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Communication Equivalence Classes in Networks

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Abstract

The quotient structure defined by a specific equivalence relation allows to simplify the study of complex communications between processes in computer networks. The quotient also preserves the initial algebraic structure.

This paper describes an algebraic structure modelling the communication in computer networks; it continues the study initiated in [1]. Given two communicating processes, we consider the translations between their communication ports, and introduce an equivalence relation between them in order to express that semantics of the communication between processes does not depend on the permutations of the communication ports. Considering a property P preserved by this abstract semantics of the communication, we first define P-translations, and then provide conditions for permutation-independent communication to preserve the property P. The quotient structure defined by this equivalence relation allows to simplify the study of a complex communication between processes to the study of the equivalence classes representatives. The quotients preserve the initial algebraic structure.

A **communication structure** is given by the following elements:

(i) $\mathcal{P}$ – the set of processes; $p, q, s, \dots$ range over $\mathcal{P}$;

(ii) for every process $p \in \mathcal{P}$ we associate the set of communication ports $H(p)$, and a permutation group $S^p_{H(p)}$;

(iii) for every process $p \in \mathcal{P}$, and for every subset of handles $K \subseteq H(p)$, we associate a permutation subgroup over K, $S^p_K = S(K) \cap S^p_{H(p)}$, where $S(K)$ is the permutation group over K; obviously, $S(K)$ can be viewed as a subgroup of $S(H(p))$.

The reader can find more information on the algebraic notions and results in [3] and [4]. Since $H(p)$ is the set of all communication ports of a server process p, the permutations of $S^p_{H(p)}$ describe the communication symmetry of p, which expresses the possibility to change the communication ports without affecting the communication. We can remark that:

1. If $H \subseteq K \subseteq H(p)$, then $S^p_H \leq S^p_K$.
2. If $H \subseteq K \subseteq H(p)$, $\rho \in S^p_K$ and $\rho/_{K\setminus H} = id_{K\setminus H}$, then $\rho \in S(H)$. Clearly, ρ is also in $S^p_{H(p)}$; therefore $\rho \in S(H) \cap S^p_{H(p)} = S^p_H$.
3. The fact that $H \subset K$ does not imply that $S^p_H < S^p_K$; it is possible to have $S^p_H = S^p_K$.

Let $\mathcal{P}$ and $\mathcal{Q}$ be two communication structures; **a correspondence** between two processes $p \in \mathcal{P}$ and $q \in \mathcal{Q}$ is a triple $(S^p_H, \varphi, S^q_{H'})$, where $H \subseteq H(p)$, $H' \subseteq H(q)$ and $\varphi : S^p_H \to S^q_{H'}$ is a group morphism. If φ is one-to-one, we call it a **faith** correspondence. If φ is onto, we say that the correspondence is **full**.
We denote by $\mathcal{C}(p, q)$ the set of all correspondences between $p \in \mathcal{P}$ and $q \in \mathcal{Q}$.
$\mathcal{C}(\mathcal{P},\ \mathcal{Q}) = \bigcup\{\mathcal{C}(p, q) \mid p \in \mathcal{P}, q \in \mathcal{Q}$.

Let $\mathcal{P}$, $\mathcal{Q}$ and $\mathcal{S}$ be three communication structures. The composition of correspondences is a partial binary operation $\circ : \mathcal{C}(\mathcal{P}, \mathcal{Q}) \times \mathcal{C}(\mathcal{Q}, \mathcal{S}) \longrightarrow \mathcal{C}(\mathcal{P},\mathcal{S})$ defined by

$$(S^p_H, \varphi, S^q_{H'}) \circ (S^q_{H'}, \varphi', S^s_{H''}) = (S^p_H, \varphi' \circ \varphi, S^s_{H''}),$$

where $p \in \mathcal{P}$, $q \in \mathcal{Q}$, $s \in \mathcal{S}$ and $H \subseteq H(p)$, $H' \subseteq H(q)$, $H'' \subseteq H(s)$.
We can associate a category $Comm(\mathcal{P})$ to each communication structure $\mathcal{P}$ defined as it follows:

(i) the **objects** pH are pairs (p, H), where $p \in \mathcal{P}$ and $H \subseteq H(p)$;

(ii) a **morphism** $\varphi : pH \to p_1H_1$ is a correspondence $(S^p_H, \varphi, S^{p_1}_{H_1})$.

The notation $\varphi \in Comm(\mathcal{P})(pH, p_1H_1)$ means that $\varphi : pH \to p_1H_1$ is a morphism of the category. The category $Corresp(\mathcal{P}, \mathcal{Q})$ of the correspondences from the structure $\mathcal{P}$ to the structure $\mathcal{Q}$ is defined by the following elements:

(i) the **objects** are triples (pH, φ, qH'), where $pH \in |Comm(\mathcal{P})|$, $qH' \in |Comm(\mathcal{Q})|$, and $(S^p_H, \varphi, S^q_{H'})$ is a correspondence from p to q;

(ii) a **morphism** from (pH, φ, qH') to $(p_1H_1, \varphi_1, q_1H'_1)$ is a pair (α, β) of morphisms $\alpha : pH \to p_1H_1$, $\beta : qH' \to q_1H'_1$ such that $\beta\,\varphi = \varphi_1\,\alpha$.

Let $\mathcal{P}$ and $\mathcal{Q}$ be two communication structures, and $Comm(\mathcal{P})$ and $Comm(\mathcal{Q})$ their associated categories. We say that $\mathcal{P}$ and $\mathcal{Q}$ are isomorphic, and we denote this by $\mathcal{P} \simeq \mathcal{Q}$, if their associated categories are isomorphic, namely there is a functor $\mathcal{F} : Comm(\mathcal{P}) \to Comm(\mathcal{Q})$ that is an isomorphism of categories.

If $\mathcal{P}$ and $\mathcal{Q}$ are two communication structures, a **communication map** between $\mathcal{P}$ and $\mathcal{Q}$ is a mapping $\mathcal{F}$ from $|Comm(\mathcal{P})|$ to $|Corresp(\mathcal{P}, \mathcal{Q})|$. We say that a communication map $\mathcal{F}$ is **compatible with the correspondences** from $\mathcal{P}$ if $\mathcal{F}$ is a (covariant) functor from $Comm(\mathcal{P})$ to $Corresp(\mathcal{P}, \mathcal{Q})$. This means that

1. for each pH there is a unique $\mathcal{F}(pH) = (pH, \varphi, qH') \in$ $\in |Corresp(\mathcal{P}, \mathcal{Q})|$,
2. for each morphism $\alpha : pH \to p_1H_1$ of $Comm(\mathcal{P})$ there is a unique morphism $\mathcal{F}(\alpha)$ in $Corresp(\mathcal{P}, \mathcal{Q})$, and
3. $\mathcal{F}(\alpha\,\alpha_1) = \mathcal{F}(\alpha) \circ \mathcal{F}(\alpha_1)$ for all $\alpha, \alpha_1 \in Comm(\mathcal{P})$.

Let $\mathcal{P}$ be a structure, and $Comm(\mathcal{P})$ its communication category.

(i) For two given processes $p, q \in \mathcal{P}$, a **translation** $\delta : H \to H'$ between their communication ports is a bijection from pH to qH', where pH and qH' are objects in $Comm(\mathcal{P})$. We denote such a translation by $p_H \xrightarrow{\delta} q_{H'}$ (or simply by δ whenever there is no confusion), and by $\mathcal{T}(\mathcal{P})$ the set of all translations.

(ii) two translations $p_H \xrightarrow{\delta_1} q_{H'}$ and $p_H \xrightarrow{\delta_2} q_{H'}$ are equivalent (and we denote this by $\delta_1 \sim \delta_2$) if there exist two permutations $\rho \in S^p_H$, $\rho' \in S^q_{H'}$ such that $\delta_2 \circ \rho = \rho' \circ \delta_1$.

For each communication structure $\mathcal{P}$ we can define the category $Trans(\mathcal{P})$ of its translations, with the same objects pH as in $Comm(\mathcal{P})$ (namely pairs (p, H) with $p \in \mathcal{P}$ and $H \subseteq H(p)$), and the translations between communication ports as morphisms between objects.

Let us consider a property P of the translations, and $\Re = \{\delta \in \mathcal{T}(\mathcal{P}) \mid \delta \text{ satisfies } P\}$. We say that $\Re$ is a P**-translation** over the communication structure $\mathcal{P}$ if for each translation δ in $\Re$, its equivalence class $[\delta] \in \Re$, and if $p_H \xrightarrow{\delta} p_H \in \Re$, then $\delta \in S^p_H$. The following result provides a condition for $\approx_\Re$ to become an equivalence. In this way $|Comm(\mathcal{P})|/_{\approx_\Re}$ defines classes of objects having similar communication.

Theorem 1 *If $\Re$ is a P-equivalence over the communication structure $\mathcal{P}$, then $\approx_\Re$ is an equivalence relation over $|Comm(\mathcal{P})|$.*

Theorem 2 *If $pH \approx_\Re qH'$, then the groups S^p_H and $S^q_{H'}$ are isomorphic.*

Let $\mathcal{P}$ be a communication structure, $\Re$ a P-equivalence over the communication structure $\mathcal{P}$, and $\approx_\Re$ the corresponding equivalence relation. More results of the quotient set $|Comm(\mathcal{P})|/_{\approx_\Re}$ are presented in [2]; for instance, we prove that a communication structure defined on the equivalence classes does not depend on the classes representatives. Thus any two communication structures determined by two different representative choices are isomorphic. It makes sense to talk about the quotient communication structure with respect to a P-equivalence. Let $\Re$ be a P-equivalence and $\approx_\Re$ the associated equivalence relation. The following communication structure on $|Comm(\mathcal{P})|/_{\approx_\Re}$ is well-defined:

- processes are given by the equivalence classes $\{[pH] \mid p \in \mathcal{P},\ H \subseteq H(p)\}$;
- the set of handles $H([pH])$ is H;
- the group of permutations $S^{[pH]}_{H([pH])}$ is S^p_H.

This communication structure is called the **quotient communication structure** of $\mathcal{P}$ with respect to the P-equivalence $\Re$, and we denote it by $\mathcal{P}/_{\Re}$.

We can affirm that such a structure reflects the complexity of the communication along the computer networks. The need of a conceptual and formal framework which make possible the study of network communication problems and properties is necessary (in our opinion). This paper is a step to such a suitable framework. It defines and studies the communication structures, formal tools which are complementary to process algebras, and can describe, analyze, and provide a semantic framework to complex distributed systems involving process communication. The algebraic structure suggests constructive solutions for communication problems by considering the group actions on the communication ports, the existence of limits in our communication categories, and so on. A wealth of research topics remains to be considered.

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