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# Logical recognition in the space of multivalued attributes

A.Zakrevsky

**Abstract**

A logical approach to the problem of recognition is proposed, that is based on searching for implicative regularities, constructing a knowledge base and its applying to the computation of the goal attributes values. Sectional Boolean vectors and matrices have been introduced for representation of data and knowledge and used in operations of inductive and deductive inference.

## 1 Introduction

The approach to the recognition problem, suggested in this paper, is based on the following principles.

A space of discrete attributes is introduced for the description of a given subject area in terms of data and knowledge. The data presents some information concerning individual subjects, affirming, for example, that there exists a subject with a definite set of properties. The knowledge, on the contrary, presents information about qualities of the whole subject area and establishes some ties between attributes. It asserts the non-existence of subjects with some definite combinations of properties (corresponding to intervals of the space), declaring bans on them.

The data is obtained experimentally, the knowledge — either from an expert, or by means of inductive inference from the data presenting information about elements of some reliable selection from the subject area. This inference consists in looking for empty (not containing elements of the selection) intervals, putting forward corresponding hypotheses (suggesting emptiness of the intervals in the whole subject area), evaluating plausibility of these hypotheses and accepting the more plausible of them as regularities.

A set of regularities forms the contents of a knowledge base and can be subjected to equivalence transformations directed towards increasing the effectiveness of its using when solving different recognition problems.

The main recognition problem relates to a situation, in which some subject is considered with known values of some attributes and unknown ones of some others, including goal attributes. The possible values of the latters are to be calculated on the base of knowledge. By that a sufficiently high plausibility of such forecasting has to be guaranteed. This problem is solved by means of deductive inference of the type of theorem proving. The problem of minimizing of inference chains can be solved additionally, that arises when explanatory modules of expert systems are worked out.

The beginning of this approach was marked at the seminar on logical-combinatorial methods of pattern recognition, held near Kishinev in 1978 [1]. For the case of Boolean attributes there was proposed in [2–6] a united vector–matrix formalizm of data and knowledge representation and also of inductive and deductive inference, that was based on the notions and methods of the theory of Boolean functions. It was extrapolated in [7,8] on the case of multivalued attributes: sectional Boolean vectors and matrices were suggested for data and knowledge representation, and some results of the Boolean functions theory were transferred on the finite predicates. The proposed methods were used for working out several experimental recognition expert systems [9–12].

## 2 A general framework

When working out a recognition system one has, first of all, to choose a suitable world model, i.e. an abstraction, that can be used for the representation of some natural subject area $A$, preferably without great losses for solutions of the recognition problems.

Elaboration of such a model begins with a selection of attributes $S = \{s_1, s_2, \ldots, s_n\}$, typical for the area, and determining the sets $V_1, V_2, \ldots, V_n$ of their alternative values. The cartesian product of these sets forms the space of multivalued attributes $M = V_1 \times V_2 \times \ldots \times V_n$, and its elements serve as models of corresponding real subjects from $A$.

## 2.1 Data

It was suggested in [7] to represent elements of $M$ by sectional Boolean vectors having one 1 in each section: sections (domains) correspond to attributes, and their components — to attribute values. Quite naturally, the considered sets are ordered and the brackets { } used for their description are changed for ( ). In such a way the components of introduced vectors are put in the 1-1 correspondence with elementary sentences of the type "the attribute $s_i$ has the value $v_j$" and take the value 1 when the corresponding sentences are true.

For instance, when $S = (a, b, c)$, $V_1 = (1, 2, 3)$, $V_2 = (1, 2, 3)$, $V_3 = (1, 2, 3, 4)$, the vector 001.100.0100 represents a subject, for which $a = 3, b = 1$ and $c = 2$. This example illustrates a situation of complete information about the considered subject. However, in general case such information can turn out to be partial. We shall present it, inserting 1's into components corresponding to possible values of attributes and 0's — to impossible ones. By that choice a quantum of information is expressed by a sentence of the type "the value of the attribute $s_i$ does not coinside with $v_j$" and is presented in the vector by the value 0 of the corresponding component. For example, the vector 001.111.0110 means that for a considered subject $a = 3$, the value of $b$ is completely unknown (i.e. every of 1,2 and 3 is believed to be possible), and $c = 2$ or $c = 3$.

So when the information about some subject is complete, the subject is represented by a point in the space $M$; when the information is partial, some region of possible existence of the subject is determined. That region turns out to be an interval of the space $M$ — the cartesian product of non–emty subsets, taken by one from each $V_i$. For example, the vector 001.111.0110 represents an interval having six elements and obtained by multiplying the subsets $(3)$, $(1,2,3)$ and $(2,3)$. It can be regarded, too, as the characteristic function of this interval, represented by the conjunction

$$(a=3) \wedge ((b=1) \vee (b=2) \vee (b=3)) \wedge ((c=2) \vee (c=3)),$$

reduced to $(a=3) \wedge ((c=2) \vee (c=3))$, since $(b=1) \vee (b=2) \vee (b=3)=1$, and to the more compact expression $(a=3)(c=2,3)$.

Sectional Boolean vectors interpreted in such a way will be called vector–conjuncts further.

A situation is rather typical, when the initial information about a subject area is given as a list of some subjects constituting a random (but reliable) selection $F$ from $A$, $|F|=m$, that can be presented in the form of a sectional Boolean matrix $\underline{F}$ with rows describing individual subjects (having exactly one value 1 in each section). It is natural to call this information data.

## 2.2 Knowledge

The most popular recognition problem is to find the value of the goal attribute of some subject, when the values of some other attributes are given. It can be solved only on the base of knowledge — information about some regularities that influence all subjects of the area. These regularities forbid some sets of attributes values, forming together a forbidden region in the space $M$.

It was proposed in [3], where a world model with binary attributes has been considered, to present knowledge in the form of implicative regularities, connecting some attributes (prohibiting certain sets of their values) of each subject belonging to the investigated area. These regularities are expressed with sequents of the type $a\overline{c} \vdash b \vee \overline{d}$ (if some subject has the attribute $a$ and has not $c$, then it has $b$ or has not $d$), that can be easily transformed into degenerated forms: a ban–conjunct $a\overline{b}\overline{c}d$ (the sequent $a\overline{b}\overline{c}d \vdash 0$ is supposed) and a disjunct $\overline{a} \vee b \vee c \vee \overline{d}$ (i.e. $1 \vdash \overline{a} \vee b \vee c \vee \overline{d}$, or $\overline{a} \vee b \vee c \vee \overline{d} = 1$).

The proposed forms of regularity expressions were generalized in [7,8] for a world model with multivalued attributes. For example, when $S = (a, b, c)$, $V_1 = (1, 2, 3)$, $V_2 = (1, 2, 3)$ and $V_3 = (1, 2, 3, 4)$, the same regularity can be put down as the sequent $(a = 1)(b = 2, 3) \vdash (c = 2)$, or the ban–conjunct $(a = 1)(b = 2, 3)(c = 1, 3, 4)$, or the disjunct $(a = 2, 3) \vee (b = 1) \vee (c = 2)$.

The latter form is preferable, since it is more convenient for using in systems of deductive inference [13], and can be expressed in the vector form as 011.100.0100 (the vector–disjunct).

# 3 Inductive inference

The inductive inference of knowledge from data consists in the analysis of the initial information about a chosen subject area, putting forward some hypotheses concerning the character of connections between different attributes, evaluating their plausibility and taking some of them as regularities.

## 3.1 Putting forward hypotheses

Let us assume that data is given in form of the matrix $\underline{F}$, representing some set of points in the space $M$. Their distribution reflects regularities inherent in the subject area: to each regularity there corresponds an empty (not having any of these points) region of the space $M$. So, when some empty region is discovered, it can be supposed that it reflects a corresponding regularity. But such a conclusion is only a hypothesis, that can be accepted only when it is plausible enough.

The role of regions, mentioned above, will be played further by intervals of the attributes space, as precisely they are characteristic sets of ban–conjuncts, used for knowledge representation. That is why putting forward some hypotheses is preceded by the search for empty intervals in the space $M$. It is reasonable to consider by that only such intervals that are big enough, because it is intuitively evident that when an interval decreases the chances for it to be empty quite accidentally — increase. It is clear also that the probability of such an event decreases when the volume of the selection increases.

Let $g_i$ be an elementary statement of the type "the attribute $s_l$ has the value $v_k$" corresponding to the $i$–th column of $\underline{F}$, and $Q$ — the set of such statements corresponding to all columns. Let $F_i$ and $\overline{F}_i$ be the sets of such elements of the selection $F$, for which the statement $g_i$ is accordingly true or false, and $A_i$, $A_i$ — defined in the same way subsets of the subject area $A$.

The most strong regularities corresponding to the biggest intervals and connecting the values of one only attribute, can be expressed by the relations $F_i = F$ or $F_i = \emptyset$. They are easily detected, being reflected in the matrix $\underline{F}$ by degenerated columns having only 1's or only 0's. Let us call trivial the sentences that correspond to such columns.

Easily enough can be detected connections between two attributes. They are reflected on $Q$ by binary relations that can be expressed, in its turn, by corresponding characteristic Boolean functions of two variables.

**Theorem 1** *The variety of relations between non–trivial sentences $q_i$ and $q_j$ reduces to five relations, represented by corresponding Boolean functions:*

1) *disjunction relation* $R^{\vee}(g_i \vee g_j)$,

2) *implication relation* $R^{\rightarrow}(g_i \rightarrow g_j)$,

3) *Sheffer relation* $R^{|}(g_i | g_j = \neg(g_i g_j))$,

4) *dichotomy relation* $R^{\oplus}(g_i \oplus g_j)$,

5) *equivalence relation* $R^{\sim}(g_i \sim g_j)$.

Indeed, the four combinations of possible values of $g_i$ and $g_j$ give rise to 16 Boolean functions, but only six of them tie non–trivial $g_i$ and $g_j$. Two of them, $g_i \rightarrow g_j$ and $g_j \rightarrow g_i$, are evidently reduced to one: $g_i \rightarrow g_j$.

The same relations can be expressed in a different way:

1. $F_i \cup F_j = F$,
2. $F_i \subseteq F_j$,
3. $F_i \cap F_j = \emptyset$,
4. $F_i + F_j = F$,
5. $F_i = F_j$.

These relations can be represented by square Boolean matrix of the size $|Q| \times |Q|$ and easily (with the complexity $0(m|Q|^2)$) computed. The matrix $\underline{R}^{\vee}$ is computed in correspondence with the formulae $\underline{r}_i^{\vee} = \wedge\{\underline{f}_j / f_j^i = 0\}$, i.e. its $i$–th row is equal to componentwise conjunction of rows of $\underline{F}$ having 0 as a value of the $i$–th component. The other four matrices are found similarly: $\underline{r}_i^{\rightarrow} = \wedge\{\underline{f}_j / f_j^i = 1\}$, $\underline{r}_i^{|} = \neg(\vee\{\underline{f}_j / f_j^i = 1\}$, $\underline{r}_i^{\oplus} = \underline{r}_i^{\vee} \wedge \underline{r}_i^{|}$, $\underline{r}_i^{\sim} = \underline{r}_i^{\rightarrow} \wedge \neg(\vee\{\underline{f}_j / f_j^i = 0\})$.

Vector–disjuncts used for presentation of these relations contain 1's only in two domains corresponding to connected attributes. By that the relations $R^{\vee}$, $R^{\rightarrow}$ and $R^{|}$ are presented by one disjunct each, whereas $R^{\oplus}$ and $R^{\sim}$ — by two.

In more general case regularities can tie more than two attributes and be presented by vector–disjuncts having 1's in several domains.

Let us say that two Boolean vectors intersect if their componentwise conjunction has some 1's. It is evident, that if some interval of $M$ is empty, then the corresponding vector–disjunct intersects with every row of $\underline{F}$, i.e. it does not contradict $\underline{F}$. Just such a vector can present a hypothesis put forward when analysing the matrix $\underline{F}$.

It is reasonable to search for hypotheses beginning with small rancs of disjuncts (measured by the number of 1's in the vectors) and restricting the search with some threshould rank value, that depends, specifically, on the required level of hypotheses plausibility. By that for every considered rank value there are generated all possible vector–disjuncts (excludung trivial ones having some domain without 0's) and checked up for non–contradicting the matrix $\underline{F}$.

**Theorem 2** *A vector–disjunct does not contradict the matrix $\underline{F}$, iff the set of matrix columns, marked with 1's in the vector, forms a cover for $\underline{F}$ (i.e. contains at least one 1 in each row).*

### 3.2 Plausibility evaluation

Let us consider first the case of $n$ binary attributes and $m$ elements in a selection presenting the data. And let the Boolean space of attributes have some empty interval of the rank $r$ (having $2^{n-r}$ elements). It was proposed in [2] to evaluate the plausibility of the corresponding hypothesis by the function $W(n,m,r)$ representing the mathematical expectation of the number of empty intervals of the rank $r$ when the selection is taken by accident from the whole space $M$. It was shown that

$$W(n,m,r) = C_n^r\, 2^r\, (1-2^{-r})^m.$$

This quantity depends powerfully on $r$, and that facilitates the acceptance of decisions. For example, when $n = 100$ and $m = 200$, $W$ takes values $1.24 \cdot 10^{-58}$ for $r = 1$, $2.04 \cdot 10^{-21}$ for $r = 2$, $3.26 \cdot 10^{-6}$ for $r = 3$, $1.56 \cdot 10^{2}$ for $r = 4$, $4.21 \cdot 10^{6}$ for $r = 5$ and $3.27 \cdot 10^{9}$ for $r = 6$. It becomes clear that the search for empty intervals (and putting forward corresponding hypotheses) can be restricted in this case by the values $r < 4$.

This method was generalized later on the case of multivalued attributes [11].

Let $\underline{d}$ be a vector–disjunct, and $p$ — the probability of its being satisfied by an arbitrary (chosen by accident) element of the space $M$:

$$p = 1 - \prod_{i=1}^{n} (k_i/r_i),$$

where $r_i$ is the number of all values of the attribute $s_i$, and $k_i$ — the number of such of them, that do not belong to the disjunct (for instance, when $\underline{d} = 00.1000.101$, $p = 1 - 2/2 \cdot 3/4 \cdot 1/3 = 3/4$). The set of all possible disjuncts can be divided into classes $D_i$, containing disjuncts with equal values of $p$. Let us number the classes in order of increasing $p$ and introduce the following designations: $q_i$ — the number of disjuncts in the class $D_i$, $p_i$ — the value of $p$ for elements of $D_i$.

The mathematical expectation $W_i$ of the number of disjuncts from the class $D_i$, not conradicting the considered accidental selection is determined by the formula $W_i = q_i p_i^m$. The similar quantity for the union of the classes $D_1, D_2, \ldots, D_k$ is

$$W_k^+ = \sum_{i=1}^{k} W_i$$

Exactly this function has been proposed in [11] for evaluating hypotheses plausibility.

# 4 Deductive inference

The simplest problems of deductive inference arise when checking the matrices $\underline{D}$ and $\underline{F}$ for mutual non–contradictoriness and the matrix $\underline{D}$ alone — for inner non–contradictoriness, or consistence. The following statements are rather evident.

**Theorem 3** *The matrices $\underline{D}$ and $\underline{E}$ do not contradict each other, iff every row of $\underline{D}$ intersects with every row of $\underline{F}$.*

The same condition can be stated in a different way: any subset of columns of $\underline{D}$, marked with 1's in some row of $\underline{F}$, has to be a cover of $\underline{D}$ (having at least one 1 in each row).

**Theorem 4** *The matrix $\underline{D}$ is consistent iff there exists its cover, having not more than one column from each domain (let us call such cover a restricted one).*

## 4.1 Satisfiability problem for finite predicates

The latter theorem has much in common with the famous satisfiability problem expanded now toward finite predicates.

Indeed, the quality of consistency of $\underline{D}$ is equivalent to that of its satisfiability. The problem is $NP$–hard, but the time spent for its solving can be shortened considerably on the basis of reduction rules.

Let $\underline{u}$ and $\underline{v}$ be some rows of $\underline{D}$; $\underline{p}, \underline{q}$ — some of its columns, and $\underline{a} \geq \underline{b}$ ($\underline{a}$ and $\underline{b}$ — Boolean vectors of an equal size) denotes that $\underline{a}$ has 1's in all components where $\underline{b}$ has 1's. The following rules can be used for the reduction of $\underline{D}$ by eliminating some of its rows and columns:

1. if $\underline{u} \geq \underline{v}$, then the row $\underline{u}$ is removed;

2. if a row contains some full (not having 0's) domain, it is removed;

3. if a column is empty (without 1's), it is removed;

4. if there exists a row, having 1's in one domain only, then all columns of this domain, which have 0's in the row, are removed.

   These rules present equivalence transformations preserving the characteristic set of $\underline{D}$ (formed of those elements of $M$ that do not contradict $\underline{D}$). One more rule can be recommended for using, with one warning. The application of this rule can change the characteristic set, but does not violate the quality of satisfiability: any satisfiable matrix remains satisfiable, and any unsatisfiable one remains unsatisfiable.

5. if $\underline{p} \geq \underline{q}$ and both $\underline{p}$ and $\underline{q}$ belong to some common domain, then the column $\underline{q}$ is removed.

## 4.2 Some problems of deductive inference

Let $\underline{D}$ and $\underline{C}$ be some matrices of disjuncts, $\underline{u}$ and $\underline{v}$ — some individual disjuncts, and $E(\underline{D})$, $E(\underline{C})$, $E(\underline{u})$ and $E(\underline{v})$ — their characteristic sets.

We shall speak that $\underline{v}$ follows from $\underline{u}$ (i.e. $\underline{v}$ is a logical consequence of $\underline{u}$) iff $E(\underline{u}) \subseteq E(\underline{v})$, and denote that by $\underline{u} \to \underline{v}$. The relations $\underline{D} \to \underline{u}$ and $\underline{D} \to \underline{C}$ are determined in a similar way.

It is easy to show that $\underline{u} \to \underline{v}$ , iff $\underline{v} \geq \underline{u}$ (for example, $10.101.01 \geq 10.100.00$).

Let some $\underline{D}$ and $\underline{u}$ are given and the problem is to find out if $\underline{u}$ follows from $\underline{D}$.

We denote by $\overline{\underline{u}}$ the componentwise inversion of $\underline{u}$, and by $\underline{D} \wedge \overline{\underline{u}}$ — the matrix obtained by the componentwise conjunction of rows from $\underline{D}$ with $\overline{\underline{u}}$.

**Theorem 5** *$\underline{D} \to \underline{u}$, iff the matrix $\underline{D} \wedge \overline{\underline{u}}$ is inconsistent.*

For example, if

$$\underline{D} = \begin{bmatrix} 001.0010.00 \\ 110.0011.01 \\ 010.1100.10 \\ 001.0100.01 \end{bmatrix}$$

$$\underline{u} = \quad 011.1000.00,$$

then

$$\overline{\underline{u}} = \quad 100.0111.11$$

$$\underline{D} \wedge \overline{\underline{u}} = \begin{bmatrix} 000.0010.00 \\ 100.0011.01 \\ 000.0100.10 \\ 000.0100.01 \end{bmatrix}$$

It can be seen that the latter matrix is inconsistent, and that means that $\underline{u}$ follows from $\underline{D}$.

A more general problem can be formulated: to find all prime disjuncts that follow from $\underline{D}$. A disjunct is called prime if it does not follow from another disjunct that follows from $\underline{D}$. This problem can be solved by means of resolution operations determined as follows.

A disjunct $\underline{w}$ is called a resolvent for $\underline{u}$ and $\underline{v}$, if it follows from the pair $(\underline{u}, \underline{v})$, but does not follow either from $\underline{u}$ or from $\underline{v}$ taken separately. Disjuncts $\underline{u}$ and $\underline{v}$ are $i$–adjacent, iff their $i$–th domains are incomparable (their componentwise disjunction differs from each of them) and for each of the rest domains there exists a component that has 0 in both $\underline{u}$ and $\underline{v}$.

Let us introduce the operation $\underline{u}\langle i\rangle\underline{v}$ of the componentwise conjunction of the $i$–th domains of $\underline{u}$ and $\underline{v}$ and the similar disjunction of others, in pairs.

**Theorem 6** *If $\underline{u}$ and $\underline{v}$ are $i$–adjacent and $\underline{w} = \underline{u}\langle i\rangle\underline{v}$, then the disjunct $\underline{w}$ is a resolvent for $\underline{u}$ and $\underline{v}$.*

For example, the disjuncts $\underline{u}$ = 100.10.0011 and $\underline{v}$ = 010.00.0110 give rise to two resolvents: $\underline{u}\langle 1\rangle\underline{v}$ = 000.10.0111 and $\underline{u}\langle 3\rangle\underline{v}$ = 110.10.0010.

**Theorem 7** *The successive looking for adjacent pairs of rows on $\underline{D}$, adding their resolvents and removing those disjuncts which are not prime ones terminates in receiving the matrix $\underline{D}^+$ of all prime disjuncts.*

For instance, the matrix $\underline{D}$, shown above, in accordance with this theorem converts into

$$\underline{D}^+ = \begin{array}{cc} \left[\begin{array}{c} 001.0010.00 \\ 010.1100.10 \\ 000.0011.01 \\ 000.1110.10 \\ 011.0000.00 \\ 001.0000.01 \end{array}\right] & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \end{array}$$

## 4.3 Minimization of knowledge representation

Finding some matrix of disjuncts equivalent to $\underline{D}$, but having minimum number of rows, we minimize the contents of the knowledge base. This problem can be solved by means of a method, proposed in [14] as an extrapolation of the Quine's method [15] on finite predicates.

The solution is formed by some of prime disjuncts and is presented, in our example, by the matrix

$$\left[\begin{array}{c} 001.0010.00 \\ 000.0011.01 \\ 010.1100.10 \\ 001.0000.01 \end{array}\right]$$

In this particular case the optimal solution is unique, and it can be found rather easily, by checking in turns the rows of $\underline{M}^+$. It turns out that each of the rows except 4 and 5 is obligatory: it can not be excluded from the matrix without violating the characteristic set. As to the rows 4 and 5, they are consequences of the rest (see Theorem 4) and that is the reason for their exclusion.

## 4.4 Algorithms of recognition

The knowledge matrix $\underline{D}$ represents a system of disjunctive logical equations, solutions of which can be interpreted as admitted combinations of attributes values. In the recognition situation one more equation (a conjunctive one this time, presented by a vector–conjunct $\underline{r}$) is added to the system, cutting off some solutions.

**Theorem 8** *The joint system of $\underline{D}$ and $\underline{r}$ is equivalent to the disjunctive matrix $\underline{D}^* = \underline{D} \wedge \underline{r}$.*

In other words, $\underline{D}^*$ is obtained by means of changing 1's for 0's in all columns of $\underline{D}$ corresponding to components of $\underline{r}$ with value 0. The subsequent reduction of $\underline{D}^*$ can lead us to the solution of the recognition problem.

For example, if

$$\begin{array}{c} \quad a \qquad\quad b \quad c \\ \underline{D} = \begin{bmatrix} 001.0010.00 \\ 010.0011.01 \\ 010.1100.10 \\ 001.0000.01 \end{bmatrix} \end{array}$$

$$\underline{r} = 111.1111.10$$

(it is known only that c=1 for the subject), then

$$\underline{D}^* = \begin{bmatrix} 001.0010.00 \\ 000.0011.00 \\ 010.1100.10 \\ 001.0000.00 \end{bmatrix}$$

Reducing $\underline{D}^*$ according to the rules formulated in 4.1, we get the equivalent matrix

$$\begin{bmatrix} 000.0011.00 \\ 000.0000.10 \\ 001.0000.00 \end{bmatrix}$$

and it follows, that the initial vector-conjunct $\underline{r} = 111.1111.10$ can be changed for 001.0011.10 : $a = 3$ and $b = 3$ or 4.

In the general case the recognition problem can be solved by putting forward a series of questions of the type "can the subject presented by the vector–conjunct $\underline{r}$ have the value $v_j$ of the attribute $s_i$?" The answers are given by

**Theorem 9** *The subject can not have the value $v_j$ of the attribute $s_i$, iff the matrix $\underline{D}^* \wedge \overline{\underline{s}}(i,j)$ is inconsistent, where $\underline{s}(i,j)$ is the vector, that has the value 0 in all domains besides the $i$–th and the value 1 in all conponents of the $i$–th domain exept the $j$–th component.*

When predicting the value of some goal attribute, four different types of answers can be met:

1. the attribute receives a unique value (full success);
2. it turns out that all values are possible (failure);
3. several (but not all) values are possible (partial success);
4. no value is possible (inconsistence, i.e. $\underline{r}$ contradicts $\underline{D}$).

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A.D.Zakrevsky,
Institute of Technical Cybernetics,
Academy of Sciences, Belorussia,
6, Surganov str., Minsk,
220605, Belorussia,
phone: (017+2) 34-44-31

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