] and iterations number is $\frac{\pi\sqrt{N}}{4}$. The second extension is a well known one: existence of several solutions. It is proved that Grover search algorithm converges with the same number of iterations even when the number of solutions is unknown, but only if any arbitrary solution is suited [9]. This is our case because presence of any contour point in the given neighborhood is enough for a positive answer.

The next subset $\mathcal{R}_q$ of $\mathcal{R}$ prepares the input for the quantum system. $\mathcal{R}_q = (2, A_{ij}/\lambda,)5, (4, B_{ij}/\lambda,)5$, for $1 \le i \le n, 1 \le j \le m$. Inp contains $A_{ij} \cup B_{ij}$, $Outp$ is $\{yes, no\}$ i.e. similar or not. The activation of H_Q membrane has to wait until segmentation of both images finishes.

The calculation inside H_Q membrane is implemented by the following way. Both user and database images contours are coded by integers $i, 0 < i < N_{cntr}$ that represent index number of the corresponding pixel in the image left-right and top-down. The $|x\rangle$ and $|y\rangle$ registers of H_Q membranes are activated by these tuples of integer. The search is provided for each integer in $|x\rangle$ having $|y\rangle$ as work register. Starting from description in work [7] where the close task is solved, the following Grover oracle is build:

$$f(x) = \begin{cases} 1, & \text{if } i_{leftnbh} < i < i_{rightnbh}; \\ 0, & \text{otherwise.} \end{cases}$$

In general the algorithm is:

1. Obtain the integer $|x_i\rangle$ from register $|x\rangle$
2. Use Grover Algorithm with oracle described above on the elements of $|y\rangle$ to search the contour point in given neighborhood.

3. **if** Grover search gives the positive answers, **then** $N_{sim}++$
4. **if** $i++ < N_{ucont}$, **then** return to step 1, else
5. the criterion $C_{sim} = N_{sim}/N_{ucont}$ is evaluated, then the answer yes/no is generated and passed into membrane $\mu_{collect}$.

This calculation can be repeated in parallel for all database images, collecting the answers in $\mu_{collect}$.

6 Conclusions

This work introduces our version of computational paradigm that combines both quantum and biological approaches. In the field of unconventional computing, quantum and biological paradigms were developed mostly in parallel but both are considered as the tools for hard tasks solutions. Solutions of some, mostly practical, hard tasks by pure quantum or pure bio-inspired methods could be inefficient. The idea of hybrid model springs from necessity of efficient computational models for such problems. We use P systems as the starting point at the hybrid model development. The proposed computational model applies the classical P system framework in which the quantum-style algorithms are interned.

We concentrated in this paper on demonstration of functioning of the proposed hybrid model. In our following works we will provide more detailed basis as well as the quantitative characteristics of the effectiveness of hybrid calculations. Simulation of the hybrid model will be also discussed. Here and now we just intend to show viability of membrane-quantum hybrid model. For this, we choose two problems of different patterns: the first problem belongs to theoretical computing, while the second is strongly practical one. Both problems show good applicability of the proposed hybrid model.

We presented here the hybrid model where the P system (macro) level is the main frame while quantum (micro) level is represented by membrane with quantum computation. The mutual accepting of input/output by computation models consists, on the current stage, in mapping of P system multisets to quantum basis states. Quantum computation is reduced in these problems to quantum oracle that answers *yes* or *no*.

We would provide in our further research mutually-inspired development of formal hybrid model description and practical solutions for more emerging and complicated tasks that can demonstrate the advantages of hybrid model.

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