Computer Science Journal of Moldova, vol.**13**, no.3(39), 2005

CGEM for Moldova with Factor Markets and Intermediate Inputs

Elvira Naval

Abstract

In this article the attempt to examine the effects of tax policy in the framework of two-sector model with factor markets and intermediate goods is considered. In this connection the difference between intermediate and final goods is introduced. Model was extended to introduce the imported and domestic intermediate goods and production functions to produce domestic goods and exports.

The basic model refers to one country with two producing sectors and three goods. Two commodities that the country produces are an export good, which is sold to foreigners and is not demanded at home, and a domestic good, which is only sold domestically. The third good is an import, which is not produced domestically. There is one consumer who receives all income. The country is small in world markets, facing fixed world prices for exports and imports. Three economic agents operate in the model: a producer, a household, and the rest of the world.

Adjustments in the real exchange rate in response to exogenous shocks to the economy and three price-wedge government policy instruments: an import tariff, an export subsidy, and an indirect tax on domestic sales are included. The single household saves a fraction of its income. Real government expenditure is assumed fixed and the government deficit or surplus is subtracted or added to aggregate savings. The balance of trade is assumed to represent foreign savings.

In order to estimate efficiency effects of tax policy, it is very important to distinguish between intermediate and final goods. In this

scope the Calculated General Equilibrium Model (CGEM) is extended to introduce the imported and domestic intermediate goods and also production functions that use primary factors of production (capital and labor) to produce domestic and exports goods.

The model contains most features of the basic model, although for simplicity the savings and investment have been eliminated, and the balance of trade is assumed fixed. The separate production functions for exports and domestic goods with associated demand functions for capital, labor, and intermediate goods as inputs are introduced. Equilibrium conditions are included for each factor that induces an assumption of full employment of fixed aggregate stocks of capital and labor. There is the distinction of two types of imports (each with its own tariff rate): consumption imports, which together with domestic output provide the composite good, and intermediate imports, which in combination with domestic output produce composite intermediate goods required to produce domestic and exported goods. The domestically produced and imported intermediate goods, as with consumption, are assumed to be imperfect substitutes, so the demand for each depends on its relative price and the elasticity of substitution. It is assumed that all tax revenue is rebated to consumers in a lump-sum fashion, so the government spending appears as a component of total income. And the foreign capital inflow is also added to private income. Now this model will be used to define the optimal structure of the tariff rates.

Further the equations of the CGEM with factor markets and intermediate inputs are presented. In the case of Republic of Moldova there were examined: production functions for exported and domestically fabricated goods which use primary factors of production (capital and labor).

Presented model was used for the Moldova economy which is highly depended on imported consumption and intermediate goods in order to evaluate tariffs policy when composite consumption Q^S is maximized.

Let's describe briefly all equations of the model. First, two equations (eq.1-2) are the production functions for exports and domestic goods. Equations (7-12) are the associated demand functions for cap-

ital, labor and intermediate goods as inputs. Equilibrium conditions are included for each factor (eq. 26-28) supposing full employment of fixed aggregate stocks of capital and labor.

Flows

$$E = K_e^{1-\alpha_e} L_e^{\alpha_e} \quad (1)$$

$$D = K_d^{1-\alpha_d} L_d^{\alpha_d} \quad (2)$$

$$Q^S = Q(M_q, D_q) \quad (3)$$

$$N = N(M_n, D_n) \quad (4)$$

$$M_q/D_q = f_1(P_q^m, P^d) \quad (5)$$

$$M_n/D_n = f_2(P_n^m, P^d) \quad (6)$$

$$N_e = a_e \cdot E \quad (7)$$

$$N_d = a_d \cdot D \quad (8)$$

$$W_K = \partial E/\partial K_e \cdot P_e^v \quad (9)$$

$$W_K = \partial D/\partial K_d \cdot P_d^v \quad (10)$$

$$W_L = \partial E/\partial L_e \cdot P_e^v \quad (11)$$

$$W_L = \partial D/\partial L_d \cdot P_d^v \quad (12)$$

$$Y = W_K \cdot \overline{K} + W_L \cdot \overline{L} + R \cdot \overline{B} + G \quad (13)$$

$$Q^D = Y/P^q \quad (14)$$

Prices

$$P^e = (1 + t^e) \cdot R \cdot P_w^e \quad (16)$$

$$P^q = (P_q^m M_q + P^d D_q)/Q^S \quad (17)$$

$$P^n = (P_n^m M_n + P^d D_n)/N \quad (18)$$

$$P_q^m = (1 + t_q^m) \cdot R \cdot P_{w_q}^m \quad (19)$$

$$P_n^m = (1 + t_n^m) \cdot R \cdot P_{w_n}^m \quad (20)$$

$$P^t = (1 + t^d) \cdot P^d \quad (21)$$

$$P_e^v = P^e - a_e \cdot P^n \quad (22)$$

$$P_d^v = P^d - a_d \cdot P^n \quad (23)$$

Equilibrium Conditions

$$D_n + D_q - D = 0 \quad (24)$$

$$Q^D - Q^S = 0 \quad (25)$$

$$N_e - N_d - N = 0 \quad (26)$$

$$K_e + K_d - \overline{K} = 0 \quad (27)$$

$$L_e + L_d - \overline{L} = 0 \quad (28)$$

$$P_{w_q}^m \cdot M_q + P_{w_n}^m \cdot M_n - P_w^e \cdot E = \overline{B} \quad (29)$$

$$T - G = 0 \quad (30)$$

$$R \equiv 1 \quad (31)$$

Identities

$$P_q \cdot Q^Q \equiv Y$$

$$P_e^v \cdot E \equiv W_K \cdot K_e + W_L \cdot L_e$$

$$P_d^v \equiv W_K \cdot K_d + W_L \cdot L_d$$

$$P^q \cdot Q^s \equiv P_q^m \cdot M_q + P^d \cdot D_q$$

$$P^n \cdot N \equiv P_n^m \cdot M_n + P^d \cdot D_n$$

Table 1: **Endogenous Variables**

E	Export good	D	Supply of domestic good
N_e	Interm. demand by E	N_d	Interm. demand by D
N	Supply of comp. interm. input	Q^S	Supply of comp. final good
Q^D	Demand for comp. final good	D_q	D used in consumption
D_n	D used in interm. inputs	K_e	Capital used to produce E
L_e	Labor used to produce E	K_d	Capital used to produce D
L_d	Labor used to produce D	M_q	Imports used in consum.
M_n	Imports used in N demand	P^e	Export price
P^d	Domestic producer price	P^t	Domestic consumer price
P_q^m	Price of consumption imports	P_n^m	Price of interm. imports
P_e^v	Value added price of exports	P_d^v	Value added price of D
P^n	Price of interm. input	P^q	Price of composite good
W_K	Return to capital	W_L	Wage rate
R	Exchange rate	Y	Total income
T	Tax revenue	G	Government transfers

Table 2: **Exogenous Variables**

$\overline{L}$	Total labor supply	$\overline{K}$	Total capital stock
$\overline{T}$	Total tax accumulation	$\overline{B}$	Balance of trade
P_w^e	World price of E	$P_{w_q}^m$	World price of M_q
$P_{w_n}^m$	World price of M_n	t^e	Export subsidy rate
t_q^m	Tariff on M_q	t_n^m	Tariff on M_n
t^d	Indirect tax rate		

There are two types of imports, each with its own tariff rate. Consumption imports (M_q) which in combination with domestic output (D_q) produce composite good (Q^S). Intermediate imports (M_n) in combination with domestic output (D_n) produce composite intermediate goods (N) required to fabricate domestic and exported goods. The domestically produced and imported intermediate goods (N) as well as consumption are assumed to be imperfect substitutes, so the demand for each depends on its relative price and the elasticity of substitution (eq. 6).

The production structure may be represented as follows. Real value added and intermediate inputs are combined in fixed proportions to produce output. There are two CES functions: one for labor and capital to produce real value added and one for imported and domestic intermediates to produce the composite intermediate input. Fixed coefficients are used for the demand for composite intermediate input and allow to define value added price for each sector.

The year 2004 was selected as base year, and all macroeconomic indicators are presented both in real and nominal prices. All prices are equal to unity in this base. Using base year data and known values for the elasticity of substitution between imports and domestic goods (σ) equal to 2 for consumer goods and 0.5 for intermediate goods there are calculated the analytical expressions for export (E), demand for domestic goods (D), supply of composite goods (Q^S), volume of intermediate goods (N), ratios between imported consumption goods and domestic consumption good and ratio between imported intermediate inputs and domestic inputs. So, for six functions enumerated earlier the explicit analytical expressions with constant coefficients are obtained. The obtained expressions for these functions are annexed here:

$$E = K_e^{0.702} \cdot L_e^{0,398}, \quad D = K_d^{0.66} \cdot L_d^{0,4545},$$

$$Q^S = 1,136 \cdot (0,5148 \cdot M_q^{-0,5} + (1 - 0,5418) \cdot D_q^{-0,5})^{-1/0,5},$$

$$N = 2,001 \cdot (0,9854 \cdot M_{nq}^1 + (1 - 0,9854) \cdot D_n^1)^1,$$

$$M_q/D_q = (0,5148 \cdot P_q^m/((1 - 0,5418) \cdot P^d)^2,$$

$$M_n/D_n = (0,9854 \cdot P_n^m/((1 - 0,9854) \cdot P^d)^{0,5}.$$

In order to evaluate efficiency resulting from taxation just as in [1], it is assumed that all tax revenue is returned to consumers as a unique sum, so that government spending appears as a component of total income in (eq.13). Finally, foreign capital inflow is also added to private income.

Further, this model is explored for answering the question about optimality of the uniform tariff rates. It is well known view that differences among tariff rates create a distortion in the economy. Different

Table 3: **Optimal tariffs with fixed indirect tax**

%	BV	15%	10%	5%	Opt.	-5%	-10%	-15%
t^d		13,5	12,9	12,3	11,7	11,1	10,5	10,0
Optimum tariff rates								
t_m^q		13,5	14,7	17,0	18,0	19,7	23,5	24,0
t_m^n		28,4	25,9	22,6	18,0	15,6	8,8	9,2
Opt. quant.(Ratio(%) to base value)								
E	64,0	99,20	99,19	99,17	100,0	99,12	99,10	99,08
D	51,0	101,31	101,33	101,36	100,0	101,40	101,42	101,44
M_q	52,0	99,10	99,08	99,06	100,0	99,01	98,99	98,97
M_n	28,0	100,31	100,31	100,31	100,0	100,31	100,31	100,31
Q	100,0	100,77	98,39	96,28	100,0	96,28	98,39	100,77

sectors tariff rates imply that the relative domestic prices of two traded goods are not equal to their relative world prices. If world prices are viewed as the appropriate "shadow prices" of this traded good, a varied tariff structure represents a distortion.

As stabled in [2] there are other distorting taxes in the economy, then the shadow prices of this traded good in this environment may not equal to world prices. In particular, if the domestic indirect tax structure is not optimal, the optimal tariff structure will generally not be uniform. Starting with various assumptions about the level of domestic indirect taxes the optimal patterns of tariffs will be determined. The numerical solution of the formulated optimization problem using statistical data for the year of 2004 will be obtained because the introducing of the intermediate goods complicates the model so much that the model cannot be solved analytically.

Base year data for numerical application are presented in Table 4. Here all data are calculated in % to GDP which is equal to 100%, and with the exported national economy equal to 64%. The export sector is capital-intensive and uses domestic and imported intermediates. The domestic sector (D) also uses domestic and imported intermediates.

Table 4: **SAM for Moldova Numerical Model**

Moldova	**Expenditures**							
	E	D	Q	N	L	K	C	Rw
Export (E)								64
Domestic (D)			48	3				
Final Good (Q)							100	
Interm. Good (N)	15	16						
Labor (L)	10	8						
Capital(K)	39	43						
Consumer					18	82		
Rest of World			52	28				
Total	64	67	100	31	18	82	100	64

The balance of trade is equal to 17% and the single consumer thus demanded 100% units of composite consumer good Q. Since all prices and wages are equal to one, the SAM (Social Accounting Matrix, see Table 4) indicates real as well as nominal magnitude. The value added production functions are assumed to be Cobb-Douglas.

It is supposed that government receives revenue from tariffs on final and intermediate goods and on indirect tax on domestic sales. Export subsidy is equal to zero. Government requires total tax revenue of 25%. The base year data given in the Table 4 represent a unique solution of the model with all tax rates set to zero. In the optimal tax experiments tax rates are redefined as variables offering freedom of choice. Optimal taxes are obtained by maximizing Q, which satisfies the equations of the model in examination plus the government revenue constraint. Alternative scenario is obtained by fixing one tax, the indirect tax on domestic goods (T^d), and solving for optimal level for remaining tax rates.

The results of calculus are given in Table 2. The optimal pattern of tariffs is uniform only in this case when all tariff rates including the indirect tax on domestic goods is also set optimally. When the indirect tax on domestic goods is set below its optimal value then optimal tariff structure consists in higher tariff on the final goods than

on the intermediate goods. When the indirect tax is above its optimal, the opposite is true. In this case when domestic indirect taxes are too low rather than too high the appropriate policy rule is that tariff rates on imported intermediate goods should be lower than the rates for imported consumer goods.

So it is no reason to move toward equal rates by raising the lowest tariffs and lowering the highest ones. Yet in our country tariffs on intermediate goods are lower than those on final goods so in this circumstances moving toward equal rates would lower welfare.

Along with a highly variable tariff structure the second scenario shows smaller variations in real variables. But in this case aggregate welfare demonstrates small changes across both scenarios. This result is consistent with results from a large number of empirical studies.

Given that, the model is solved as a nonlinear programming problem using SOLVER package from Excel. The solution generates dual or shadow prices for all constrains. The solution value for the shadow price on the government revenue constraint (eq. 30) directly measures the welfare cost of raising an additional unit of tax revenue (T). In Table 3, there are presented these shadow prices for the cases in which the indirect tax rate is set below its constrained optimal value. Results for two alternative models are calculated. One model is solved for optimal changed tariffs - as in table two. And the other model in which tariffs are constrained to be the same for both goods. In this case, the single tariff rate is uniquely determined by the government revenue constraint and the fixed indirect tax rate. There are no policy degree of freedom.

Let's comment the results obtained in Table 3. The results of calculus show that in both scenario there are observed the diminishing of the marginal welfare cost when additional government revenue is received from increasing tax revenue. So in the case of Moldova there exist reserves in rising indirect tax rate. Yet in the case of equal tariffs there are more possibilities in reducing welfare cost by absolute value, than in the case of differential tariffs.

These results, as mentioned in [1], are only suggestive because of the stylized nature of the model, but they create some theoretical under-

Table 5: **Welfare Cost of increasing Tax revenue**

	Marginal welfare cost of increasing tax revenue as a % of additional revenue		
Indirect tax rate (%)	Sc.I (equal tariffs)	Sc.II (differential tariffs)	Ratio (%) of Sc.I to Sc.II
11,7 (optimal)	0,0	0,0	
11,1 (-5%)	-6,42	-6,0	106,5
10,5 (-10%)	-6,54	-5,3	123,4
9,9 (-15%)	-6,67	-5,2	127,8

pinning for the common policy rule that countries should unify their tariff structure. And the results from this simple two sector model appear vigorous in comparison with larger applied models [2].

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Elvira Naval,

Received December 3, 2005

Institute of Mathematics and Computer Sciences,
Academy of Sciences, Moldova
5, Academiei Str., Kishinev,
MD–28, Moldova
E–mail: *nvelvira@math.md*