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Evaluation of the traffic coefficient in priority queueing systems

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Abstract

Methodology and algorithms of evaluation of the traffic coefficient in priority queueing systems with zero and nonzero switchover times are presented. In the case of zero switchover times the calculation of the traffic coefficient is straightforward. In contrast, it relies heavily on the efficient numerical evaluation of the busy period's Laplace-Stieltjes transform in the case when switchover times are not all degenerate zero. Examples for both cases are provided.

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Keywords: priority queues, switchover times, traffic coefficient.

1. Introduction

It is a usual practice to represent and to study real world phenomena processes by mathematical models. Among the latter there are models of priority queueing systems. The theory of priority queueing systems is concerned with the phenomena of prioritized servicing—the incoming requests should be classified by their importance and served according to assigned priority labels. In comparison with other queueing models the priority queueing systems have a more complicated structure, which limits the possibility of their exact analytical analysis. Thus, many results are derived considering the stationary behaviour of the system.

The traffic coefficient is an important measure of the performance of a queueing system and it is responsible for the workload of the system. Analysis of queueing systems delivers formulae for system performance characteristics—many of such analytical expressions involve the traffic coefficient ρ. In the case of priority queueing systems $M|G|1|\infty$ and $M_r|G_r|1|\infty$ with zero switchover times one can easily evaluate ρ via analytic formulae using the rates of incoming flows and mean values of corresponding service times. However, in the case of priority systems with random switchover times, one should be able to evaluate the Laplace-Stieltjes transforms (LST's) of the system busy period in order to estimate the value of the traffic coefficient. Generally, this can only be done numerically.

2. Priority Queueing Systems with Switchover Times

2.1 Description

Consider a queueing system with a single server and r classes of incoming requests, each having its own flow of arrival and waiting line. We call the requests from the i^{th} queueing line L_i i-requests. The i-requests have a higher priority than j-requests if $1 \leq i < j \leq r$. The server gives a preference in service to the requests of the highest priority among those presented in the system.

Suppose that the time periods between two consecutive arrivals of the requests of the class i are independent and identically distributed with some common cumulative distribution function (cdf) $A_i(t)$ with mean $\mathbb{E}[A_i]$, $i = 1, \ldots, r$. Similarly, suppose that the service time of a customer of the class i is a random variable B_i with a cumulative distribution function $B_i(t)$ having mean $\mathbb{E}[B_i]$, $i = 1, \ldots, r$.

It is assumed that the server needs some additional time to proceed with the switching from one priority waiting line of requests to another. This time is considered to be a random variable, and we say that C_{ij} is the time of switching from the service of i-requests to the service of j-requests, if $1 \leq i, j \leq r$, $i \neq j$.

We adopt the classification and the terminology introduced in [1, 2]. We also explain some additional notions and notations.

Definition 1. *By a kk-busy period call the period of time which starts when a k-request enters the empty system and finishes when there are no longer k-requests in the system. Denote the kk-busy period by Π_{kk}.*

Definition 2. *By a k-busy period call the period of time which starts when an i-request enters the empty system, $i \le k$, and finishes when there are no longer k-requests in the system. Denote the k-busy period by Π_k.*

Note, that an r-busy period is nothing but the system's busy period Π, i.e. $\Pi \equiv \Pi_r$.

The following two notions are due to the fact that both servicing and switching can be interrupted under preemptive service and switching policies.

Definition 3. *By a k-cycle of service call the period of time which starts when server begins the servicing of a k-request, and finishes when this request leaves the system. Denote the k-cycle of service by H_k. If the servicing of a certain k-request is not interrupted then the corresponding realization of H_k coincides with the time B_k this request was being serviced.*

Definition 4. *By a k-cycle of switching call the period of time which starts when the server begins the switching to the line of k-requests, and finishes when the server is ready to provide service to these requests. Denote the k-cycle of switching by N_k. If the switching from the i^{th} line to the k^{th} line is not interrupted, then the corresponding realization of N_k coincides with the time C_{ik} this ik-switching lasted.*

Let $\Pi_{kk}(t)$, $\Pi_k(t)$, $H_k(t)$ and $N_k(t)$ be the cumulative distribution functions of kk-busy periods, k-busy periods, k-cycle of service and k-cycle of switching, correspondingly. Let also $\pi_{kk}(t)$, $\pi_k(t)$, $h_k(t)$ and $\nu_k(t)$ be their Laplace-Stieltjes transform, i.e.

$$\pi_{kk}(s) = \int_0^\infty e^{-st} d\Pi_{kk}(t), \ldots, \ \nu_k(s) = \int_0^\infty e^{-st} dN_k(t).$$

Finally, let $\beta_i(s)$ be the Laplace-Stieltjes transform of $B_i(t)$, i.e.

$$\beta_i(s) = \int_0^\infty e^{-st} dB_i(t).$$

From now on throughout the text we assume that C_{ij} do not depend on i and only depend on j, i.e. $C_{ij} \equiv C_j$, $i = 1, \ldots, r$. Denote the Laplace-Stieltjes transform of C_j with cdf $C_j(t)$ by $c_j(s)$:

$$c_j(s) = \int_0^\infty e^{-st} dC_j(t).$$

2.2 Priority Queueing Systems with Poisson incoming flows

The queueing systems with Poisson incoming flows are of great importance in the theory and practice. In this case the interarrival times are exponentially distributed, i.e. $A_i(t) = 1 - e^{-a_i t}$, $i = 1, \ldots, r$, where a_1, a_2,..., a_r are some positive real numbers with the physical meaning of the flow arrival rates. The compound flow of the requests with priority not greater than k is Poisson with the arrival rate $\sigma_k = \sum\limits_{i=1}^{k} a_i$.

Using the extended Kendall notation we write $M_r|G_r|1|\infty$ to denote a priority queueing system with Poisson incoming flows of requests and random switchover times.

3. Traffic coefficient and its calculation

Analysis of queueinq systems delivers formulae for systems performance characteristic—many of such analytical expressions involve the traffic coefficient ρ.

3.1 Zero switchover times

In the case of priority queueing systems $M_r|G_r|1|\infty$ with degenerated switchover times one can easily evaluate ρ via analytic formulae using the rates of incoming flows and mean values of corresponding service times.

For instance, the traffic coefficient of the system $M_r|G_r|1|\infty$ can be calculated as follows [1]:

$$\rho = \sum_{i=1}^{r} a_i b_i, \tag{1}$$

where

- for the service scheme "repeat again"

$$b_i = \frac{1}{\sigma_{i-1}} \left[\frac{1}{\beta_i(\sigma_{i-1})} - 1 \right] \tag{2}$$

- for the service scheme "resume"

$$b_i = \mathbb{E}[B_i] \tag{3}$$

- for the service scheme "loss"

$$b_i = \frac{1}{\sigma_{i-1}} \left[1 - \beta_i(\sigma_{i-1}) \right]. \tag{4}$$

In the case when $\rho > 1$, the following takes place: $\pi(0) < 1$, and $\Pi(t)$ is an improper cumulative distribution function, i.e.

$$\lim_{t \to \infty} \Pi(t) < 1,$$

which means that the busy period is of indefinite length with a positive probability. However, if $\rho < 1$, then $\pi(0) = 1$ and the cdf $\Pi(t)$ of the busy period Π is proper. These comments motivate the presence of the quantity $\pi(0) \equiv \pi_r(0)$ in our further examples.

Remark 1. *It is easy to see that the value of the traffic coefficient in the system $M_r|M_r|1|\infty$ running under the service scheme "repeat again" is the same as the value of the traffic coefficient of the same system running under the service scheme "resume". To see this, one* *should compare (2) and (3). The LST's of the exponential service times* B_i *can be written as follows:*

$$\beta_i(s) = \frac{1}{s\mathbb{E}[B_i]+1}, \ i = 1, \ldots, r. \tag{5}$$

Calculation of b_i *in (2) using (5) shows that*

$$b_i = \frac{1}{\sigma_{i-1}}\left[\frac{1}{\beta_i(\sigma_{i-1})} - 1\right] = \frac{1}{\sigma_{i-1}}\left(\sigma_{i-1}\mathbb{E}[B_i] + 1 - 1\right) = \mathbb{E}[B_i],$$

i.e., the traffic coefficients for the priority systems with exponential switchover times under the service schemes "resume" and "repeat again" coincide (for both schemes ρ *should be calculated using (1) with the same values of* a_i*,* $i = 1, \ldots, r$*).*

Example 3.1.1. *Consider the system* $M_{10}|M_{10}|1|\infty$ *with zero switchover times. In this case*

$$\beta_i(s) = \frac{1}{s\mathbb{E}[B_i]+1}, \ i = 1, \ldots, 10.$$

Let $a_i = 1$ *and* $\mathbb{E}[B_i] = 0.05$*,* $i = 1, \ldots, 10$*. The results of our calculations of the traffic coefficient* ρ *and the LST of the busy period using the given algorithms can be found in Table 1. The approximation error* ϵ *is taken to be equal to* 0.001.

preemptive service schemes:	*repeat again*	*resume*	*loss*
ρ	0.5	0.5	0.413914
$\pi_{10}(0)$	0.999959	0.999959	0.999989

Table 1. Calculation of the traffic coefficient ρ and $\pi_{10}(0)$ in Example 3.1.1.

Example 3.1.2. *Consider* $M_{10}|G_{10}|1|\infty$ *with zero switchover times running under the service scheme "repeat again". Let* $a_i = 45$, *i=1,...,10. The service times of the requests from the queueing priority lines L_1, L_2, L_3, L_4 are exponential Exp*(20)*, the service times of the requests from the lines L_5, L_6, L_7, L_8 are uniformly distributed on the interval [0,1], and, finally, the service times of the requests from the lines L_9 and L_{10} are of Erlang type Er*$(2, 20)$*. The approximation error ϵ is taken to be equal to* 0.001*. Thus, the following holds:*

$$\beta_i(s) = \frac{20}{20+s},\ i = 1,2,3,4,$$
$$\beta_i(s) = 1 - e^{-s},\ i = 5,6,7,8,$$
$$\beta_i(s) = \left(\frac{20}{20+s}\right)^2,\ i = 9,10.$$

For this system the traffic coefficient ρ was numerically estimated to be equal to $104.0625 \gg 1$*, whereas* $\pi(0) \equiv \pi_{10}(0) = 0.429542 < 1$*. This clearly shows that the system is under the heavy traffic regime.*

3.2 Nonzero switchover times

Let us assume that at least one random variable C_j is not constant zero. In this case the the traffic coefficient can be calculated using the following formula [1]:

$$\rho = \sum_{k=1}^{r} a_k b_k,$$

where

$$b_1 = -\frac{\beta'(0) + c_1'(0)}{1 - a_1 c_1'(0)},$$

and

- for the service scheme “repeat again”

$$b_k = f_1 \dots f_{k-1} \frac{1}{\sigma_{k-1} c_k(\sigma_{k-1})} \left[\frac{1}{\beta_k(\sigma_{k-1})} - 1\right] \tag{6}$$

- for the scheme "loss"

$$b_k = f_1 \dots f_{k-1} \frac{1}{\sigma_{k-1} c_k(\sigma_{k-1})} \left[1 - \beta_k(\sigma_{k-1})\right] \qquad (7)$$

- for the service scheme "resume"

$$b_k = f_1 \dots f_{k-1} \frac{1}{c_k(\sigma_{k-1})} \mathbb{E}[B_k]. \qquad (8)$$

Here

$$f_1 = 1,$$
$$f_k = 1 + \frac{\sigma_{k-1} - \sigma_{k-1}\pi(a_k)}{\sigma_{k-1}} \left(\frac{1}{c(\sigma_{k-1})} - 1 \right),$$

where $\sigma_0 = 0$ and $\pi_k(t)$ is the Laplace-Stieltjes transform of the busy period $\Pi(t)$.

Value of ρ can be evaluated numerically. In what follows we present algorithms of numerical evaluation of the LST of the k-busy periods and the traffic coefficient ρ. For simplicity we only give the algorithms for the systems $M_r|G_r|1|\infty$ under the service scheme "repeat again" and preemptive switchover policy. In order to calculate ρ one needs to be able to evaluate the LST of the busy period (r-busy period). This can be done numerically using the following sample algorithm [3].

Algorithm 1 $BPLST_E$ ***(for the system*** $M_r|G_r|1$ ***with switchover times under the preemptive service scheme "repeat again")***

Input: r, s^*, $E > 0$, $\{a_k\}_{k=1}^r$, $\{\beta_k(s)\}_{k=1}^r$, $\{c_k(s)\}_{k=1}^r$.
Output: $\pi_k(s^*)$
Description:
IF (k==0) THEN $\pi_0(s^*) := 0$; *RETURN*
$k := 1$; $q := 1$; $\sigma_0 := 0$;
Repeat
inc(q);
$\sigma_q := \sigma_{q-1} + a_q$;
Until $q == r$;

Repeat

$\nu_k(s) := c_k(s^* + \sigma_{k-1})\{1 - \frac{\sigma_{k-1}}{s^*+\sigma_{k-1}}\,[1-$

$-c_k(s^* + \sigma_{k-1})]\,\pi_{k-1}(s^*)\}^{-1};$

$h_k(s^*) := \beta_k(s^* + \sigma_{k-1})\{1 - \frac{\sigma_{k-1}}{s^*+\sigma_{k-1}}\,[1-$

$-\beta_k(s^* + \sigma_{k-1})]\,\pi_{k-1}(s^*)\nu_k(s^*)\}^{-1};$

$\pi_{kk}^{(0)}(s^*) := 0;\ n := 1;$

Repeat

$\pi_{kk}^{(n)}(s^*) := h_k(s^* + a_k - a_k\pi_{kk}^{(n-1)});$

$inc(n);$

Until $|\pi_{kk}^{(n)}(s^*) - \pi_{kk}^{(n-1)}(s^*)| < E;$

$\pi_{kk}(s^*) = \pi_{kk}^{(n)}(s^*);$

$$\pi_k(s^*) := \frac{\sigma_{k-1}\pi_{k-1}(s^* + a_k)}{\sigma_k} + \frac{\sigma_{k-1}}{\sigma_k}(\pi_{k-1}(s^* + a_k - a_k\pi_{kk}(s^*)))-$$

$$-\pi_{k-1}(s^*+a_k)\nu_k(s^*+a_k[1-\pi_{kk}(s^*)])+\frac{a_k}{\sigma_k}\nu(s^*+a_k-a_k\pi_{kk}(s^*))\pi_{kk}(s^*);$$

$inc(k);$

Until $k == r;$

End of Algorithm 1 $BPLST_E$**.**

Remark 2. *Algorithm 1 is convergent. However, it does not provide one with the absolute error of the approximation. In this algorithm some quantity* E *is used to judge on the convergence of the Cauchy sequence* $\{\pi_{kk}^{(n)}(s^*)\}_{n=0}^{\infty}$.

Improved algorithms which evaluate the LST of the busy period with a certain precision are discussed in [3, 4]. We present next Algorithm $BPLST_\epsilon$, which was introduced and discussed in [3]. It is an improved algorithm of numerical evaluation of the LST of the busy period with a given precision and it is based on the acceleration scheme of solving the Kendall equation [5].

Algorithm 2 $BPLST_\epsilon$ ***(for the system*** $M_r|G_r|1$ ***with switch-over times under the preemptive service scheme "repeat again")***

Input: r, s^*, $\epsilon > 0$, $\{a_k\}_{k=1}^r$, $\{\beta_k(s)\}_{k=1}^r$, $\{c_k(s)\}_{k=1}^r$
Output: $\pi_k(s^*)$
Description:
IF (k==0) THEN $\pi_0(s^*) := 0$; *RETURN*
$k := 1$; $q := 1$; $\sigma_0 := 0$;
Repeat
inc(q);
$\sigma_q := \sigma_{q-1} + a_q$;
Until $q == r$;

Repeat

$\nu_k(s^*) := c_k(s^* + \sigma_{k-1})\{1 - \frac{\sigma_{k-1}}{s^*+\sigma_{k-1}}[1-$
$-c_k(s^* + \sigma_{k-1})]\,\pi_{k-1}(s^*)\}^{-1}$;

$h_k(s^*) := \beta_k(s^* + \sigma_{k-1})\{1 - \frac{\sigma_{k-1}}{s^*+\sigma_{k-1}}[1-$
$-\beta_k(s^* + \sigma_{k-1})]\,\pi_{k-1}(s^*)\nu_k(s^*)\}^{-1}$;

$\pi_{kk}^{(0)}(s^*) := 0$; $n := 1$;
Repeat
$\underset{\sim}{\pi}_{kk}^{(n)}(0) := 0$; $\widetilde{\pi}_{kk}^{(n)}(0) = 1$;

Repeat
$\widetilde{\pi}_{kk}^{(n)}(s^*) = h_k(s^* + a_k - a_k\widetilde{\pi}_{kk}^{(n-1)}(s^*))$;
$\underset{\sim}{\pi}_{kk}^{(n)}(s^*) = h_k(s^* + a_k - a_k\underset{\sim}{\pi}_{kk}^{(n-1)}(s^*))$;
inc(n);
Until $\frac{\widetilde{\pi}_{kk}^{(n)}(s^*) - \underset{\sim}{\pi}_{kk}^{(n-1)}(s^*)}{2} < \epsilon$;

$\pi_{kk}(s^*) := \frac{\widetilde{\pi}_{kk}^{(n)}(s^*)+\underset{\sim}{\pi}_{kk}^{(n-1)}(s^*)}{2};$

$$\pi_k(s^*) := \frac{\sigma_{k-1}\pi_{k-1}(s^*+a_k)}{\sigma_k} + \frac{\sigma_{k-1}}{\sigma_k}(\pi_{k-1}(s^*+a_k-a_k\pi_{kk}(s^*)))-$$

$$-\pi_{k-1}(s^*+a_k)\nu_k(s^*+a_k[1-\pi_{kk}(s^*)])+\frac{a_k}{\sigma_k}\nu(s^*+a_k-a_k\pi_{kk}(s^*))\pi_{kk}(s^*);$$

inc(k);
Until $k == r$;

End of Algorithm 2 $BPLST_\epsilon$**.**

The following is the model algorithm of calculation of the traffic coefficient for the priority queueing systems with switchover times [3].

Algorithm 3 WLCOEEF (*for the system* $M_r|G_r|1$ *with switchover times under the preemptive "repeat again" service scheme*)

Input: r, $\{a_k\}_{k=1}^r$, $\{\beta_k(s)\}_{k=1}^r$, $\{c_k(s)\}_{k=1}^r$.
Output: ρ
Description:
$k := 1$; $\rho := 1$; $\sigma_0 := 0$; $\sigma_1 := a_1$;
$f_1 := 1$; $p := 1$;
$b_1 := -(\beta'(0) + c_1'(0))/(1 - a_1 c_1'(0))$;
$\rho := a_1 b_1$;

Repeat
inc(k);
$\sigma_k := \sigma_{k-1} + a_k$;

$b_k := p\frac{1}{\sigma_{k-1}c_k(\sigma_{k-1})}\left(\frac{1}{\beta_k(\sigma_{k-1})} - 1\right)$;

$\rho := \rho + a_k b_k$;

$f_k := 1 + \frac{\sigma_k - \sigma_{k-1}\pi_{k-1}(a_k)}{\sigma_{k-1}} \left(\frac{1}{c_k(\sigma_{k-1})} - 1 \right)$;

$p := f_k p$;

Until $k == r$;

End of Algorithm 3 WLCOEEF.

Remark 3. *Calculation of ρ in Algorithm 3 requires calculation of $\pi_{k-1}(a_i)$, $k = 2, \ldots, r$, which, in turn, can be realized using Algorithm 1 or Algorithm 2. For a different scheme of service or switching policy one should employ the corresponding formulae for the LST's $h_k(s)$, $\nu_k(s)$, $\pi_{kk}(s)$, $\pi_k(s)$ (see [1]).*

Example 3.2.1. *Consider the system $M_5|M_5|1$ with all interarrival times being distributed exponentially Exp(10) and all service times being distributed exponentially Exp(200). The switchover times C_k are all distributed as Exp(100), $k = 1, \ldots, 5$; the service scheme is "repeat again". The results of calculations for such systems can be found in Table 2. The quantity ϵ is taken to be* 0.001.

	$BPLST_E$	$BPLST_\epsilon$	*difference*
$\pi_1(10)$	0.859847	0.858993	0.000854
$\pi_2(10)$	0.839391	0.838905	0.000486
$\pi_3(10)$	0.813851	0.813453	0.000398
$\pi_4(10)$	0.781973	0.781613	0.000360
ρ	0.442225	0.442271	-0.000046
$\pi_5(0)$	0.999874	0.999385	0.000489

Table 2. Calculation results for k-busy periods and the traffic coefficient for the system from Example 3.2.1

Example 3.2.2. *Consider the system $M_{10}|M_{10}|1$ with all interarrival times being distributed exponentially Exp(a_k), $k = 1, ..., 10$, and all* *service times being distributed exponentially Exp(200). The switchover times C_k are all distributed as Exp(100), $k = 1, \ldots, 10$. The results of calculations for such systems can be found in Tables 3, 4. The quantity ϵ is taken to be* 0.001*. In this case:*

$$\beta_k(s) = \frac{1}{s\mathbb{E}[B_k] + 1},$$

$$c_k(s) = \frac{1}{s\mathbb{E}[C_k] + 1},$$

$$k = 1, \ldots, 10.$$

a_k	$\rho(BPLST_E)$	$\rho(BPLST_\epsilon)$	*difference*
1	0.064175	0.064175	0.000000
10	1.806396	1.807565	-0.001169
100	7213724.500000	7212584.000000	1140.500000

Table 3. Calculation results for the traffic coefficient for the systems from Examples 3.2.2 (scheme "repeat again").

a_k	$\rho(BPLST_E)$	$\rho(BPLST_\epsilon)$	*difference*
1	0.062948	0.062948	0.000000
10	1.375752	1.376590	-0.000838
100	1323211.375000	1322987.875000	223.500000

Table 4. Calculation results for the traffic coefficient for the systems from Example 3.2.2 (scheme "loss").

Example 3.2.3. *Consider the system $M_{10}|G_{10}|1$ with all interarrival times being distributed exponentially Exp(a_k), $k = 1, ..., 10$, and all service times being distributed Er$(3, 200)$. The switchover times C_k are all distributed as Exp(100), $k = 1, \ldots, 10$. The results of calculations* *for such systems can be found in Tables 5, 6. The quantity* ϵ *is taken to be* 0.001*. In this case:*

$$\beta_k(s) = \left(\frac{1}{s\mathbb{E}[B_k]+1}\right)^3,$$

$$c_k(s) = \frac{1}{s\mathbb{E}[C_k]+1},$$

$$k = 1, \ldots, 10.$$

a_k	$\rho(BPLST_E)$	$\rho(BPLST_\epsilon)$	*difference*
1	0.176818	0.176821	-0.000003
10	12.101274	12.102625	-0.001351
100	348237984.000000	348191168.000000	46816.000000

Table 5. Calculation results for the traffic coefficient for the systems from Example 3.2.3 (scheme "repeat again").

a_k	$\rho(BPLST_E)$	$\rho(BPLST_\epsilon)$	*difference*
1	0.165731	0.165734	-0.000003
10	4.723875	4.724642	-0.000767
100	2153345.5000000	2153149.750000	195,750000

Table 6. Calculation results for the traffic coefficient for the systems from Example 3.2.3 (scheme "loss").

Example 3.2.4. *Consider the system* $M_{10}|G_{10}|1$ *with interarrival times being all distributed as Exp(10) and with the times of service of the requests from the lines* L_1*,* L_2 *and* L_3 *being distributed exponentially Exp(200), from the lines* L_4*,* L_5 *and* L_6 *being distributed uniformly* $U[0,1]$*, and from the lines* L_7*,* L_8*,* L_9*,* L_{10} *being distributed as*

Er$(3, 200)$. *The switchover times* C_k *are all distributed as Exp(100),* $k = 1, \dots, 10$. *The quantity* ϵ *is taken to be* 0.001.

In this case:

$$\beta_k(s) = \frac{1}{s\mathbb{E}[B_k] + 1}, \; k = 1, 2, 3;$$

$$\beta_k(s) = \frac{e^{-as} - e^{-bs}}{b - a}, \; a = 0, \; b = 1, \; k = 4, 5, 6;$$

$$\beta_k(s) = \left(\frac{1}{s\mathbb{E}[B_k] + 1} \right)^3, \; k = 7, 8, 9, 10;$$

$$c_k(s) = \frac{1}{s\mathbb{E}[C_k] + 1}, \; k = 1, \dots, 10.$$

scheme	$\rho(BPLST_E)$	$\rho(BPLST_\epsilon)$	*difference*
repeat again	5.184965	5.188527	-0.003562
loss	2.020857	2.022199	-0.001342

Table 7. Calculation results for the traffic coefficient for the systems from Example 3.2.4.

4. Conclusions

We presented a model algorithm of the numerical evaluation of the traffic coefficient in priority queueing systems (Algorithm 3 WLCOEFF). This algorithm makes use of the LST of busy period of the system—this should also be calculated numerically, using algorithms similar to the model Algorithm 1 $BPLTS_E$ and Algorithm 2 $BPLTS_\epsilon$. However, it was found from our experience that (i) the number of priority flows r should not exceed 10-12 for satisfactory fast calculations, and (ii) the calculation of the LST periods with algorithm Algorithm 1 $BPLTS_E$ was performed without clear idea about the absolute error of the evaluation. Therefore, there was a necessity of further optimization of this numerical algorithm in order to achieve fast performance and high level of precision of calculations. Algorithm 2 $BPLST_\epsilon$ served this purpose. However, one should also notice that for some systems the algorithm $BPLTS_E$ calculates the busy period's LST with the same precision as $BPLTS_\epsilon$ does (throughout the considered examples for the systems for which 'difference' in tables is of the same order as E and ϵ).

There is a necessity of further optimization in order to consider greater number of priority waiting lines. Such work is being done currently [3, 4, 6].

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