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Traveling Salesman Problem with Transportation*

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Abstract

Traveling Salesman Problem (TSP) is a generic name that includes diverse practical models. Motivated by applications, a new model of TSP is examined — a synthesis of classical TSP and classical Transportation Problem. Algorithms based on Integer Programming cutting-plane methods and Branch and Bound Techniques are obvious.

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1 Introduction

The TSP gained notoriety over the past century as the prototype of problem that is easy to state and hard to solve practically. It is simply formulated: *a traveling salesman has to visit exactly once each of n cities and to return to the start city, but in such order that the respective tour (Hamiltonian cycle) has the minimal total cost (it is supposed that the cost c_{ij} of traveling from every city i to every city j is known).* There are other related formulations of this problem and a lot of methods for their solving [1-5].

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TSP is representative for a large class of problems known as NP-complete combinatorial optimization problems. This class (of NP-complete problems) has an important property that all of them simultaneously have or don't have polynomial-time algorithms [6]. To date, no one has found efficient (polynomial-time) algorithm for the TSP solving. But, over the past few years many practical problems of really large size are solved. Thus, at present, a record exactly solved problem (with branch-and-cut algorithm) has 24 978 cities (Applegate, Bixby, Chvatal, and Cook - 2004).

The Transportation Problem is the well known classical problem [7]. There are several efficient methods for its solving [8, 9, 10].

TSP with Transportation and Fixed Additional Payments [11] generalizes these two problems.

2 Formulations of the TSPT.

Given an orgraf $G = (V, E), |V| = n, |E| = m$. Each node $j \in V$ has its own capacity δ_j (demand, if $\delta_j < 0$, supply, if $\delta_j > 0$), such that $\sum_{j=1}^{n} \delta_j = 0$. Salesman starts his traveling from node $k \in V$ with $\delta_k > 0$. A unit transportation cost through arc $(i, j) \in E$ is c_{ij}. If the arc (i, j) is active, then additional payment d_{ij} is demanded. If $(i, j) \notin E$, then $c_{ij} = d_{ij} = \infty$. Find the Hamiltonian cycle and the starting node $k \in V$ with the property that the respective salesman traveling satisfies all demands with minimal cost.

A TSPT may be formulated as integer programming problem. Let x_{ij} denotes the quantity of product which is transported via (i, j). Let $y_{ij} \in \{0; 1\}$ be equal 1 if $x_{ij} > 0$, and $x_{ij} = 0$ if $y_{ij} = 0$. In such notations the TSPT, as stated above, is equivalent to the following problem:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} x_{ij} + d_{ij} y_{ij}) \to \min, \tag{1}$$

$$\sum_{i=1}^{n} y_{ij} = 1, \quad j = \overline{1, n}, \tag{2}$$

$$\sum_{j=1}^{n} y_{ij} = 1, \quad i = \overline{2,n}, \tag{3}$$

$$\sum_{k=1}^{n} x_{jk} - \sum_{i=1}^{n} x_{ij} = \delta_j, \quad j = \overline{1,n}, \tag{4}$$

$$u_i - u_j + n y_{ij} \leq n-1, \quad i,j = \overline{2,n},\ i \neq j, \tag{5}$$

$$x_{ij} \leq M y_{ij}, \quad i,j = \overline{1,n}, \tag{6}$$

$$x_{ij} \geq 0,\ y_{ij} \in \{0;1\},\ u_j \geq 0, \quad i,j = \overline{1,n}, \tag{7}$$

where $M = \sum_{j=1}^{n} |\delta_j|$.

If $c_{ij} \equiv 0$, $\forall i,j$, then (1)-(7) is the classical TSP. If $d_{ij} \equiv 0$, $\forall i,j$, then (1)-(7) is the classical Transportation Problem.

Theorem. *TSPT and problem (1)-(7) are equivalent.*

Proof. (1)-(3), (5) and (7) define Hamiltonian cycle [8]. (1), (4), (7) state the transportation problem [7]. (6) realizes the connection between both *"facets"* of the TSPT. The starting node $k \in V$ is determined by elementary searching. □

If each arc $(i,j) \in E$ has upper bound capacity $u_{ij} > 0$, we obtain another capacitated model of the problem. In this case inequalities

$$x_{ij} \leq u_{ij} y_{ij},\ (i,j) \in E,$$

are substitute for (6).

If each arc $(i,j) \in E$ has also a lower bound capacity $l_{ij} > 0$, then inequalities

$$l_{ij} y_{ij} \leq x_{ij} \leq u_{ij} y_{ij},\ (i,j) \in E,$$

are substitute for (6).

Kuhn [8] restrictions (5) may be substituted by equivalent restrictions:

$$\sum_{i \in K} \sum_{j \in K} y_{ij} = |K| - 1,\ \forall K \subset V,$$

where K is any proper subset of V.

3 Algorithms

It is obvious that the solution of the classical TSP does not solve TSPT.

The branch-and-bound algorithm may be constructed on backtracking technique for branch generation and the value of 1-tree plus T_0 for lower bound estimation, where T_0 is calculated at first step and represents the value of minimal cost flow problem obtained in relaxed problem without Hamiltonian cycle requirement. For efficient bounding, T_0 may be substituted at every step with exact cost of transportation through respective fragment of the cycle.

Direct solving (1)-(7) with Gomory type cutting-plane algorithms is rational for problem with modest size. In recent vogue opinion, the branch-and-cut super-algorithm [9, 10] may be much more recommended for TSPT.

Finally, note that dynamic programming approach comports difficulty as the TSPT optimal value depends really (practically) on first node from which the travel is starting. This fact may be simply taken in consideration in previous methods, but not in dynamic programming method.

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