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Involving d-Convex Simple and Quasi-simple Planar Graphs in $\mathbb{R}^3$

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Abstract

The problem of finding dimension of d-convex simple and quasi-simple planar graphs is studied. Algorithms for involving these graphs in $\mathbb{R}^3$ are described.

1 Prelinimary Considerations

A connected graph can be considered as a metric space. Elements of this metric space are vertexes of the graph; distance between two vertexes is minimal length of chains between them. If we denote through $d(x,\, y)$ distance between two vertexes $x,\, y$ of graph $G = (X;\, U)$, then this graph defines a discrete metric space $(X;\, d)$. Let X' be a finite space, where a metric of whole values ρ is defined. It is known that independently of metric ρ there always exists a graph $G = (X;\, U)$ such that $X' \subseteq X$ and distance among vertexes of subset X' in G coincides with distance ρ of a space X' (see [1]). In this case we say that metric ρ is involved in graph G or metric ρ is realized in graph G. A lot of results about realization of metric in special graphs are exposed in [2] and [3].

It has also practical and theoretical interest the mutual problem. We will say that a graph $G = (X;\, U)$ is involved in a metric space X', if there exists an application $\varphi : X \to X'$, such that any two adjacent vertexes $x,\, y$ in G have the images $\varphi(x)$ and $\varphi(y)$ in X', that are of distance equal to 1. In other words, adjacent vertexes of graph G are transformed into elements of distance 1 of space X'. The minimal dimension of an Euclidean space, where a graph G can be involved is called *dimension of graph* G and denoted *dim* G. The problem itself can be formulated as follows: for any graph $G = (X;\ U)$ to find its dimension. Of course in this case we can talk about elaboration of an algorithm that will be allowed to compute dimension of a non-oriented graph.

In general case, the formulated problem is hard enough. In this article we will give some results that refer to finding dimension of some special graphs: *d-convex simple and d-convex quasi-simple graphs.*

2 Dimension of d-Convex Simple Planar Graphs

By definition, a subset of vertexes $A \subset X$ of a graph $G = (X;\ U)$ is called *d-convex* if for each two vertexes $x,\ y \in A$, the following relation is true:

$$< x,\ y >= \{z \in X;\ d(x,\ z) + d(z,\ y) = d(x,\ y)\} \subset A\ (see[4]).$$

Definition 1 [5]. *A non-oriented graph* $G = (X, U)$ *is called d-convex simple if any subset of vertexes* $A \subset X,\ 2 < |A| < |X|$ *is not d-convex.*

Since in any graph the empty set, the set formed of 1 vertex, the set formed of two adjacent vertexes and the set of all vertexes are d-convex, the result is that d-convex graphs are graphs with minimal number of d-convex sets. If we denote the family of all d-convex sets of cardinal k by D_k, and the set of all d-convex sets of a graph with n vertexes and m edges by D, so that

$$D = \bigcup_{k=0}^{n} D_k,$$

then

$$|D| \geq |D_0| + |D_1| + |D_2| + |D_n| = 1 + n + m + 1 = n + m + 2.$$

In case of d-convex graph here we have always equality $|D| = n+m+2$. Let us denote by $\Gamma(x)$ the neighborhood of vertex x, i. e. $\Gamma(x) = \{y \in X | x \sim y\}$.

Definition 2 [5]. *A vertex y is called copy for vertex x ($x \neq y$), in graph $G = (X;\, U)$ if $\Gamma(x) = \Gamma(y)$.*

Let T be a tree with at least 3 vertexes and T_0 a sub-graph of T, that consists of all vertexes and edges of T, without those suspended. So, for each unsuspended vertex x from T, we have a uniquely correspondent vertex $\bar{x}$ from T_0, and for each vertex of T_0 we have a uniquely correspondent vertex from T. Let $L(T,\, T_0)$ be a graph obtained from T, T_0 and by adding the following edges: every vertex $\bar{x}$ of T_0 will be adjacent with all vertexes from $\Gamma(x)$ from T, where x and $\bar{x}$ are correspondent vertexes. It is easy to see that in graph $L(T,\, T_0)$ every vertex of degree at least 3 has a unique copy and there are no suspended vertexes.

The next theorem is true:

Theorem 1 [6]. *If T is a tree with at least 3 vertexes, then graph $G = L(T,\, T_0)$ is d-convex simple and planar.*

As follows from [6] the graph described above has deal with a class of d-convex simple planar graphs.

Theorem 2 [6]. *For any d-convex simple planar graph $G = (X,\, U)$, $|X| > 3$, there exists a tree T such that $G = L(T,\, T_0)$.*

The theorem 2 determines a structure for d-convex simple planar graphs, that we are going to use for finding the dimension of these graphs.

All d-convex simple graphs with number of vertexes $n < 5$ are in fig. 1.

It is easy to see that $dim\, K_1 = 0$, $dim\, K_2 = dim\, P_3 = 1$, $dim\, K_3 = dim\, C_4 = 2$ (see fig. 1.). Let now $G = (X;\, U)$ be a d-convex simple planar graph, with $|X| \geq 5$. In this case G has at least one vertex x with degree at least 3, which has also a copy $\bar{x}$. Let us suppose that $\Gamma(x) = \Gamma(\bar{x}) = \{y_1,\, y_2,\, y_3, \ldots, y_p\}$, $p \geq 3$. If we suppose that $dim\, G = 2$ then elements of set $\Gamma(x)$ are placed on circle of radius 1 with centre in one point that corresponds to vertex x. On the other

Figure 1. K_1, K_2, P_3, K_3 and C_4 respectively.

hand, the same vertexes should be on circle of radius 1 with centre in one point, that corresponds to vertex $\bar{x}$. This means that in $\mathbb{R}^2$ we can draw two different circles of radius 1, which intersect each other in $p,\ p \geq 3$ points, that is impossible. So we have $dim\, G \geq 3$. We will show now that $dim\, G = 3$.

Theorem 3 *If* $G = (X, U),\ |X| \geq 5$ *is a d-convex simple planar graph then* $dim\, G = 3$.

Proof: Let $G = (X,\ U),\ |X| \geq 5$ be a d-convex simple planar graph. According to the theorem 2 there exists a tree T such that $G = L(T;\ T_0)$. We will prove theorem using mathematical induction by number of vertexes $n = |X| \geq 5$. The unique d-convex simple planar graph with $n = 5$ is graph in fig. 2. Let us choose in $\mathbb{R}^3$ two points A, B at distance less than 2 and draw two spheres of radius 1, with centers in A and B. The intersection of these spheres is a circle $\mathcal{C}$. Let us fix on circle $\mathcal{C}$ three points $D_1,\ D_2,\ D_3$. If we place vertexes x and $\bar{x}$ of graph from fig. 2 in points A and B, and $y,\ y',\ y''$ in points $D_1,\ D_2,\ D_3$ and join them by segments of length one to x and $\bar{x}$, then we obtain an inclusion of this graph in $\mathbb{R}^3$.

Let us suppose that assertion of theorem is true for any d-convex simple planar graph G with $n \leq k,\ k \geq 5$ vertexes.

We will examine now case $n = k + 1$. According to the theorem 2, for graph G there exists a tree T such that $G = L(T;\ T_0)$. For any

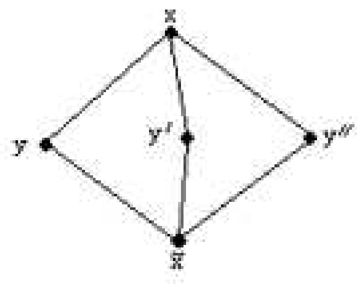


Figure 2. d-Convex simple planar graphs with 5 vetrtexes.

arbitrary vertex x from T we will denote by $deg_T\ x$ and $deg_G\ x$ the degree of x in tree T and graph G respectively.

Let x, y be two adjacent vertexes in tree T, such that $deg_T\ x = 1$. It is obvious that $deg_G\ x = 2$. Since according to the conditions of this theorem the graph G has at least 5 vertexes, we have the result that $deg_T\ y \geq 2$. We will analyze two cases.

I $deg_T\ y \geq 3$. If we eliminate the suspended vertex x from T, then we obtain a tree T', where y is not a suspended vertex. This implies that tree T'_0 coincides with T_0. According to the theorem 1 the graph $G' = L(T', T'_0)$ is d-convex simple and planar. Because G' has exact one vertex less than G, by mathematical induction we have $dim G' = 3$. Let $\bar{y}$ be copy of y and its image in T_0. In G' these two points are also copies one for another and theirs neighborhoods coincide. Let $\Gamma(y) = \{\omega_1,\ \omega_2, \ldots,\ \omega_p\} = \Gamma(\bar{y}),\ p \geq 3$. When we involve the graph G' in $\mathbb{R}^3$, then vertexes from $\Gamma(y)$ are placed on the circle that is at intersection of two spheres of radius 1, with centers in y and $\bar{y}$. If we place on this circle the vertex x, then distances from it to y and $\bar{y}$, as centers of spheres, are equal to 1. As the result, the graph G can be involved in $\mathbb{R}^3$, so $dim\ G = 3$.

II $deg_T\ y = 2$. So in T there exists a vertex $z \neq x$ and adjacent to y. Because G has at least 5 vertexes, then $deg_T\ z \geq 2$. This means that tree T_0 contains vertexes $\bar{y}$ and $\bar{z}$, which are images of vertexes y and z. So, in graph G there are true relations:

$$\Gamma(y) = \Gamma(\bar{y}),\ |\Gamma(y) \geq 3|$$

$$\Gamma(z) = \Gamma(\bar{z}),\ |\Gamma(z) \geq 3|$$

When we eliminate the vertex x from T, we obtain a new tree T', where y is a suspended vertex ($deg_{T'}\ y = 1$). In this case the tree T_0', which is obtained from T' by elimination of all suspended vertexes, differs from T_0 by one vertex $\bar{y}$. So $T' = T - x$, $T_0' = T_0 - \bar{y}$. We denote $G = L(T', T_0')$. By mathematical induction $dim\, G = 3$. Since $|\Gamma(z) \geq 3|$, then when we involve the graph G' in $\mathbb{R}^3$, the set of vertexes $\Gamma(z)$ is placed on the circle, that is at intersection of two spheres of radius 1, with centers in z and $\bar{z}$. On this circle we can place the vertex $\bar{y}$ (the vertex y is already on this circle as a vertex of graph G'). Now since y and $\bar{y}$ are placed on circle of diameter less than 2, then intersection of two unitary spheres with centers in y and $\bar{y}$ is a new circle, where we can place the vertex x. As the result, the graph G can be involved in $\mathbb{R}^3$, so $dim\, G = 3$.

By mathematical induction the assertion is true for any natural n. Theorem is proved. □

Now we are going to study a class of graphs with a wider family of d-convex sets.

3 Dimension of d-Conves Quasi-Simple Planar Graphs

Definition 3 [6]. *A graph $G = (X, U)$ is called d-convex quasi-simple graph if any d-convex set $A \subset X$ forms in G a complete sub-graph.*

It is obvious that each d-convex simple graph is d-convex quasi-simple. D-convex quasi-simple graphs with $|X| < 5$ are graphs in fig. 1 and fig. 3.

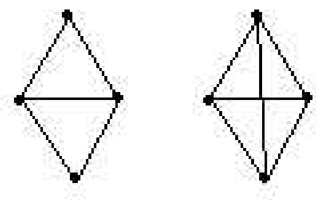


Figure 3. Graphs P and K_4.

Dimension of graphs in fig. 3 is 2 and 3 respectively.

Definition 4 *An edge $u = (x, y)$ of a graph G is called diagonal edge if in G there exist at least 2 vertexes s, t adjacent with x, y.*

Theorem 4 [6]. *If $G = (X, U)$ is d-convex quasi-simple planar graph then there exists a d-convex simple planar graph $G_b = (X, U_b)$ (called basis graph) such that G is obtained from G_b through adding some diagonal edges.*

In order to describe the structure of d-convex quasi-simple planar graphs we will describe next class of graphs. Let T be a tree with at least 3 vertexes. We will denote by $\mathcal{R}(T)$ the set of graphs that is obtained from T through adding some new edges, which will respect next conditions.

1. each new edge is incident to two old vertexes in T, the distance between which is equal to 2;
2. each new edge is incident to at least one suspended vertex;
3. in new graph the degree of each suspended vertex in T will be equal to at most 3;
4. if the new edge (x, y) join one suspended vertex with one unsuspended vertex from T and z is that vertex of T, that was between x and y, then $deg_T\ z = 2$;

5. a) if T is not a star, then in new graph there do not exist simple cycles, that are formed from suspended vertex of T;
   b) if T is a star and in new graph there exists a cycle that consists only from suspended vertexes, then this cycle passes through all suspended vertexes of T;

Like above we will denote by T_0 a sub-graph of T that consists of all vertexes and edges of T without those suspended. Let R be a graph from $\mathcal{R}(T)$. Let us construct graph $L(R,\ T_0)$ like above.

Theorem 5 [7]. *The graph $G = (X,\ U)$, $|X| > 4$, is d-convex quasi-simple planar graph if and only if there exists a tree T and a graph $R \in \mathcal{R}(T)$ such that $G = L(R,\ T_0)$.*

It is easy to see that for the d-convex quasi-simple planar graph $L(R,\ T_0)$ one basis graph is $L(T,\ T_0)$. The question is, on how many the dimension of d-convex quasi-simple planar graphs changes comparatively with the dimension of d-convex simple planar graphs. We are going to answer it.

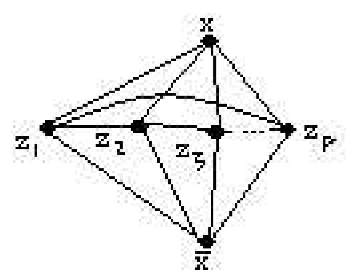


Figure 4. Class Q.

Condition 5. b) gives a special class of d-convex quasi-simple planar graphs. Any graph from this class is formed from two vertexes copies x and $\bar{x}$ with $\Gamma(x) = \Gamma(\bar{x}) = \{\ y_1,\ y_2, \ldots,\ y_p\ \}$, $p \geq 3$, such that $z_1 \sim z_2 \sim z_3 \sim \ldots \sim z_p \sim z_1$(see fig. 4.). Let us denote this class by Q. First, we are going to examine the class of graphs Q. These graphs differ by number of suspended vertexes in the star T. Let us denote by $Q_n,\ n > 2$ a graph of this class with n suspended vertexes in T. The problem is to inscribe in a circle of radius less than 1 a regular, closed polygonal line with unit length of edges. If we would be able to do this then the graph Q_n will be placed in space like in the fig. 5., where there in the circle will be the closed polygonal line we are talking about.

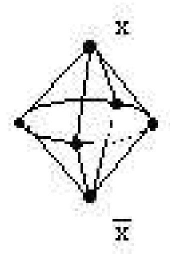


Figure 5. Q_n in 3-space.

For $n = 3,\ 4$ and 5 the circles are just those we inscribe a equilateral triangle, a square and a regular pentagon, because radiuses of these circles are less than 1. The circle where a unit regular hexagon can be inscribed has radius exactly equal to 1. That is why the graph Q_6 remains as a limit case. In order to study general case we need to prove first 3 lemmas.

Lemma 1 *The natural odd numbers $n = 2k + 1$ are relatively prime with k.*

Proof: Let us assume that the greater common divisor $(2k+1,\ k) = m$, then $\exists\ p,\ q \in \mathbb{N}$ such that:

$$2k + 1 = mp; \quad k = mq$$

or 2mq+1=mp, equivalent with m(p-2q)=1. The product of 2 natural numbers is 1 if and only if each of them is equal to 1, so $m = 1$. □

Lemma 2 *The natural even numbers of kind $n = 4k$ are relatively prime with $2k - 1$.*

Proof: Let us assume that the greater common divisor $(4k,\ 2k-1) = m$, then $\exists\ p,\ q \in \mathbb{N}$ such that:

$$4k = mp;\ \ 2k-1 = mq$$

or $2(mq+1) = mp$, we have $m(p-2q) = 2$. The product of 2 natural numbers is 2, so either $m = 1$, or $m = 2$. The number m could not be equal to 2 because it is divisor for an odd number $2k-1$, the result is $m = 1$. $\square$

Lemma 3 *The natural even numbers of kind $n = 4k+2$ are relatively prime with $2k-1$.*

Proof: Let us assume that the greater common divisor $(4k+2,\ 2k-1) = m$, then $\exists\ p,\ q \in \mathbb{N}$ such that:

$$4k+2 = mp;\ \ 2k-1 = mq$$

or $2(mq+1)+2 = mp$, we have $m(p-2q) = 4$. The product of 2 natural numbers is 4, so either $m = 1$, or $m = 2$, or $m = 4$. The number m can not be equal to 2 because it is divisor for an odd number $2k-1$, m also can not be equal to 4, as divisor of the number $4k+2$, the result is $m = 1$. $\square$

From algebra we know that every number g, that is relatively prime with n $(g < n)$, is a generator for the group $\mathbb{Z}_n$, i. e. we have $\mathbb{Z}_n =< \bar{g} >= \{\ \bar{g},\ \overline{g+g},\ \overline{g+g+g}, \ldots,\ \underbrace{\overline{g+g+\ldots+g}}_{n-times}\ \} = \{\bar{0},\ \bar{1},\ \bar{2}, \ldots, \overline{n-1}\ \}$. Now let us take a circle and place there all natural numbers from 0 till $n-1$, at equal distances one from another (see fig. 6.).

Let g be that number respectively prime with n from lemmas. If we will draw line from 0 to g, then from g to $g+g(mod n)$, and so on, then we will draw a polygonal line, that will be closed, because g is a cyclic generator of all these numbers and regular, because every line is based on the portion of circle of the same length. If we ask for the length between two numbers be equal to 1, then we obtain that

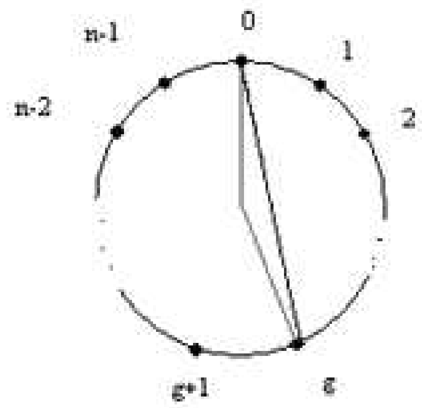


Figure 6. Closed polygonal regular line inscribed in circle.

radius of the circle in fig. 6. is $r = \frac{1}{2\sin(\alpha)}$, $\alpha = \frac{\pi g}{n}$. For $n = 2k+1$, $\alpha = \frac{\pi k}{2k+1} = \frac{\pi}{2}\frac{2k}{2k+1}$. For $n = 4k$, $\alpha = \frac{\pi(2k-1)}{4k} = \frac{\pi}{2}\frac{2k-1}{2k}$. For $n = 4k+2$, $\alpha = \frac{\pi(2k-1)}{4k+2} = \frac{\pi}{2}\frac{2k-1}{2k+1}$. Since we are interested in $n > 6$, then we have $\frac{\pi}{6} < \alpha < \frac{\pi}{2}$, or $\frac{1}{2} < \sin(\alpha) < 1$, finally $\frac{1}{2} < r < 1$. The last fact implies that we can place vertexes $x,\ \bar{x}$ at distance $d = 2\sqrt{1-r^2}$, $(0 < d < \sqrt{3})$ and in this case the circle, that is at intersection of unit spheres with centres in x and $\bar{x}$, will have radius r.

We have already proved next theorem:

Theorem 6 *If $Q_n,\ n > 2,\ n \neq 6$ is a d-convex simple planar graph from Q, then dim $Q_n = 3$.*

Theorem 7 *If $G = (X,\ U)$, $|X| > 4$, $G \neq Q_6$, is a d-convex quasi-simple planar graph, then dim $G = 3$.*

Proof: Let G be a graph that satisfies conditions of theorem. If $G \in Q$ then according to the last theorem $dim G = 3$. If $G \notin Q$ then according to the theorem 5 there are a tree T, a graph $R \in \mathcal{R}(T)$ and T_0 such that $G = L(R,\ T0)$. According to the theorem 1, the graph $G_b = L(T;\ T_0)$

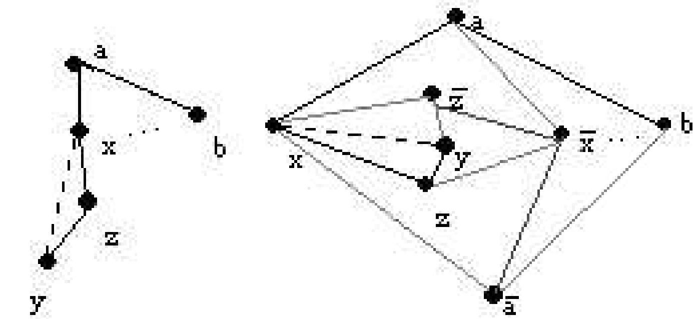
is d-convex simple and planar, so according to the theorem 3, it has dimension equal to 3. Let us place in space the graph G_b like in the theorem 3.

There can be two cases.

1. All new edges in R join only suspended vertexes of T. Then according to condition 5. a), we have either some separated edges, that join some suspended vertexes at distance 2 and according to condition 1 they have the same predecessor, or some chains, that join also suspended vertexes of the same predecessor, according to conditions 1 and 3. So, if we have that y is a predecessor in T for suspended vertexes, that are adjacent in R, then we can replace vertexes y and $\bar{y}$ at distance d, $(d < \sqrt{(3)})$ for example $d \leq 0,02$ and try to inscribe our chains in the circle formed between unit spheres with centres in y, $\bar{y}$. If we can't do this, because some vertexes pretend at the same place, then try again to modify distance between y and $\bar{y}$. We are sure that we can find the distance for our copies, for which we can place the chains. We can place these copies at distance at least $d = \sqrt{(1-r^2)}$, where like above, $r = \frac{1}{2\sin(\frac{\pi g}{n})}$, for $n = |\Gamma(y)|$ and g-computed from lemmas. We will draw the chains here in the same order like we do it above for cycles. So, we have that this graph is 3-dimensional.

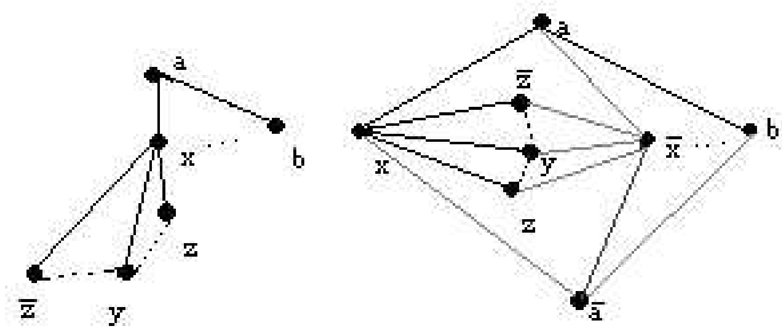
2. Through new edges there exist some, that join a suspended vertex with one unsuspended vertex, for example $(x,\ y)$ is a new edge with this property (see fig. 7. a)). Then in $L(R,\ T_0)$ this part of graph will be like in fig. 7. b).

   Now we will consider a new tree T^1, that will be identical with T, except the portion from fig. 7. a), where it will be like in the fig. 8. a), i. e. the vertexes y, z and $\bar{z}$ are suspended in T^1 and edges $(y,\ z)$, $(y,\ \bar{z})$ will be new edges, which will be added to R^1. Then the graph $L(R^1,\ T_0^1)$ looks like the graph $L(R,\ T_0)$, except the part of graph, where it looks like in the fig. 8. b). It is easy to see that graph $L(R,\ T_0)$ is a sub-graph of $L(R^1, T_0^1)$,



a) b)

Figure 7. Portion of graphs R, G, where an edge joins a suspended vertex with the unsuspended one.



a) b)

Figure 8. Portion of graphs R^1, G^1, after transformations.

because of edge $(\bar{x}, y)$. If we will do the same transformations for each edge that joins unsuspended vertexes, then we will obtain a graph, that is d-convex quasi-simple planar graph and all new edges join only suspended vertexes. According to the first part of the proof, this graph can be involved in 3-dimensional space, after that we can exclude the additional edges. So this graph is also 3- dimensional.

Theorem is proved. □

4 Algorithm for Involving d-Convex Simple Planar Graphs in $\mathbb{R}^3$

Let $G = (X; U)$ be a d-convex simple planar graph with $|X| \geq 5$. According to the theorem 2, we can consider $G = L(T; T_0)$. Algorithm for involving G in $\mathbb{R}^3$ is an iterative algorithm, at each step of which the place of all vertexes from neighborhood of two copies-vertexes is found. In process of algorithm two special sets of vertexes are formed:

1. R - the set of all vertexes of the graph G, which we have not yet placed in $\mathbb{R}^3$. Of course, initially $R = X$.
2. S - the set of all pairs of copies-vertexes, which was already placed in $\mathbb{R}^3$, but their neighborhoods were not yet researched.

Description of algorithm:

1. The sets $R = X,\ S = \emptyset$ are formed;
2. Find any vertex $x \in R$, of degree greater than 2 and its copy $\bar{x}$. Include them in S, i. e. $S = S \cup \{(x, \bar{x})\}$. Place the vertexes $x, \bar{x}$ in $\mathbb{R}^3$, arbitrary at distance less than 2.
3. Take any pair of copies $(x; \bar{x}) \in S$ and change the sets:

$$S = S - \{(x; \bar{x})\}; \quad R = R - \{x; \bar{x}\}.$$

4. For every $y \in \Gamma(x) \cap R$:
   (a) if $degy = 2$, then place arbitrary the vertex y on circle that is at intersection of spheres with radius 1 and centres in x, $\bar{x}$. Modify the set R:
   $$R = R - \{y\};$$
   (b) if $deg\ y > 2$, then find $\bar{y} \in \Gamma(x) \cap R$, place arbitrary the vertex y, $\bar{y}$ on circle that is at intersection of spheres with radius 1 and centres in x, $\bar{x}$. Modify the sets S and R:
   $$S = S \cup \{(y;\ \bar{y})\}; \quad R = R - \{y;\ \bar{y}\};$$
5. If $S \neq \emptyset$ then go to 3, else STOP.

5 Algorithm for Involving d-Convex Quasi-simple Planar Graphs in $\mathbb{R}^3$

Let $G = (X; U)$ be a d-convex quasi-simple planar graph with $|X| \geq 5$, $G \neq Q_6$. According to the theorem 4 we have that any d-convex quasi-simple planar graph is obtained from the d-convex simple one by adding some new edges. These edges form either a family $\mathcal{L}$ of simple chains, or a simple cycle and then the graph is one from family Q. Algorithm for involving G in $\mathbb{R}^3$ is also an iterative algorithm, and in order to discribe it we will use the same sets of vertexes S and R.

Description of algorithm:

1. If $G \in Q$ then the pair of copies-vertexes is placed in $\mathbb{R}^3$ at distance
   $$d = 2\sqrt{1 - r^2}, \quad where\ \ r = \frac{1}{2\sin\frac{\pi g}{n-2}}$$
   and
   $$g = \begin{cases} k, & if\ n-2 = 2k+1 \\ 2k-1, & if\ n-2 = 4k, \quad or\ n-2 = 4k+2 \end{cases}$$
   The other vertexes of G are placed in $\mathbb{R}^3$ like it is described in proof of theorem 6. If $G \notin Q$ then go to 2.

2. The sets $R = X$, $S = \emptyset$ are formed and find a pair of copies-vertexes x, $\bar{x}$ from G, set d=0,01; Change the set:
$$R = R - \{x;\ \bar{x}\}.$$

3. Put d=d+0,01. If $d \geq \sqrt{3}$ then put $d = \sqrt{1 - r^2}$, where r is computed like in 1. for $n = |\Gamma(x)|$. Place the vertexes x, $\bar{x}$ in $\mathbb{R}^3$ at distance d.

4. From vertexes $\Gamma(x) \cap R$ form two new sets W_1, W_2:
$$W_1 = \{y \in \Gamma(x) \cap R \,|\, \Gamma(x) \cap \Gamma(y) = \emptyset\}; \quad W_2 = (\Gamma(x) \cap R) - W_1;$$
If $W_2 \neq \emptyset$ then go to 5, else go to 6.

5. The vertexes from W_2 generate in G a family of simple chains. Try to place all vertexes from W_2 on the circle between x, $\bar{x}$, such that adjacent vertexes to be placed at distance 1. If we can do this then $R = R - W_2$ and go to 6, else go to 3.

6. If $W_1 = \emptyset$ then go to 7, else for every $y \in W_1$
   (a) if $deg y = 2$, then place arbitrary the vertex y on circle that is at intersection of spheres with radius 1 and centres in x, $\bar{x}$. Modify the set R:
   $$R = R - \{y\};$$
   (b) if $deg\ y > 2$, then find $\bar{y} \in \Gamma(x) \cap R$, place arbitrary the vertex y, $\bar{y}$ on circle that is at intersection of spheres with radius 1 and centres in x, $\bar{x}$. Modify the sets S and R:
   $$S = S \cup \{(y;\ \bar{y})\}; \quad R = R - \{y;\ \bar{y}\};$$

7. If $S \neq \emptyset$ then take any pair $(x;\ \bar{x})$, put $S = S - \{(x;\ \bar{x})\}$, $R = R - \{x,\ \bar{x}\}$, d=0,01 and go to 3, else STOP.

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