The solution of problem of objects classification as the method of restoration of objects images

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Abstract
This paper deals with problems of restoration of images on incomplete information of objects. In present paper the solution of the problem of restoration of defective images by using classification of objects is suggested.

1 Introduction
An important property of human brain is the figurative perception of the world. This property allows on the basis of acquaintance with final number of objects to find out with the certain reliability an infinite number of their variations, for example, by the incomplete, deformed or defective images to restore the true image of object. Another interesting property of human brain is the classification of input information. This property means the ability of a brain to react to the infinite set of conditions of external world by finite number of reactions. A person breaks data into groups of similar, but not identical phenomena. Different persons, training on a various material of supervision, equally and independently from each other classify the same objects. This is the objective character of images.

2 Restoration of images with the help of Kohonen networks.
An image or class is considered to be the classification grouping, uniting certain group of objects by some sign. Objects of the same image...
can differ greatly enough from each other. For example, following portrayals can make the image of a coin: free of defects; with semi-effaced emblem; with semi-effaced value of a coin; with semi-effaced face of emperor; with scratches, etc. Other examples of images – triangular prism \( A = \{a_1, a_2, \ldots, a_n\} \) and rectangular triangle \( B = \{b_1, b_2, \ldots, b_m\} \) – are presented in fig.1.

![Figure 1](image_url) Image of triangular prism and rectangular triangle.

On the basis of the specified properties it is supposed to reduce problem of restoration of defective objects to problem of classification of objects. Thus, the problem of objects classification is a method of solution of the problem of restoration of defective images.

The problem of classification consists in division of objects into classes on the basis of vector of object parameters. Objects within one class are considered to be equivalent according to a criterion of division. Frequently, classes are unknown beforehand, and formed dynamically (for example, in Kohonen networks). In this case, classes depend on shown objects and the consequent addition of a new one demands the correction of system of classes.

Let objects be characterized by a vector of parameters \( x_n \in X \), which has \( K \) components: \( x_n = (x_1, x_2, \ldots, x_K) \), where \( X \) is a space of objects. Set of classes \( C_q = \{C_1, C_2, \ldots, C_Q\} \) is in the space of classes \( C : C_1 \cup C_2 \cup \ldots \cup C_Q \subset C_q \). Let’s define nucleuses of classes \( c_q = \{c_1, c_2, \ldots, c_Q\} \) in the space of classes \( C \) as true objects for its
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class. For example, if for the classification of geometrical figures the
following parameters are chosen:
{quantity of corners, types of corners and their number, quantity
of direct lines},
then the nucleus of the class "triangular prism" will have the following
values of parameters:
{18 corners, 6 sharp corners, 12 right angles, 9 lines} (fig. 2).

Figure 2. Nucleus of the class “triangular prism”.

It is possible to relate to this class the defective object with the
following values of parameters:
{13 corners, 5 sharp corners, 9 right angles, 11 lines},
as from nucleuses ”cylinder”, ”parallelepiped”, ”pyramid”, ”triangular
prism” the parameters of considered object are most of all similar to
the nucleus “triangular prism” (fig. 3). The quantitative estimation

Figure 3. Defective object of class “triangular prism”.

of affinity of an object to a nucleus is defined by a measure of affinity
d(\(x_n, c_q\)) of the object and the nucleus of the class which is as less,
as more this object is similar to the nucleus of the class. Measure of
affinity of two nucleuses of classes is: d(\(c_1, c_2\)). The Euclid measure
represents geometrical distance between objects in many-dimensional space of attributes:
\[ d(x, y) = \sqrt{\sum_{i=1}^{N} (x_i - y_i)^2}. \]

A class is formed by group of vectors, the distance between which inside the group is less, than the distance up to the next groups. Inside classes objects should be closely connected among themselves, but objects of different classes should be far from each other (the requirement of compactness of classes). Distribution of objects inside classes should be uniform (fig. 4).

![Figure 4. Distances between object and a nucleus (1) and two nucleuses (2).](image)

The problem of classification for given number \(Q\) of classes is formulated as: to find \(Q\) nucleuses of classes \(\{c_q\}\) and to break objects \(\{x_n\}\) into classes \(\{C_q\}\), i.e. to construct the function \(q(n)\) so that to minimize a total measure of affinity for the whole set of input objects \(\{x_n\}\):
\[
\min\{D = \sum_{n=1}^{N} \sum_{i=1}^{K} (x_{ni} - c_{q(n)i})^2 \}.
\]

The function \(q(n)\), which determines the class number by index \(n\) of set of objects \(\{x_n\}\), sets the splitting into classes and is the solution of the problem of classification. In the elementary case \(\mathbf{X} = \mathbf{C}\), the space
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of objects $X$ is broken into areas $\{C_q\}$, and if $x_n \in C_q$, then $q(n)=q$ and the object is ascribed to the class $q$.

Presence of nucleuses of classes allows using the neural networks trained without teacher (Kohonen networks) for restoration of defective images. For input data the vector of parameters of object $x_n$ (the defective image) is used. The vector values of nucleuses of classes are given to synapse weights of neurons. Every neuron remembers one nucleus of a class, being responsible for definition of the objects in its class, and gives the sum $y_i$ on the output. The total number of classes coincides with the number of Kohonen neurons. The neuron output value is the greater, the closer object is to the given nucleus of the class. The vector of defective image, input to the Kohonen network, makes active one of the neurons. Neuron with maximal output defines the class to which the object presented on input belongs.

Synapse weights of every neuron represent the n-dimensional vector-column $w = [w_1, w_2, \ldots, w_K]^T$, where $K$ is the dimension of input vectors. Before the beginning of training a network it is required to initialize weight coefficients of neurons. Usually to synapse weights of neurons the normalized small uniformly distributed random numbers are given initially.

The problem of training consists in teaching the network to activate the same neuron for similar input vectors. If the number of input vectors is equal to the number of nucleuses (neurons), then training is not required. It is enough to give to nucleuses the values of input vectors, and each vector will activate its own Kohonen neuron. If the number of classes is less than the number of input vectors, then the training will consist in consecutive correction of synapse weights of neurons. On each step of the training one of the vectors is selected randomly from initial data set, and then the search of the vector of neuron coefficients, which is most similar to it, is carried out. The most similar coefficient vector is defined by the neuron-winner which has the maximal output value. Similarity is understood as distance between vectors, calculated in Euclidean space. For i-th neuron-winner we have:
\[ |x_n - w_i| = \min_j \{|x_n - w_j| \}. \]

Updating the weight coefficients is made according to the expression:

\[ w_i^{t+1} = w_i^t + h_i^t(x_n^t - w_i^t), \]

where

- \( w_i^{t+1} \) - new value of weight which the i-th neuron has gained;
- \( w_i^t \) - previous value of this weight;
- \( h_i^t \) - function of neuron neighborhood;
- \( x_n^t \) - randomly chosen input vector on t-th iteration.

The training of Kohonen network with uniformly distributed random vectors of weights (nucleuses of classes) is graphically presented in fig. 5.

![Figure 5. Training of Kohonen network: on the left - untrained network; on the right - trained network.](image)

In the fields of space \( \mathbf{X} \) in which the nucleuses are far from all training vectors, neuron \( c_1 \) will never win, and its weights will not be corrected by training. In those areas where there are a lot of input vectors, the density of nucleuses is small, and the unlike objects will
activate the same neuron \( c_k \). These lacks are connected with the initial assignment of the uniformly distributed random numbers to the synapse weights of neurons. The problem is solved by allocation of nucleuses according to the density of input vectors. But the distribution of input vectors frequently happens to be unknown beforehand. In this case at training the method of convex combination is used, which allows to distribute the nucleuses of classes (vectors of weights) according to the density of input vectors in the space \( X \). Method is realized as follows:

- assignment of the identical initial value to all weights:

\[
\begin{align*}
\omega_{ij} &= \frac{1}{\sqrt{\dim X}},
\end{align*}
\]

where \( \dim X \) – diameter of a class;

- setting of training set \( \{x_n\} \) and carrying out the training with vectors:

\[
\beta(t)x_n + \frac{1 - \beta(t)}{\sqrt{\dim X}}
\]

where \( t \) – time of training;

\( \beta(t) \) – monotonously growing function in an interval \([0,1]\).

In the beginning of training the function \( \beta(t) = 0 \) and all the vectors of weights and of training set have the same value (fig. 6.1). In the process of training the function \( \beta(t) \) grows, the training vectors diverge from a point with coordinates \( 1/\sqrt{\dim X} \) and approach to their true values \( x_n \) (fig. 6.2) which are reached at \( \beta(t) = 1 \). Each vector of weights grasps the group or one training vector and traces it in the process of growth of the function \( \beta \). As a result in the network remains no untrained neuron and the density of weight vectors corresponds to the density of vectors of training set (fig. 6.3). The process of increasing of \( \beta \) demands many iterations that results in the increase of training time.

Method of convex combination gives correct distribution of density of nucleuses. And in network remain no untrained neurons which occur at usual training.
Kohonen Self Organizing Map is the competitive neural network in which neurons compete with each other for the right to be combined with the input vector in the best way. During self-organization the Kohonen map configures neurons according to topological representation of initial data, and vectors similar in the initial space, appear beside one another on the obtained map. Usually neurons are settled down in nodes of bidimensional network. At that neurons interact with each other. The value of interaction is defined by distance between neurons on the map. The class structure can be reflected by visualization of the distances between vectors of neuron weights. The values of distances define colors by which the node will be painted. Using gradation of gray color, the more is the distance, the darker the node is painted. For color palette the distance is defined according to a color scale (fig. 7). On presented Kohonen map two classes of objects are determined. By black points the vectors of defective images of objects, used at training, are marked. Empty cells mean the vectors of all possible defective images concerning given classes of objects. Thus, the analysis of Kohonen map allows to specify a priori with which defects the images can be restored.

3 Conclusions

From the point of view of the problem of restoration of defective images the application of method of convex combinations gives the following
advantages:

- The method does not need obligatory presence of nucleus of class (true image). The nucleus of a class is formed during the network training on the basis of available defective images of the object, and can be specified in the process of appearance of new defective images.

- When only defective images of the object are present it is possible with sufficient degree of accuracy to specify a location of class nucleus (the true image of the object) by calculation of the center of gravity of the class. It is possible as, in the case of choice of Euclid measure of closeness, the nucleus of a class, that minimizes the sum of measures of closeness for objects of this class, coincides with the center of gravity of objects:

\[
c_q = \frac{1}{N_{q,n,q[n]=q}} \sum x_n,
\]

where \( N_q \) - number of objects \( x_n \) in the class \( q \).
The carried out analysis of the problem of classification shows that the problem of restoration of defective image consists only in definition of belonging of an object to some class. The restored or true image will be the nucleus of the class.

References


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