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Macroeconomic growth determining

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Abstract

In the offered work two approaches to the problem of economic growth rate determining is considered. The first approach assumes, that economic growth rate is set exogenously, so its value is known while at the second approach economic growth rate is determined endogenously, i.e. its value is unknown. In the model there are included such components of the economic growth, which are determined by the technical progress containing the human factor during training, and also sector of research and development ($R\&D$). Research and development are estimated in two ways: one, supposing development of new technologies and another, using already available developed technologies, and on their basis developing new manufactures. Estimations of expenses both on the first, and on the second way are resulted.

Introduction

Work consists of three sections. In the first section the Solow model [11] with the exogenously set growth rate of technical progress is investigated. The equation of dynamics is brought and the golden rule of accumulation is written. Using dynamics equation for Solow model, the optimization problem is formulated and it is solved by the maximum principle. The solution of this problem determines an optimal trajectory for consumption function and, accordingly, an optimal rate of accumulation.

In the second section Aghion-Howitt [1-2] model with endogenous growth caused by uncertainty in which researches provide qualitatively improved innovations is discussed. The dynamics equation of the model is brought, and economic growth rate is determined, proceeding from the known values of the model parameters offering basic characteristics of economic development.

In the third section the Romer [13-15] endogenous growth model is examined. Estimation cost of the original invention which underlies qualitatively improved innovational processes and the cost of the license invention providing qualitatively improved innovations are brought.

1. Optimal rate of accumulation in exogenous growth economic model

Let's consider Solow exogenous growth economic model with technical progress.

$$\begin{aligned} Y &= K^\alpha (EL)^{1-\alpha} && (1)\\ Y &= C + I && (2)\\ C &= (1-s)Y, \qquad 0 < (1-s) < 1 && (3)\\ \Delta K &= I - \delta K, \qquad 0 < \delta < 1 && (4)\\ \Delta L/L &= n, \qquad n > 0 && (5)\\ \Delta E/E &= g, \qquad g > 0 && (6) \end{aligned}$$

Here Y, C, I, K are endogenous variables, L, E are exogenous variables. Y is gross domestic product; C is total consumption; I is total investments; K is fixed capital; L is total workers quantity occupied in manufacture; E is some variable named work efficiency. s, δ, n, g are parameters determining rate of accumulation, fixed capital amortization rate, growth rate of the labor occupied in manufacture, growth rate of the labor caused by technological progress, it takes into account quantity of workers L and efficiency of everyone working E. Production function (1) determines, that total output Y depends on the available capital quantity K and on the quantity of effective workers $L \cdot E$. The increase of efficiency of work E is equivalent to increase in a labor L. Using substitution, $\frac{Y}{EL} = y$; $\frac{K}{EL} = k$, we shall transform the first equation to the following one $y = k^\alpha$. From the equation (4) we shall receive

$$\dot{k} = i - \delta k,$$

where $i = \frac{I}{EL}$. Further, using (2) and (3), we shall transform (4) to the form
$$\dot{k} = sy - \delta k; \quad \dot{k} = sk^{\alpha} - \delta k.$$
After simple transformations we shall receive, that
$$\frac{\dot{K}}{EL} = \frac{I - \delta K}{EL} = \frac{sY - \delta K}{EL} = sk^{\alpha} - \delta k. \tag{7}$$
And, finally,
$$\frac{\dot{k}}{k} = sk^{\alpha-1} - \delta. \tag{8}$$
The stationary condition takes the form of:
$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} - \frac{\dot{E}}{E} = 0. \tag{9}$$
And, substituting (8) in (9), we shall receive
$$\frac{\dot{k}}{k} = sk^{\alpha-1} - (\delta + n + g).$$
Solving this equation in respect to k we shall obtain
$$k^* = \Big(\frac{s}{\delta + n + g}\Big)^{1/(1-\alpha)}. \tag{10}$$
Let's notice, that in steady state the value k is as greater as the values s and α (rate of accumulation and the interchangeability constant of work and capital) are greater and as smaller as the values δ, n and g are smaller. So the growth rate of the capital per capita is positive and is equal to $g + n$.
$$\frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta L}{L} - \frac{\Delta E}{E} = 0; \quad \frac{\Delta K}{K} = g + n > 0. \tag{11}$$
From the equation (3) we shall receive equation for consumption function in a steady state
$$c^* = y - i; \quad c^* = f(k^*) - k^*(n + g + \delta). \tag{12}$$

As $\dfrac{I}{EL} = \dfrac{\Delta K}{K} + \delta$ from (11) it is received

$$i^* = k^*(n + g + \delta). \tag{13}$$

From (12) follows, that the maximal value c^* is achieved when

$$\frac{dc^*}{dk^*} = f'(k^*) - (n + g + \delta) = 0 \Longrightarrow f'(k^*) = n + g + \delta$$

$$f'(k^*)k^* = sf(k^*); \quad s^* = \frac{f'(k^*)k^*}{f(k^*)}. \tag{14}$$

Expression (14) determines "golden rule" of accumulation. Such premises, as the constant returns to scale production function, competitive markets, exogenous savings and exogenous growth of technology, made this model an extremely convenient one to solve the case for steady state.

Let's take advantage of a maximum principle for determination of the maximal rate of accumulation which satisfies to the "golden rule". We shall consider the problem of optimal control in which the utility function on infinite orison is maximized and the restriction is the dynamics equation for the Solow model. As a control variable the consumption function takes place.

$$\begin{aligned} \int_0^\infty \frac{c(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt \quad &\Longrightarrow \quad max && (15) \\ k' \quad &= \quad [f(k) - k(n + g + \delta)] - c(t) && (16) \\ k(0) \quad &= \quad k_0 && (17) \end{aligned}$$

Profit function is written as $V = f(k) - rk - w$ where V is the maximal profit rent equal to zero and r is characterized by the following ratio: $\frac{\partial f(k)}{\partial k} = r, \quad f(k) = rk + w.$

So, let's write down the Hamiltonian of the system (15) - (17)

$$H(c(t), k(t), \pi(t)) = \frac{c(t)^{1-\theta}}{1-\theta} e^{-\rho t} + \pi(t) \cdot [f(k) - c(t) - k(n+g+\delta)]. \tag{18}$$

Further the conditions of the maximum principle follow:

$$\frac{\partial H}{\partial c} = 0; \qquad \frac{\partial H}{\partial \pi} = \dot{k}; \qquad \dot{\pi} = \rho\pi - \frac{\partial H}{\partial k}. \tag{19}$$

And the transversality condition is written down as

$$\lim_{t\to\infty} \pi(t) \cdot k(t) \cdot e^{-\rho t} = 0$$

After simple transformations we receive the following three equations:

$$e^{-\rho t} \cdot c(t)^{-\theta} - \pi(t) = 0 \tag{20}$$

$$\dot{k} = s \cdot f(k) - k \cdot (n + g + \delta) \tag{21}$$

$$\dot{\pi} = \pi \cdot (\rho - \dot{f}(k) + (n + g + \delta)). \tag{22}$$

And, since $\frac{\partial f(k)}{\partial k} = r$, the equation (21) will become $\dot{\pi} = \pi \cdot [\rho - r + (n + g + \delta]$. In consequence, we receive system of two differential equations which are necessary to solve:

$$\begin{aligned} \dot{k}(t) &= w - k(t) \cdot (n + g + \delta) - c(t) \\ \dot{\pi}(t) &= \pi(t)[\rho - r + (n + g + \delta)] \qquad (23) \\ k(0) &= 0; \quad \lim_{t\to\infty} \pi(t) \cdot k(t) \cdot e^{-\rho t} = 0. \qquad (24) \end{aligned}$$

Introducing some notations, we obtain

$$\begin{aligned} \dot{k}(t) &= -\beta k(t) - c(t) + w \\ \dot{\pi}(t) &= \pi(t) \cdot \gamma \qquad (25) \\ k(0) &= 0; \quad \lim_{t\to\infty} \pi(t) \cdot k(t) \cdot e^{-\rho t} = 0. \qquad (26) \end{aligned}$$

In order to solve this problem, the equation (22) should be differentiated on t

$$\frac{\dot{c}(t)}{c(t)} = -\frac{\gamma}{\theta} = \frac{r - (n + g + \delta) - \rho}{\theta} = \dot{\gamma}.$$

Hence, $c(t)$ grows with a constant exponential growth rate. To receive the solution in $k(t)$ we act as follows. To resolve equation (22) in accordance with $k(t)$ we shall take an integrating multiplier $e^{\beta t}$, and we shall receive

$$\dot{k}(t) \cdot e^{\beta t} + \beta \cdot e^{\beta t} \cdot k(t) = c_0 \cdot e^{-\grave{\gamma} t} + w.$$

Common solution of this equation depends on two constants. Taking into account the transversallity condition one of the two integration constants in the common solution for $k(t)$ is determined. The second constant is determined from the condition $k(0) = k_0$. As a result, the following is received:

$$k(t) = k_0 \cdot e^{\grave{\gamma} t} \quad (27)$$

$$c(t) = k_0 \cdot [\beta - \grave{\gamma}] \cdot e^{\grave{\gamma} t} + w. \quad (28)$$

2. Endogenous growth economic model

Solow neoclassical model was first modified by Cass [3] and (independently) by Koopmans in [7] following a mathematical version of Frank Ramseys model [12](RCK). The endogenous saving rate in it is deeply different from the Solow model.

The Aghion-Howitt [1-2] endogenous growth economic model represents such approach when the endogenous economic growth is examined under uncertainty conditions. In this model the economic growth is generated by a sequence of qualitatively improved innovations which are the investigations connected to economic activities in a random way. This model has natural property when the last innovation is presented out-of-date in comparison with the present innovation. In other words this model is the model of destructive innovation creation. Starting from Romers [13] the basic idea of endogenous technological progress is obtained

$$u(y) = \int_0^\infty y(\tau) e^{-r\tau} d\tau.$$

Here y is the production volume of the final consumer goods, r is the rate of preference in time equal to the interest rate.

$$y = Ax^\alpha, \quad 0 < \alpha < 1. \quad (1)$$

A is the parameter of technological progress. Innovations increase A by size of a constant factor $\gamma = \frac{A_{t+1}}{A_t}$, here t is an innovational index but not the time.

$$L = x + n. \tag{2}$$

L is the volume of labor, x are the works used by manufacture of the intermediate goods (one for one). It means that for production of one unit of the intermediate goods one unit of labor is used. If the price of intermediate goods is denoted by P, then from the profit maximization condition the demand function for the produced intermediate goods is deduced.

$$Ax^\alpha - Px = \pi; \quad \frac{\partial \pi}{\partial x} = \alpha A x^{\alpha-1} - P = 0; \quad P = \alpha A x^{\alpha-1}, \tag{3}$$

hence, it follows that the manufacturer will produce final goods until the marginal product will be equal to its price. Innovations appear casually with the Poisson rate of appearance equal to λn where λ indicates productivity of research technologies. If there is one additional unit of labor, the probability of new innovations appearance grows by value λ. And because Poisson distribution is additive, the rate expectation of n researchers appearance is equal to λn. V_{t+1} is a profit function caused by innovation $t+1$; λV_{t+1} is an expected gross revenue of each researcher for the period before appearance of the following innovation. The net expected profit from the use of n units of labor in research area for production of an innovation t is equal to:

$$\lambda n V_{t+1} - w_t n = 0, \tag{4}$$

here w_t is a wages rate resulted at the innovation t.

$$\lambda V_{t+1} = w_t, \tag{5}$$

and this equation settles that expected cost of one unit used in scientific researches with necessity is equal to its price. Value V_{t+1} is determined from the following equations

$$rV_{t+1} = \pi_{t+1} - \lambda n_{t+1} V_{t+1}, \tag{6}$$

$$V_{t+1} = \frac{\pi_{t+1}}{(r + \lambda n_{t+1})}. \tag{7}$$

The denominator from (7) can be interpreted as the interest rate adapted to disappearance of an innovation, and illustrates it creative-destructive effect. It is necessary to note, that from (7) it follows that the innovator can not improve $(R\&D)$ since, actually, λn_{t+1} determines the probability that innovator will lose exclusive innovational rent

$$\pi_t = \max_x [P(x) \cdot x - w \cdot x]. \tag{8}$$

Equation (8) defines income flow, which can be received from the following innovation. Here P and x are interconnected by means of the equation (3). As expression $\alpha A \cdot x^{\alpha} - w \cdot x$ is maximized, then $\alpha^2 \cdot A_t x_t^{\alpha-1} = w_t$ or

$$x_t = \Big(\frac{\alpha^2}{w_t/A}\Big)^{1/(1-\alpha)}. \tag{9}$$

Defining $W = \frac{w_t}{A_t}$ as the wages referred to productivity, we can express x as decreasing function of w_t, $x_t = \tilde{x}(W_t)$, $\tilde{x}' < 0$. Substituting x_t in expression for income function, we receive

$$\pi_t = A_t \cdot \alpha \cdot x_t^{\alpha} - w_t \cdot x_t = \Big(\frac{1}{\alpha} - 1\Big) \cdot w_t \cdot x_t = A_t \tilde{\pi}\Big(\frac{w_t}{A_t}\Big) \tag{10}$$

or $\pi_t = \tilde{\pi}(W_t)$, $\tilde{\pi}' < 0$.

Two basic equations.

The first equation is an arbitrary one. It turns out from (5) that $w_t = \lambda \cdot V_{t+1}$, further from (7) it is received the following:

$$\frac{w_t}{A_t} = \lambda[\pi_{t+1}/(r + \lambda \cdot n_{t+1})]/A_t$$

$$W_t = \lambda \frac{A_{t+1}}{A_t} \cdot \tilde{\pi}(W_{t+1}) \cdot (r + \lambda \cdot n_{t+1})^{-1}.$$

And, since $\frac{A_{t+1}}{A_t} = \gamma$, we have

$$W_t = \lambda \frac{\gamma \cdot \tilde{\pi}(W_{t+1})}{r + \lambda \cdot n_{t+1}}. \tag{11}$$

The second equation is the equation for labor market clearing.

$$L = n_t + \tilde{x}(W_t). \tag{12}$$

The balanced growth SS (stable state) or growth in condition of stability is defined as the stationary solution of the system (11) - (12) at $n = n_{+1} = n$ and $W = W_{+1} = W$. That gives two equations with two unknown variables n and W. In other words, SS means, that both attraction of an additional labor n in sphere of researches, and $\tilde{x}(W)$ in sphere of manufacture remain constants in time until salary, income and volume of manufacture remain at the same level $\gamma < 1$ each time when innovations are made. In SS the equations (11) and (12) are reduced to:

$$\begin{aligned} W &= \lambda \frac{\gamma \cdot \tilde{\pi}(W)}{r + \lambda \cdot n} && (11(a)) \\ L &= n + \tilde{x}(W) && (12(a)) \end{aligned}$$

Market forces.

It is easy to show, that from (12(a)) it follows

$$\tilde{\pi} = \frac{1-\alpha}{\alpha} \cdot Wx = \frac{1-\alpha}{\alpha} \cdot W(L-n)$$

and, having substituted this equation in (11(a)), dividing by W, we shall receive

$$1 = \frac{\lambda \cdot \dfrac{\gamma \cdot (1-\alpha)}{\alpha} \cdot (L-n)}{r + \lambda \cdot n}.$$

From this equation the statement for $\hat{n}$ in SS is obtained:

$$\hat{n} = \frac{\lambda \cdot \gamma \cdot (1-\alpha) \cdot L - \alpha \cdot r}{(\alpha \cdot \lambda + \lambda \cdot \gamma \cdot (1-\alpha))}.$$

Growth in the stable state.

In SS we have $y = A \cdot (\hat{x})^{\alpha} = A \cdot (L - \hat{n})^{\alpha}$, which implies $y_{+1} = \gamma \cdot y$.

3. P. Romer growth endogenous economic model

The other spark in growth theory has been provided by the advent of "New Growth Theory" attributed to the efforts of Romer [13] and Lucas [7]. New growth theory casts doubt on many results and assumptions of the neoclassical model. Romer's model included increasing returns, imperfect competition and endogenous technological growth. P. Romer in [14] determines technological progress as finding of new varieties and increasing amount of capital goods.

$$Y_i = A \cdot L_i^{1-\alpha} \cdot \sum (X_{ij})^{\alpha},$$

here X_{ij} is the implication of the specialized intermediate goods of the type j. At any point of time, the technology exists to create N specialized intermediate goods. Research sector in the economy uses the human capital (labor skills and labor capacities) and accumulates knowledge to produce designs for new intermediate capital goods. The growth rate in such a model is determined from the following equation:

$$\gamma = \frac{1}{\theta} \cdot \Big[\frac{L}{\eta} \cdot A^{\frac{1}{1-\alpha}} \cdot \Big(\frac{1-\alpha}{\alpha} \Big) \cdot \alpha^{\frac{2}{1-\alpha}} - \rho \Big],$$

where η is a cost of new product creation. Ii is supposed that total human capital accessible to use in economy is constant. If $\gamma < 0$, firms have insufficient incentive to expend resources on $R\&D$, N remains constant, and γ ultimately equals 0. The increase in growth rates can be provided by the following actions:

- desire to raise the rate of accumulation, which lowers ρ and θ;
- reducing the cost of a new product inventing, which lowers η;
- advanced technology, which is represented by expansions of N resulted from ($R\&D$).

However, constant increase in N results in the tendency of income reduction. The size of innovation income – profit value for the j-th innovator, is equal to π_j / r_j, where r_j is the rate of return. Let r_1 be the rate of return, for example, in the USA, then

$$r_1 = \Big(\frac{L_1}{\eta} \Big) \cdot \frac{(1-\alpha)}{\alpha} \cdot (A_1)^{\frac{1}{(1-\alpha)}} \cdot \alpha^{2/(1-\alpha)}.$$

Here η is the invention cost; L_1 is the working amount. While imitation or adaptation of innovational products is equal to ν and is less expensive than expenditure for an innovation η, the rate of return r_2 in this country is approximately constant

$$r_2 = \left(\frac{L_2}{\nu}\right) \cdot \frac{(1-\alpha)}{\alpha} \cdot (A_2)^{\frac{1}{(1-\alpha)}} \cdot \alpha^{2/(1-\alpha)}.$$

The conclusion

Macroeconomic models of economic growth [1-15] are investigated. Such production factors as capital and labor are examined. The last is subdivided into a manpower used in industrial sector and in $(R\&D)$ sector. Opportunities of each sector for maintenance of steady economic growth are studied.

We shall start from the fact, that the gain of capital is defined, basically, as a level of accumulation in national economy, particularly in private sector. By using the Solow model it is possible to determine optimal rate of accumulation corresponding to the given economic growth rate. Thus there are two problems, the first is connected with attraction of additional investments, and the second concerns a manpower which grows presumably with constant rates. The last can be restrictive for Moldova, taking into account the outflow of labor from the country and the fact, that the government has not developed a clear policy in the problem of professional training of high quality specialists. It can happen, that a bit later the country will face labor shortage in industry and agriculture. And also, as assumed, the economic growth in a physical world is constrained by diminishing returns and scarcity of resources. Such state policies as maintenance of services infrastructure, property rights protection and taxation of economic activities influences technological level and, simultaneously with the desire to accumulate, influence the growth rate in the long-term prospects.

And, as in [13], two basic fundaments of economic growth are the new ideas and advances in technology; and by creating appropriate economic incentives, the government can increase the rate of growth in a way that can make all citizens better off. In this connection in our country these ideas may by interpreted as clear definition of key economy development directions (whether it be the sphere of services, tourism, information technologies, etc.) and elaboration of professional training strategy in these brunches. Therefore the state should provide for specified branches the preferential development, stimulating investments, giving tax privileges and other.

In endogenous growth models [1-2, 13-15] the economic growth rate, proceeding from economic development in examined period, is determined. The problem of alternative use of scientific innovations is considered:

- whether to create scientific innovations, which for the certain period monopolize appropriate sphere of production, involving essential additional income until the new innovation in this area will appear, thus depriving a country of existing monopoly and additional profits,

- or to simulate and adapt advanced technologies, the application of which, at other equal conditions (capital and labor), is cheaper and, to a certain extent, more trustful.

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