

# On Soundness for Time Workflow Nets

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## Abstract

Workflow technology is widely used in order to offer companies a solution for managing business processes. Time management is a critical component of workflow management. In a workflow management system there is a delay between the moment an activity becomes enabled and the moment the activity is executed by a certain resource. The notion of correctness also called soundness for untimed workflow nets is extended for Time Workflow Nets and a characterisation of this property is given for two particular classes of Time Workflow Nets.

## 1 Introduction

Workflow technology has been introduced in order to model and manage business processes, but workflows have some disadvantages: they are inflexible, they do not support inter-operability, and the formal verification of their correctness is difficult. For solving these problems, workflows can be modeled using Petri Nets[6, 9], which are expressive, have a well defined semantic, a very accessible graphical representation and reach techniques for checking quantitative and qualitative properties.

A *workflow* represents the automatization of a complex process which consists of a set of interdependent activities, orientated towards the fulfilling of a certain objective. The applicability domains of workflows are: modeling, coordination, management of business processes. Workflows are based on *cases*, which are generated by external clients or they are generated internal. A case is an instance of a workflow. A

workflow process is designed to handle similar cases, specifying what action must be executed and in what order.

In this article we will define and use Time Workflow nets for modeling workflows, because Petri nets have well-known three advantages: simplicity, generality, and adaptability.

- *Simplicity* - a reduced number of elementary concepts, which can be combined in a large variety.
- *Generality* - the different kind of semantics (transition sequences, tracks, processes) are easy to associate with Petri nets.
- *Adaptability* - the modification of the basic model leads to special models which include different aspects like time, making it usable in different domains.

Workflow properties can be easily checked using the analysis techniques of Petri nets.

The correctness, effectiveness, and efficiency of the business processes supported by the workflow management system are vital to the organization. It is important to analyze a workflow process definition before it is put into production. In this article we will focus on *verification* (establishing the correctness of a workflow). For verification linear algebraic techniques and coverability graph analysis can be used. With these techniques it is known that such problem like boundness and liveness are decidable. That is why we will reduce soundness problem to boundness and liveness problems.

## 2 Workflows

As we mentioned above a set of interdependent atomic activities forms a workflow. Basic entities of a workflow are: *actions*, *agents* and activities *dependences*.

An action can take place if some preconditions are fulfilled and it yields some postconditions. For the execution of an action a *trigger*

is necessary; a trigger can be represented by the external conditions which lead to the execution of an enabled task.

We distinguish between four types of tasks: *automatic* - a task is triggered at the moment when it is enabled, *user* - a task is triggered by human participant, *message* - an external event (message) triggers an enabled task instance, *time* - an enabled task instance is triggered by a clock (we are especially interested in these types of tasks).

A task which is enabled for a specific case is a *work item*. A work item is the combination of action + case + trigger (optional).

An *activity* is the actual execution of a work item, i.e., a task is executed for a specific case = action + case + resource (optional) + trigger.

A workflow has three dimensions: the case dimension, the process dimension, the resource dimension.

1. *Case dimension* specifying that every case is treated individually.
2. *Process dimension* specifying the workflow process, i.e., actions and routing for these actions.
3. *Resource dimension* specifying the what resources are grouped in roles and organizational units.

We will focus only on process dimension.

### 3 Time Workflow Nets

In this section we model the process dimension using Petri nets.

A Petri Net is a bipartite graph with two types of nodes: places and transitions interconnected by arcs, which connect only different types of nodes.

The process dimension specifies, as we mentioned above, which actions must be performed and in what order. For modeling workflows by means of Petri Nets the transition will be done directly: actions will be modeled by transitions, work items by enabled transitions, activities by firing transitions, conditions will be modeled by places, and cases will be modeled by tokens and dependences by arcs.

Further we will consider only Petri nets which describe the life cycle of one case. A Workflow net [2] will be defined as a Petri net which models the workflow process definition.

**Definition 3.1** *A Petri net  $PN = (P, T, F, W)$  is a Workflow net iff:*

1.  *$PN$  has two additional places  $i$  and  $o$ , "start" place  $i$ , "destination" place  $o$ .*
2. *If we add a transition  $t^*$  to  $PN$  which connects  $o$  with  $i$  then the resulting Petri net is strongly connected.*

*(where  $P$  - is a finite set of places,  $T$  - is a finite set of transitions,  $F \subseteq P \times T \cup T \times P$  - is the flow relation,  $W : F \rightarrow \mathbb{N}$  - is the weight function.)*

We define the *extended net*  $PN' = (P', T', F')$  with  $P' = P, T' = T \cup \{t^*\}, F' = F \cup \{(o, t^*), (t^*, i)\}$   $W' = W \cup \{W(o, t^*) = 1, W(t^*, i) = 1\}$ .

The notion of trigger defined in the paragraph above corresponds to an additional condition which must be fulfilled before the execution of the action, so it can be modeled by a token in an supplementary input location for the action.

There are different known methods of incorporating time in Petri nets: associating time delay to transition, associating time delay to places, associating time delay with arcs, associating time delays or time intervals to different types of objects of the net, associating stochastic time. Further we consider only Petri nets which have deterministic time associated to transitions, in the form of time intervals, defined by Merlin in 1972 [9] and then studied by Berthomeu-Menasche, Popova [10, 11, 12], Berthomieu-Diaz [4, 5], Boucheneb-Berthelot. Time Petri nets are classical Petri nets where for each transition  $t$ , a time interval  $[a_t, b_t]$  is associated. The times  $a_t, b_t$  are relative to the moment at which  $t$  was last enabled. Assuming that  $t$  was last enabled at time  $c_t$ , then  $t$  may fire only after the time interval  $[a_t + c_t, b_t + c_t]$  elapses.

We define a Time Workflow net in following way:

**Definition 3.2** A Time Workflow net is a tuple  $\Sigma = (P, T, F, W, I)$  where  $PN = (P, T, F, W)$  is the Workflow net,  $I: T \rightarrow Q_0^+ \times Q_0^+$  (where  $Q_0^+$  is the set of assertive numbers) is a time function which associates timed intervals with transitions and for each transition  $t \in T$ ,  $I_1(t) \leq I_2(t)$ , where  $I(t) = (I_1(t), I_2(t))$ .

A global clock is associated with the Time Workflow net, which begins to work as soon as the first token appears in the net. After time association, the Workflow net will work in following way: from the moment when a transition  $t$  is enabled, the tokens from the input locations are stored for  $I_2(t) - I_1(t)$  time units, and after this time elapses the transition fires putting tokens in their output places. For transitions in conflict, the first transition that fires is the one which has the latest time interval smaller.

For the definition of a state and of a change of state of the net  $\Sigma$  we will follow [10, 11]:

**Definition 3.3** Let  $\Sigma = (P, T, F, W, I)$  be a Time Workflow net and  $J: T \rightarrow Q_0^+ \cup \{\#\}$ . Then  $S = (m, J)$  is the state of  $\Sigma$  iff:

1.  $m$  is a marking in skeleton net.
2.  $\forall t (t \in T \text{ and } t^- \leq m \rightarrow J(t) \leq I_2(t))$ .
3.  $\forall t (t \in T \text{ and } t^- \not\leq m \rightarrow J(t) = \#)$ .

(where symbol  $\#$  means that clock does not work,  $t^-(p) = W(p, t), \forall p \in P$  is arc weight from place  $p$  to transition  $t$ ).

One can understand the notion of state in the following way: let  $S = (m, J)$  be a state. Each transition  $t$  in the net has a watch. The watch doesn't work ( $J(t) = \#$ ) at the marking  $m$  if  $t$  is disabled at  $m$ . If  $t$  is enabled at  $m$ , then the watch of  $t$  shows the time  $J(t)$  that has elapsed since  $t$  was last enabled.

Let  $\Sigma = (P, T, F, W, I)$  be a time workflow net. The state  $S_0 := (i, J_0)$  with  $i$  the initial marking of the workflow net (the marking which

has a single token in place  $i$ ) and  $J_0(t) = \begin{cases} 0 & \text{iff } t^- \leq m \\ \# & \text{iff } t^- \not\leq m \end{cases}$   
 is considered to be the initial state of the time workflow net.

The states in a Time Workflow Net can change due to transition firings or time elapsing.

**Definition 3.4** *Transition  $t$  is enabled in the state  $S = (m, J)$ , denoted by  $S \rightarrow$  iff*

1.  $t^- \leq m$
2.  $I_1(t) \leq J(t)$ .

The resulting state is defined as follows:

**Definition 3.5** *Transition  $t$  enabled at the state  $S = (m, J)$ , will fire inducing state  $S' = (m', J')$ , denoted by  $S \rightarrow S'$  defined thus:*

1.  $m'(p) = m(p) + \Delta t(p) = m(p) + W(t, p) - W(p, t)$
- 2.

$$J'(t) = \begin{cases} \# & t^- \not\leq m' \\ J(t) & t^- \leq m \wedge t^- \leq m' \wedge F_t \cap F'_t = 0 \\ 0, & \text{otherwise} \end{cases}$$

where  $F_t = \{p | p \in P \wedge pFt\}$ ,  $F'_t = \{p | p \in P \wedge pF't\}$

**Definition 3.6** *Let  $\Sigma = (P, T, F, W, I)$  be a time workflow net. The state  $S = (m, J)$  changes into the state  $S' = (m', J')$  by the time duration  $\tau \in Q$ , denoted by  $S \xrightarrow{\tau} S'$  iff:  $m' = m$  and the time duration  $\tau$  is possible  $\forall t(t \in T \wedge J(t) \neq \# \rightarrow J(t) + \tau \leq I_2(t))$  and*

$$J'(t) = \begin{cases} J(t) + \tau & \text{iff } t^- \leq m \\ \# & \text{iff } t^- \not\leq m \end{cases}$$

**Definition 3.7**  *$RS(\Sigma, S_0)$  denotes the set of all reachable states from initial state  $S_0$ .*

**Definition 3.8** *Transition  $t$  is live at the state  $S'$  iff  $\forall S' \in RS(\Sigma, S_0) \rightarrow \exists S'' (S'' \in RS(\Sigma, S') \text{ and } t \text{ is enabled at the state } S'')$ . State  $S$  is live in the net  $\Sigma$  iff all transitions  $t \in T$  are live in  $S$  and  $\Sigma$  is live iff  $S_0$  is live in  $\Sigma$ .*

**Definition 3.9** *The state  $S$  is bounded iff  $\forall p \in P : \exists k \in N : \forall S' \in [S > S(p) \leq k]$ . The net  $\Sigma$  is bounded if  $\forall S \in [S_0 >$  is bounded.*

## 4 The Soundness Property

This section defines a notion of correctness for Time Workflow Nets - the notion of soundness and a sufficient condition for soundness is proven. This property reduces to the problem of soundness for the corresponding untimed workflow skeleton net, for two special classes of Time Workflow Nets: time interval workflow nets with immediate transitions: for these nets, a transition can fire as soon as it becomes enabled, the second class is the class of time interval workflow nets with transitions that are not forced to fire in a specific amount of time. For these classes, the soundness property can be checked by verifying the boundness and liveness property for an untimed Petri Net.

**Definition 4.1** *Let  $\Sigma = (P, T, F, W, I)$  be a Time Workflow Net.  $\Sigma$  is sound iff:*

1. *For every state  $S$  reachable from the initial state  $S_0$ , there exists a firing sequence leading from  $S$  to a final state  $(o, J)$*   
 $\forall S (S_0[*]S) \Rightarrow (S[*](o, J))$
2. *The states  $(o, J)$  are the only states reachable from state  $S_0$  with at least one token in place  $o$ :*  
 $\forall S = (M, J) (S_0[*]S \wedge M \geq o \Rightarrow (M = o))$
3. *There are no dead transitions in  $\Sigma$ :*  
 $\forall t \in T, \exists S, S' (S_0[*]S(t)S')$

Note that the soundness property relates to the dynamics of the WF - net. Given  $\Sigma = (P, T, F, W, I)$  a Time Workflow Net, we define the

extended Time Workflow Net  $\Sigma'$  as follows:  $\Sigma' = (P', T', F', W', I')$  where:

- \*  $P' = P$
- \*  $T' = T \cup \{t^*\}$
- \*  $F' = F \cup \{(o, t^*), (t^*, i)\}$
- \*  $I'(t) = I(t)$  for all  $t \in T$  and  $I'(t^*) = [0, +\infty]$
- \*  $W' = W \cup \{W(o, t^*) = 1, W(t^*, i) = 1\}$

**Lemma 4.1** *Let  $\Sigma$  be a time workflow net with the initial state  $S_0 = (i, J_0)$ . If  $\Sigma'$  is live and bounded, then  $\Sigma$  is a sound time workflow net.*

**Proof.**

$\Sigma'$  is live, i.e for each reachable state  $S$  there is a sequence which leads to another state  $S'$  in which transition  $t^*$  is enabled. Let  $S' = (m', J')$ . Since  $t^*$  can fire it results that  $t^*$  is enabled in marking  $m'$ . Place  $o$  is the input place for  $t^*$ , so  $m'(o) = 1$ . So, for any state reachable from the initial state, it is possible to reach a state with at least one token in place  $o$ . So the first condition from the definition of soundness holds.

Consider  $S$  a state reachable from  $S_0$ ,  $S = (M, J)$  with  $M \geq o$  (at least one token in place  $o$ ). This means  $M = M' + o$ . The transition  $t^*$  is fireable in this marking: Since  $M \geq o$ , then  $M(o) \geq 1$  and  $o$  is the only input place for  $t^*$ , so  $t^*$  is enabled in  $M'$ . It also holds that  $I_1(t^*) \leq J(t^*)$  because  $I_1(t^*) = 0$ . If  $t^*$  fires, a new state  $S' = (M' + i, J')$  is reached. Since  $\Sigma'$  is bounded and  $M' + i \geq i$  it results that  $M' + i = i$ , so  $M'$  should be equal to the empty state. Hence condition (2) from the definition of soundness also holds. The final condition from the definition of soundness results from the fact that  $\Sigma'$  is live.

The next lemma shows that, for time interval workflows nets with immediate transitions (i.e transitions that can fire as soon as they become enabled), the soundness property implies the boundedness of the extended time workflow net.

**Lemma 4.2** *If  $\Sigma$  is sound and  $\forall t \in T : I_1(t) = 0$  then  $\Sigma'$  is bounded.*



**Proof.**

We will first show that  $\Sigma$  is bounded. Assume that  $\Sigma$  is sound and  $\Sigma$  is not bounded. Since  $\Sigma$  is not bounded, there are two states  $S_i = (M_i, J_i)$  and  $S_k = (M_k, J_k)$  such that  $S_0[*]S_i, S_i[*]S_k$  and  $M_k > M_i$ . However, since  $\Sigma$  is sound, there is a sequence  $\sigma = \tau_0, t_0, \dots, \tau_{n-1}, t_{n-1}$  such that  $S_i[\sigma](o, J)$ . We will show that the sequence  $\sigma' = t_0, t_1, \dots, t_{n-1}$  is fireable from  $S_k$ . We prove the statement by induction on  $n$ .

If  $n = 1$ , then  $S_i \xrightarrow{\tau_0} S'_{i1} \xrightarrow{t_0} S_{i1}$ . We prove that  $t_0$  is fireable at state  $S_k$ . It holds that  $t_0^- \leq M_k$ , since  $t_0^- \leq M_i$  and  $M_k > M_i$ . It must hold that  $J_k(t_0) \geq I_1(t_0)$ . But this always holds, because  $I_1(t_0) = 0$ . It also holds that  $M_{k1} > M_{i1}$ .

Suppose the statement holds for  $n$  and we want to prove it for  $n+1$ . So, if  $S_i \xrightarrow{\tau_0} S'_{i1} \xrightarrow{t_0} S_{i1}, \dots, \xrightarrow{\tau_{n-1}} S'_{in} \xrightarrow{t_{n-1}} S_{in}$  then we have the sequence:  $S_k \xrightarrow{t_0} S_{k1} \dots S_{kn-1} \xrightarrow{t_{n-1}} S_{kn}$ . Let  $S_i \xrightarrow{\tau_0} S'_{i1} \xrightarrow{t_0} S_{i1}, \dots, \xrightarrow{\tau_{n-1}} S'_{in} \xrightarrow{t_{n-1}} S_{in} \xrightarrow{\tau_n} S'_{in+1} \xrightarrow{t_n} S_{in+1}$ . From the induction assumption:  $S_k \xrightarrow{t_0} S_{k1} \dots S_{kn-1} \xrightarrow{t_{n-1}} S_{kn}$ . It also holds that:  $M'_{ij+1} = M_{ij} < M_{kj}, j \in 0 \dots n-1$ . We prove that  $t_n$  is fireable at state  $S_{kn}$  and  $M_{kn+1} > M_{in+1}$ . We know that  $t_n$  is fireable at  $S'_{in+1}$  and  $M'_{in+1} = M_{in} < M_{kn}$  hence  $t_n^- \leq M_{kn}$ . It must hold that  $J_{kn}(t_n) \geq I_1(t_n)$ . This statement always holds, since  $I_1(t_n) = 0$ . So  $t_n$  can fire at state  $S_{kn}$  and it results a new state  $S_{kn+1}$  with  $M_{kn+1} > M_{in+1}$ .

Using the result proven above, if  $S_i[\sigma]S_o$  and  $S_i > S_k$  then there exists  $\sigma'$  such that  $S_k[\sigma']S_{ko}$  such that  $M_{ko} > o$ . This fact contradicts the condition 2 from the definition of soundness. Thus,  $\Sigma$  must be bounded. From the fact that  $\Sigma$  is bounded and sound it results that  $\Sigma'$  is bounded: if transition  $t^*$  in  $\Sigma'$  fires, then the time workflow net returns to its initial state.

The next lemma proves the same result as Lemma 4.2 for time interval workflow nets in which transitions don't have the obligation to fire at a specific moment of time.

**Lemma 4.3** *If  $\Sigma$  is sound and  $\forall t \in T : I_2(t) = +\infty$  then  $\Sigma'$  is bounded.*

**Proof.**

We will first show that  $\Sigma$  is bounded. Assume that  $\Sigma$  is sound and  $\Sigma$  is not bounded. Since  $\Sigma$  is not bounded, there are two states  $S_i = (M_i, J_i)$  and  $S_k = (M_k, J_k)$  such that  $S_0[*]S_i, S_i[*]S_k$  and  $M_k > M_i$ .

We will prove that for any sequence  $\sigma = S_i \xrightarrow{\tau_0} S'_{i1} \xrightarrow{t_0} S_{i1}, \dots \xrightarrow{\tau_{n-1}^-} S'_{in} \xrightarrow{t_{n-1}^-} S_{in}$  there exists a sequence  $\sigma'$  fireable from  $S_k$ :  $\sigma' = S_k \xrightarrow{\tau_0^*} S'_{k1} \xrightarrow{t_0} S_{k1}, \dots \xrightarrow{\tau_{n-1}^*} S'_{kn} \xrightarrow{t_{n-1}^-} S_{kn}$  with  $M_{kn} > M_{in}$ , where  $\tau_i^* = \max\{I_1(t), t^- \leq M_{i-1}\}$ .

We will prove that if  $S_{il-1} \xrightarrow{\tau_l} S'_{il} \xrightarrow{t_l} S_{il}$ ,  $M_{il-1} > M_{ik-1}$  then it holds:

$S_{kl-1} \xrightarrow{\tau_l^*} S'_{kl} \xrightarrow{t_l} S_{kl}$  and  $M_{kl} > M_{il}$ , where  $\tau_l^* = \max\{I_1(t), t^- \leq M_{il-1}\}$ . We must prove that the time duration  $\tau_l^*$  is possible at  $S_{il-1}$ , i.e  $\forall t : t^- \leq M_{il-1} \rightarrow J_{il-1}(t) + \tau_l^* \leq I_2(t)$ . This always holds, since  $I_2(t) = +\infty$ . The resulting state has  $J'_{kl}(t) = J_{kl-1}(t) + \tau_l^*, \forall t : t^- \leq M_{il-1}$ . Next, we must prove that  $t_l$  is fireable at the state  $S'_{kl}$ . We know that  $M'_{kl} = M_{kl-1} > M_{il-1}$  and  $t_l$  is fireable at  $M'_{il} = M_{il-1}$ , so it holds that  $t_l^- \leq M'_{kl} = M_{kl-1}$ . We must prove that  $I_1(t_l) \leq J'_{kl}(t_l)$ . But  $t_l^- \leq M_{kl-1} = M'_{kl}$ , so, from the definition of  $\tau_l^*$  it holds that  $\tau_l^* \geq I_1(t_l)$ . Then,  $J'_{kl}(t_l) = J_{kl-1}(t_l) + \tau_l^* \geq J_{kl-1}(t_l) + I_1(t_l) \geq I_1(t_l)$ . So,  $I_1(t_l) \leq J'_{kl}(t_l)$ . Thus we have proven that  $t_l$  is fireable at  $S'_{kl}$ . For the resulting state  $S_{kl}$  it holds that  $M_{kl} > M_{il}$ .

From the fact that  $\Sigma$  is sound, it results that there exists a sequence  $\sigma$  such that  $M_i[\sigma](o, J) = M_o$ . Then, there exists a sequence  $\sigma'$  as described above such that  $M_k[\sigma']M'_o = (M', J')$  such that  $M' > o$ . This relation contradicts the second relation from the definition of soundness, so  $\Sigma$  cannot be unbounded. From the fact that  $\Sigma$  is bounded and sound it results that  $\Sigma'$  is bounded: if transition  $t^*$  in  $\Sigma'$  fires, then the time workflow net returns to its initial state.

**Lemma 4.4** *If  $\Sigma$  is a sound time workflow net, then  $\Sigma'$  is live.*

**Proof.**

First we show that state  $S_0$  is a home state for  $\Sigma'$ , i.e  $\forall S \in [S_0]_{\Sigma'} : S_0 \in [S]_{\Sigma'}$ . From the definition of soundness, for all states  $S \in [S_0]$ ,

there exists an execution sequence  $S[\sigma](o, J)$ . We prove that  $t^*$  is enabled in state  $(o, J)$ . It holds that  $t^{*-} \leq o$ . We must prove that  $J(t^*) \geq I_1(t^*)$ . This relation is true, because  $I_1(t^*) = 0$ . The resulting state is  $S_0 = (i, J_0)$ . So, for every state  $S \in [S_0]$  there exists a sequence  $S[\sigma]t^*[S_0]$ . Now we will prove that  $\Sigma'$  is live. Let  $t$  be a transition and  $S$  a state. From the soundness (3), there exists state  $S' \in [S_0]_\Sigma$  such that  $t$  is enabled in  $S'$ . We show that  $S' \in [S]_{\Sigma'}$ . We know that  $S[*]_{\Sigma'} S_0[*]_{\Sigma'} S'$ . So  $S' \in [S]_{\Sigma'}$ , and we have proven that  $\Sigma'$  is live.

**Theorem 4.1** *If  $\Sigma = (P, T, F, W, I)$  is a Time Workflow Net, such that  $\forall t \in T : I_1(t) = 0$ . Then  $\Sigma$  is sound iff the extended Time Workflow Net,  $\Sigma'$  is live and bounded.*

**Proof.**

The proof of the theorem results immediately from Lemma 4.1, Lemma 4.2 and Lemma 4.4.

**Theorem 4.2** *If  $\Sigma = (P, T, F, W, I)$  is a Time Workflow Net, such that  $\forall t \in T : I_2(t) = +\infty$ . Then  $\Sigma$  is sound iff the extended Time Workflow Net,  $\Sigma'$  is live and bounded.*

**Proof.**

The proof of the theorem results immediately from Lemma 4.1, Lemma 4.3 and Lemma 4.4

**Proposition 4.1** *Let  $\Sigma = (P, T, F, W, I)$  be a Time Workflow Net such that  $\forall t (t \in T \rightarrow I_1(t) = 0)$  and  $S(\Sigma)$  the skeleton of  $\Sigma$ , then it holds:*

1.  $S(\Sigma)$  is unbounded iff  $\Sigma$  is unbounded.
2.  $S(\Sigma)$  is live iff  $\Sigma$  is live.

**Proof.**

Demonstration is similar to the demonstration from [11].

**Proposition 4.2** *Let  $\Sigma = (P, T, F, W, I)$  be a Time Workflow Net such that  $\forall t (t \in T \rightarrow I_2(t) = \infty)$  and  $S(\Sigma)$  the skeleton of  $\Sigma$ , then it holds:*

1.  $S(\Sigma)$  is unbounded iff  $\Sigma$  is unbounded.
2.  $S(\Sigma)$  is live iff  $\Sigma$  is live.

It can be noticed that the two classes of Time Workflow Nets defined above have the same boundedness and liveness behaviour as the corresponding classical Petri Nets (their skeletons). Now, using theorems 4.1 and 4.2 and proposition 4.1 and 4.2, the following results regarding the soundness of these classes of Time Workflow Nets can be proven.

**Theorem 4.3** *Let  $\Sigma = (P, T, F, W, I)$  be a Time Workflow Net such that  $\forall t (t \in T \rightarrow I_1(t) = 0)$  and  $S(\Sigma)$  the skeleton of  $\Sigma$ , then it holds:  $\Sigma$  is a sound Time Workflow Net iff  $S(\Sigma)$  is a sound workflow net.*

**Proof.**

According to Theorem 4.1,  $\Sigma$  is sound iff  $\Sigma'$  is live and bound. Since  $\Sigma'$  has for all  $t : I_1(t) = 0$ , then  $\Sigma'$  is live and bound iff  $S(\Sigma')$  is live and bound. But  $S(\Sigma') = S(\Sigma)'$ , so  $S(\Sigma')$  is live and bound iff  $S(\Sigma)'$  is live and bound. For untimed workflow nets we know that  $WF$  is sound iff  $WF'$  is live and bounded. So  $S(\Sigma)$  is sound.

**Theorem 4.4** *Let  $\Sigma = (P, T, F, W, I)$  be a Time Workflow Net such that  $\forall t (t \in T \rightarrow I_2(t) = \infty)$  and  $S(\Sigma)$  the skeleton of  $\Sigma$ , then it holds:  $\Sigma$  is a sound Time Workflow Net iff  $S(\Sigma)$  is a sound workflow net.*

Using theorem 4.3, theorem 4.4 and the fact that soundness is decidable for untimed workflow nets, it results that:

**Corollary 4.1** *Let  $\Sigma = (P, T, F, W, I)$  be a Time Workflow Net such that  $\forall t (t \in T \rightarrow I_1(t) = 0)$ . The soundness property is decidable for  $\Sigma$ .*

**Corollary 4.2** *Let  $\Sigma = (P, T, F, W, I)$  be a Time Workflow Net such that  $\forall t (t \in T \rightarrow I_2(t) = \infty)$ . The soundness property is decidable for  $\Sigma$ .*

For the two classes of Time Workflow Nets described above, the soundness property is decidable and it can be checked by verifying the boundness and liveness property of the underlying untimed net of the extended net  $\Sigma'$ .

## 5 Conclusions

In this paper we have introduced a new class of Petri Nets for modelling workflows with time delays associated to tasks. We have defined the notion of soundness for Time Workflow Nets, extending the notion of soundness defined in [2] for untimed workflow nets. It was shown for a Time Workflow Net  $\Sigma$  that, if the extended Time Workflow Net  $\Sigma'$  is live and bounded, then  $\Sigma$  is sound. There were identified two subclasses of Time Workflow Nets (Time Workflow Nets with immediate transition firing and Time Workflow Nets with no obligation to fire for transitions) for which the soundness property reduces to the soundness property of the skeleton net. Thus, the soundness can be verified using the liveness and the boundness properties of an untimed workflow net. Therefore, the soundness property is decidable in these two particular cases. Further we research aims at finding a characterisation for the soundness property for all Time Workflow Nets and finding interesting subclasses of Time Workflow Nets for which the soundness is decidable.

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