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A Stochastic Map Model of Phase Transition in Internet Traffic

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Abstract

We analyse the macroscopic dynamics of packet flow density in a WAN from the standpoint of statistical physics. Our results show the existence of a phase transition in macroscopic Internet traffic flow, which is characterized by three typical behaviors. We propose a simple autoregressive map model based on empirical phase transition phenomena, and show that it can reproduce the basic properties of actual traffic.

1 Introduction

In traditional methods of modeling network traffic, packet arrival is often assumed to be Poisson process; namely, the arrival of each packet is assumed to have no correlation to those of the others. However, high time-resolution measurements from a variety of computer networks have shown that the packet traffic generally has a self-similar nature where the fluctuation of flow density or round trip time (RTT) exhibits correlations over a wide range of time scales [2, 3, 9, 11, 15, 19, 21]. In other words, packet arrivals are strongly correlated, so actual traffic behavior deviates greatly from the Poisson traffic model. Theoretical and experimental studies show that the queueing length distribution decays following a long-tail distribution when the input source is self-similar [5, 10, 12], though the distribution is exponential under Markovian traffic which includes Poisson traffic. Similarly, several research efforts are focusing on chaotic behavior of network traffic and its performance evaluation [1, 5, 18]. From the viewpoint of statistical physics, many phenomena that are self-similar can be recognized as the critical behavior of underlying phase transitions [13]. We have already found that the end-to-end network delay behavior can be well described by a framework of phase transition [7, 15, 16]. There are two phases in this phase transition phenomenon - the non-congested and congested phases - and a system will only be self-similar at the critical point between them. In the non-congested phase the total amount of traffic is low and the statistics of end-to-end delay fluctuation can be approximated by the Markovian process. Around the critical point the statistics clearly deviate from the Markovian process and we can observe self-similar behavior such as the 1/f power spectrum and a power law distribution of congestion duration times. In the congested phase, the probability of everlasting congestion occurring is non-zero, and the effect of large-scale congestion is not negligible. Thus, the distribution deviates from the Markovian process, and its temporal fluctuation also loses self-similarity. Namely, in this sense the phase transition formulation combines both Markovian and self-similar statistics as special cases and also introduces a new statistic for the congested phase. The end-to-end network delay reflects the fluctuation of queues in all routers along the observed path, so phase transition can be expected to be a global phenomenon involving many routers. Another type of phase transition, a local phase transition observed in the traffic flow of an output link on a router, is derived from theoretical analysis of the classical queue model [16]. It has been confirmed by numerical simulation that there are two phases just like the case where the global phase transition and self-similar statistics can be found at the critical point. In order to observe the local phase transition, we analyzed packet flow density fluctuations of an intermediate link in a WAN, and confirmed very recently that the flow density behavior in actual WAN traffic, whose data were collected at the entrance to a university, network also shows phase transition behavior [17]. Namely, the packet flow data in the link are categorized into three typical states: the non-congested state whose statistics are more like a Markovian process, the critical state in which typical critical behavior of self-similarity is confirmed, and the congested state, which follows neither Markovian nor self-similar behavior. In this paper we first review our recent observation results on the local phase transition of packet flow density illustrating the existence of the three states in the following section [17]. Based on these experimental findings we introduce a dynamic model of traffic in the third section. The model is the simplest autoregressive map with random noise and it reproduces the most basic properties of phase transition. The last section is devoted to a discussion on the validity of the map model and possible applications to flow control.

2 Observation of traffic flow fluctuations

In order to observe fluctuations of packet flow in wide-area Internet traffic, we set monitoring hosts on two measurement points in the Japanese Internet (Figure 1). One was located on an Ethernet cable that connects the gateway of Keio Univ. and the WIDE Internet backbone. Keio Univ. is one of the largest universities in Japan, so we expected to be a large amount of traffic. The data-link layer of the campus backbone is 20 Mbps ATM, and the university gateway is connected to the network operating center of the WIDE backbone by Ethernet (10 Mbps). Our monitoring host (indicated by monitor (B) in Figure 1) was connected to this segment via a non-intelligent hub. The other monitoring host was located at the entry point between the edge router of the WIDE Internet backbone and the router connected directly to San Francisco. This full-duplex point-to-point 1.5 Mbps link is one of the most heavily used links from Japan to other countries, and it is reported that about 60% of the traffic is http [20]. The monitoring host was connected on the 10Mbps Ethernet cable of the Japanese edge router side as indicated by monitor (A) in Figure 1. Note that there was no other traffic sources in the Ethernet cables in our experiment, thus we were able to collect only traffic passing through these links. The measurement hosts collected data for every packet that goes through these links (both upstream and downstream) into their local hard-disks via the "tcpdump" command. In all of the traces, each record consisted of a timestamp and the first 56 bytes of raw data including the link-level, IP, and TCP/UDP headers. We collected packet traces using "tcpdump" for 4 hours for each experiment. In this way, we obtained flow density and packet arrival density time sequences (unit size = 100 ms) for 144000 samples. In our analysis, we ignored the higher-level header information, since our objective was simply to investigate the statistical behavior of network traffic. At monitor (B) we collected 9 traces in the period from Nov. 1997 to Feb. 1998. These were categorized into three 4-hour periods, at night, morning, and afternoon where average mean flow densities (mean packet number densities) for each case were 284 kbyte/s (800 packets/s), 365 kbyte/s (1118 packets/s), 463 kbyte/s (1374 packets/s), respectively.

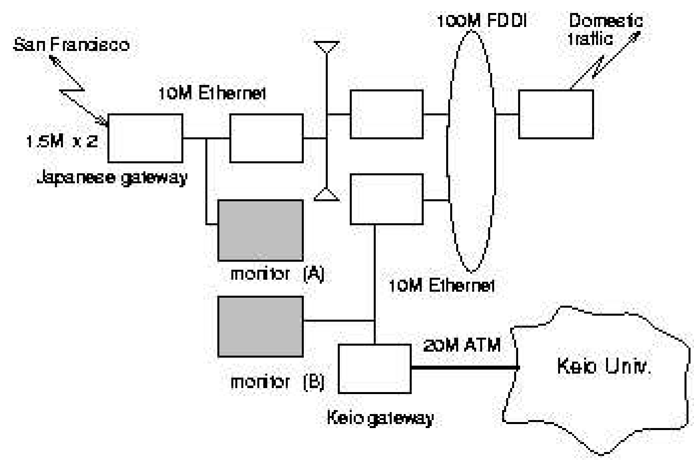


Figure 1. Network configuration

It should be noted that the mean flow density is physically limited by the link capacity, which is 10 Mbps (= 1250 kbyte/s) for the Ethernet, and the mean flow density is always smaller than this maximum value. Also the probability of "tcp dump" missing some packets was less than 0.5% in the worst case for these measurements. An example of temporal fluctuation of packet low at monitor (B) is shown in Figure 2. The fluctuation is usually very large but if we pay attention to shorter time behavior, we can find very crowded periods and sparse periods.

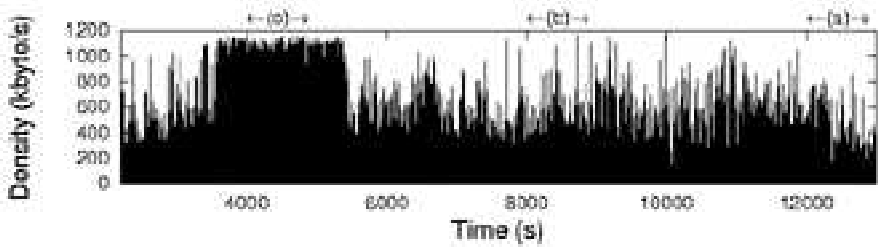
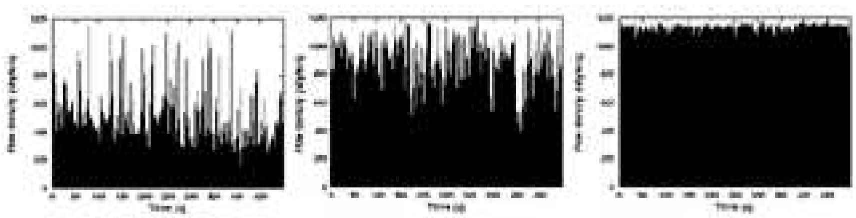


Figure 2. Example of flow density fluctuation



(a) Non-congested period (b) Intermediate period (c) Congested period

Figure 3. Typical traffic fluctuations

Figure 3, shows three typical examples with observation time units of 0.1 s: (a) Non-congested period (the flow density averaged over 500 s was about 200 kbyte/s), (b) intermediate period (about 500 kbyte/s), and (c) congested period (about 1000 kbyte/s) in the trace for monitor B. In the non-congested and intermediate periods, we can find large fluctuations, whereas, in the congested period the fluctuations in packet flow are obviously saturated due to the limited link capacity (10 Mbps = 1250 kbyte/s). In order to characterize these periods more clearly, we illustrate a histogram or the probability density of the flow in time units of 0.1 s. in Figure 4. Each plot in this figure was derived from 5000 samples; namely, the length of each observation period was set to 500 s.

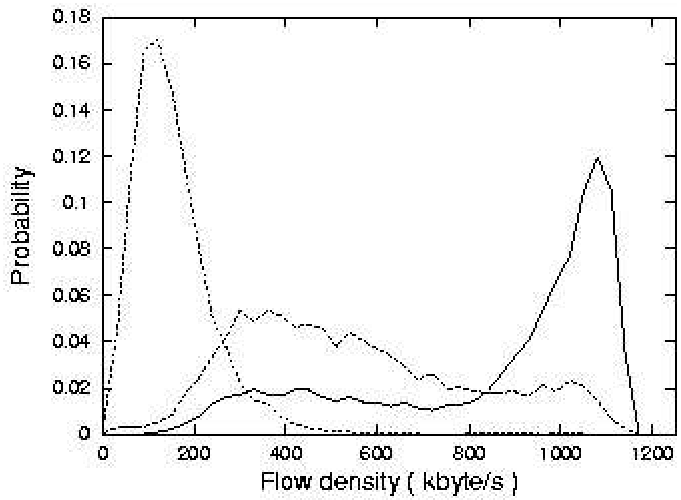


Figure 4. Probability density of the packet flow. The non-congested period averaged 150 kbyte/s (dotted line), the intermediate period averaged 500 kbyte/s (dashed line), and the congested period averaged 800 kbyte/s (solid line). The bin size was 50 kbyte/s.

In the non-congestion period, the distribution had a single peak around 100 kbyte/s. As the mean flow density increased, the distribution widely expanded. In particular the distribution was most widespread and became nearly flat around the density of 500 kbyte/s. The distribution for a higher mean flow period again had a single peak but its position was very different. We divided the original 9 time sequences of 4 hours into boxes of size 500 s. and confirmed that the shape of the probability density for each box changed smoothly as a function of the mean flow density interpolating the three curves in Figure 4. As described in the previous section, phase transition is accompanied by drastic changes in several macroscopic behaviors at the critical point. In order to confirm these dynamical changes, we focus on two quantities: peak position and half height. The peak position analysis depicts the transition of the mode in the probability density when the mean flow density increases. If given time sequences contain a phase transition, the peak position should not change linearly. The half width is used to quantify this variation in the variance of time sequences and is defined as the peak width at the half height of the probability density. The half width value takes a higher value when the fluctuation has larger variance, and we observed divergence of the half width value at the critical point, the same as for the correlation length [17]. In this analysis, the variance itself was not used because it tends to be affected by the small number of samples that deviate greatly from the mean value. The peak position and the peak width at half height of the probability density as functions of the mean flow density are plotted in Figures 5 and 6. The number of boxes observed was 252.

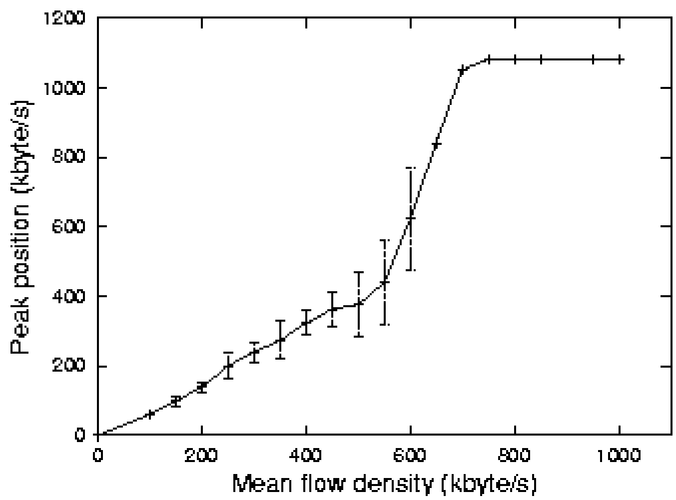


Figure 5. Transition of the peak flow position

When the mean flow density was less than 500 kbyte/s, the peak position was nearly proportional to the mean flow density, and the peak width increased gradually. This region corresponds to the non-congested period in which the probability density can be characterized by the peak position and peak width. Around 600 kbyte/s, the distribution drastically changed its peak position and the peak width was largest meaning that the fluctuation was largest. This case corresponds to the intermediate period. When the mean flow density was more than 700 kbyte/s, the peak position was nearly fixed at a value slightly smaller than the theoretical maximum flow density and the half width of the distribution also nearly converged to a certain value. The changes in probability densities can be viewed as a dynamic phase transition. Typical critical behavior,

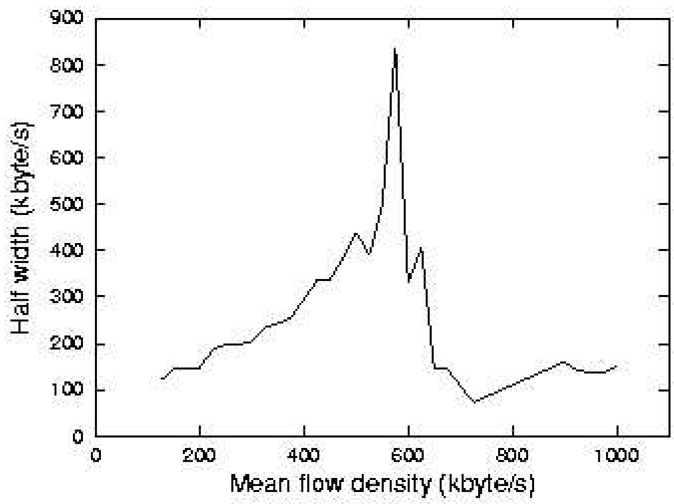


Figure 6. Peak width at half height

such as divergence of correlation or a power law cluster size distribution, has been confirmed for the same traces [17].

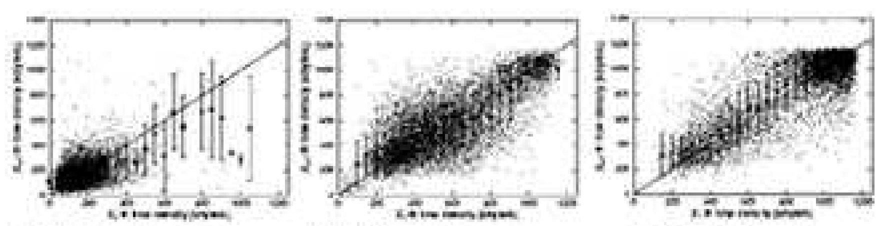
All of these results were qualitatively the same for other observation traces on monitor A and at a LAN inside the laboratory. In the case of monitor A, the flow density was generally large and finding non-congested periods was more difficult than in the case of monitor B. On the other hand our observation of LAN traffic showed that the flow density was always small. At monitor A we could estimate the critical flow density by above mentioned methods to be 200 kbyte/s which is about 60% of 1.5×2 Mbps (up and down streams), the physical upper-limit. However, in the case of LAN traffic all the traces we observed belonged to the non-congested period and we could not find the critical flow density.

3 Traffic model based on autoregressive map

In this section we introduce a new traffic model based on an autoregressive map. Pruthi and Erramilli proposed a self-similar traffic model based on chaotic autoregressive maps, which aimed to explain the property that aggregated data of ON/OFF sources obey a long-tailed distribution [12]. Our approach is different from theirs and our goal is to find the simplest model that reproduces the basic properties of actual traffic described in the preceding section. We assume a first-order autoregressive map with external random force,

$$X_{n+1} = F(X_n) + \eta_n \tag{1}$$

where X_n denotes the flow density at time step η_n, and the map function $F(X)$ is empirically constructed from the measurement data. The term η_n denotes an external noise. Figures 7 (a), (b), and (c) depict the flow density maps of X_n vs. X_{n+1} from real data with time step 0.1 s. for non-congested, intermediate, and congested periods, respectively. The data belong to the same boxes as used in Figure 4 obtained at monitor B.



(a) Non-congested period (b) Intermediate period (c) Congested period

Figure 7. Empirical maps for the three periods

In each figure there are 4999 samples and the squares with error bars represent the mean values and standard deviations for each flow density. In the non-congested period, both X_n and X_{n+1} values tend to concentrate around the peak point at 150 kbyte/s. For $X_n \geq 150$ the samples are lower than the line of $X_{n+1} = X_n$. The large fluctuations in the middle and higher flow density are due to the shortage of data samples. In the intermediate period, the samples are distributed over a wide range. They are roughly on the line $X_{n+1} = X_n$ with nearly equal variance in the middle flow density range, $400 < X_n < 1000$. For the smaller flow density range they are above the line $X_{n+1} = X_n$, and for very large X_n the samples are below the straight line. In the congested period the mean values are a little above the lines of $X_{n+1} = X_n$ in the middle range and many samples concentrate around $X_n = X_{n+1} = 1100$. We also checked other periods with different mean densities and confirmed that the map shapes changed continuously with the mean density. From these direct observations, we can roughly approximate $F(X)$ by a piece-wise linear function with two bending points.

The functional form of $F(X)$ we propose is given by

$$F(X) = \begin{cases} a_1 X + b_1, & \text{for } 0 < X < c_1 \\ X + b, & \text{for } c_1 \leq X \leq c_2 \\ a_2(X - 1.0) + b_2, & \text{for } c_2 < X \leq 1.0 \end{cases} \tag{2}$$

where the maximum flow is normalized to be 1.0 (see also Figure 8). The parameters c_1 and c_2 indicate the bending points, and b_1 and b_2 are the minimum and maximum values of X_{n+1}. Here, b is the most important parameter controlling the mean flow density. For $b < 0$ the generated traffic has a low mean flow density and belongs to the non-congested period; for $0 < b$, the generated traffic belongs to the congested period. The critical point is given by $b = 0$, which means that all the points over the range of $c_1 \leq X \leq c_2$ are fixed points, thus the variance of the produced time sequences is large. The values of a_1 and a_2 indicate the line slopes in the small and large flow density ranges and are determined by continuity condition of $F(X)$ as follows.

$$a_1 = \frac{(c_1 - b_1) + b}{c_1}, \quad a_2 = \frac{(b_2 - c_2) - b}{1.0 - c_2} \tag{3}$$

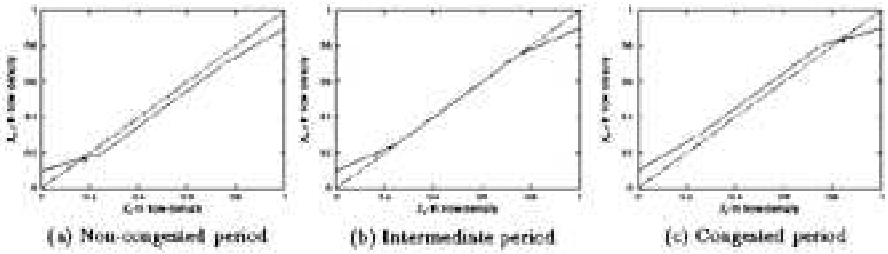


Figure 8. Map function of flow density

The external noise term η_n is an independent random variable with zero mean, and in the numerical simulation its distribution is uniform in the range of [-0.1, 0.1]. We can tell immediately from Figure 8 that the function $F(X)$ does not produce any chaotic behavior. When case $b \neq 0$ there is only one fixed point and it is attractive, namely, without the noise term η_n, X_n simply converges to a fixed point. At the critical case of $b = 0$, the range $c_1 \leq X \leq c_2$ is attractive and dynamic evolution is also trivial without η_n. To be realistic the value of b should also fluctuate, but we assume that it changes much more slowly than η_n and we fix its value in each numerical simulation in the following discussion. We set the parameters $c_1 = 0.25$, $c_2 = 0.75$, $b_1 = 0.1$, and, $b_2 = 0.9$, and generate the traffic for some b values. Figure 9 shows typical time evolutions for three different values of $b : -0.01$, 0.0, and 0.005, respectively. The corresponding probability densities are plotted in Figure 10. Comparing these figures with Figures 3 and 4, it is clear that numerically generated time sequences have the basic properties of a real flow.

The peak position plot (corresponding to Figure 5) and the half-width plot (corresponding to Figure 6) can also be reproduced, but fine tuning of the parameters c_1, c_2, b_1, and b_2 is required and we must also introduce small temporal fluctuations in the parameters, especially for b. In Figure 11 we plot a level set analysis [14] of the numerically produced data. We set the threshold value to $X_c = 0.3$ and regard $X_n > X_c$ as the "congested" state. We observe the length distribution

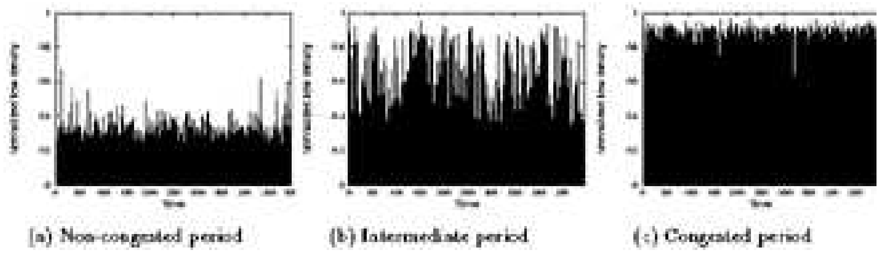


Figure 9. Examples of generated traffic

of time interval in which all X_n values are "congested". $P(\geq L)$ denotes the cumulative probability that a randomly chosen congested interval length is longer than L. It is known that the distribution approximately obeys a power law at the critical point. Away from the critical point, the distribution decays exponentially in the non-congested phase, and it deviates from the power law distribution again in the congested phase due to large-scale congestion.

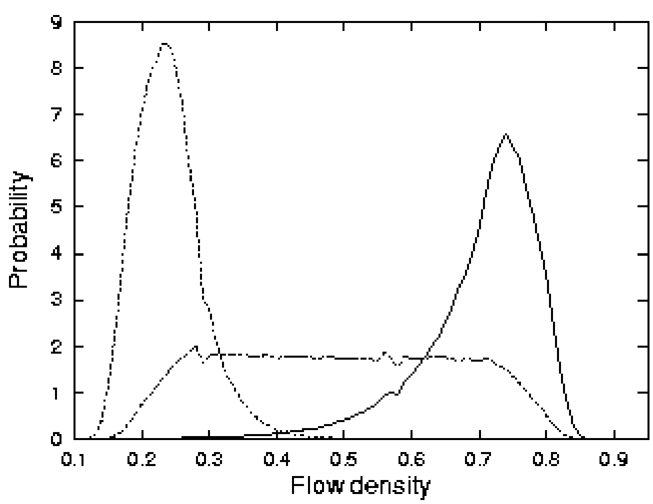


Figure 10. Probability density of generated traffic. Dotted line $b = -0.01$, dashed line $b = 0.0$, and solid line $b = 0.005$.

Also, the appearance of the power law distribution is independent of the selection of the threshold value X_c, because the power law distribution satisfies the scaling property. We checked several value of X_c, and obtained power law distributions with the same exponent at the critical point unless X_c was too high or too low. We can see from the figure that $P(\geq L)$ decays exponentially for negative b, meaning that the occurrence of congestion can be approximated by a Markovian process. At $b = 0.0$ we have a power law distribution, demonstrating that this case corresponds to the critical point of a phase transition. The exponent $1/2$ comes directly from the property of Brownian random walk due to the uncorrelated random noise η_n, and the range of L satisfying the power law is determined by the range of $[c_1, c_2]$ and the variance of η_n. For positive b a plateau is observed in the distribution of $P(\geq L)$, meaning that very long congestion occurs with finite probability. The exponent of the power law in the congestion duration distribution corresponds to the one in the power spectra distribution. We analyzed the power spectrum of the fluctuation at the critical point, and obtained the same exponent (1/2). The following relationship exists between the exponent and Hurst parameter, which indicates degree of self-similarity.

$$H = \frac{\alpha + 1}{2} \tag{4}$$

where α is the exponent of the power spectrum and H is the Hurst parameter.

The estimated Hurst parameter is 0.75 at the critical point. It is important to note that the Hurst parameter is not defined away from the critical point, because it can be defined only when the fluctuation is statistically self-similar like FGN (Fractal Gaussian Noise). Although all of these properties are qualitatively consistent with our observation of phase transition in a real network [17], there is a discrepancy in the value of the exponent of $P(\geq L)$. In our numerical simulation the exponent was $1/2$, but, the real value was very close to 1 according to our observations [17]. We consider that the exponent can be modified by introducing certain temporal fluctuations in b. This will be presented in a future paper.

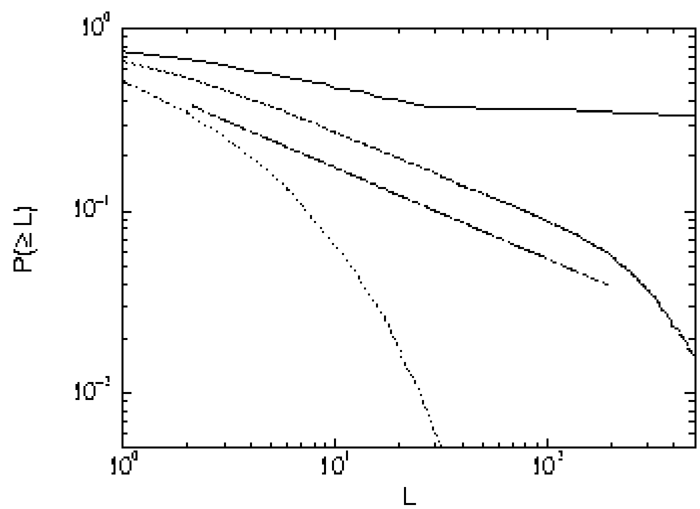


Figure 11. Distribution of congestion duration time of generated traffic on a log-log scale. The straight line shows the power law distribution, $P(\geq L) \propto L^{-1/2}$.

4 Discussion

We have shown experimental evidence that the fluctuations in flow density of inter network traffic can be interpreted as phase transition phenomena in statistical physics. We clarified that the value of the mean flow density, i.e., the control parameter in statistical physics terminology, changes in time. The low mean flow density traffic belongs to the non-congested period characterized by short-time dependency, for example, the congestion duration length obeys an exponential distribution. As the mean flow density increases, the traffic has larger fluctuation and the congestion duration exhibits self-similarity at the critical point. When the mean flow density exceeds the critical flow density, the traffic loses this self-similarity again. Thus, the self-similarity or the fractal property only represents a special case of complex real traffic behavior.

It is important to discuss the difference in time scales between packet forwarding time and congestion duration time. A router processes each packet very quickly on the order of less than a millisecond. On the other hand, congestion duration lasted up to the order of 100 s in our analysis.

Namely, the congestion state of a router and link affects a huge number of packets. This number is so large that the properties of individual packets become negligible and the statistical physics approach becomes meaningful. In the flow density analysis we observed the critical behavior at 550 kbyte/s at monitor B and at 200 kbyte/s at monitor A. Monitor B traces were obtained from the Ethernet (10 Mbps) link in a WAN, and the estimated critical flow density was nearly 50% of the link capacity (550 kbyte/s = 4.4 Mbps).

At monitor A located near the bottleneck link between Japan and U.S. with a bandwidth (including upstream and downstream) of 3 Mbps, traces determined the critical point to be 60% of the link capacity as mentioned in Section 2. So far we have only two examples but it is likely that the critical flow density is determined by many factors such as link bandwidth, potential performance of routers, link topology, and user applications. At present we have no answer to why the critical flow density is about half the maximum link capacity. One of the reasons, that network traffic, observed on long time scales, tends to show self-similarity, may be that network traffic spontaneously stays near the critical point because of the network control scheme.

One example of such an effect is that end-to-end flow control scheme like TCP can estimate the current congestion level using information about RTT and dropped packets, and the transmission rate is set adaptively to decrease the congestion level of a congested path. Another example is the general human response when the network is congested. Users tend to refrain from web browsing because of the excessive waiting times, and when the waiting time is very short people tend to use the network more.

Another possibility of finding self-similar behavior on a long time scale observation is that longer correlation periods may prevail over short ones. Assume that we have a long sequence of artificial data that is a composite of white noise periods and correlated periods having strong and long correlation. If we analyze the data as a whole we will find a long correlation, however, if we divide the data into small boxes as in Section 2 we will find the true property that the data is a composite of short and long correlations. A non-trivial problem in this type of analysis is what is the right size for the boxes. If the box size is too large we cannot separate the different periods, but, if it is too small the number of samples in each box is limited and no conclusive result can be derived from the statistics. Our choice of 500 s was determined as follows:

(1) The estimated maximum local correlation length was about 100 s [17] and the box size should be larger than this.

(2) In a probability density plot such as Figure 3, the number of data samples, 5000, is nearly the lower limit for distinguishing the shape difference.

There are several tasks to be solved concerning the map model of traffic density fluctuations. In the present paper we showed results for a fixed value of control parameter b, but, as described in Section 3, its value should fluctuate with time. The correlation of b values in different boxes is a completely open problem. In order to solve it we must get many traces to establish the statistics. Also as we discussed in Section 3 we may need to introduce short-time fluctuations in b to achieve a better fit with experimental traces.

From the theoretical viewpoint it is important to derive the map model from microscopic realistic dynamics. Intuitively the map function form is very natural in the sense that the dominant part of the model is linear plus the constant (b values) which is negative in the non-congested period and positive in the congested period. Nonlinear effects are effective only for very small or very large flow densities. An interesting application of the map model is a simulation of a virtual network of such maps. Interaction or coupling of maps may lead to some new results that are important for understanding the global properties of the Internet. From our data analysis and map model simulations we can derive some implications for congestion control. In phase transition theory, the critical point can be interpreted as a balance point between the total *input* and *output* of the system [16]. Intuitively, input means the total packet flows generated by users' requests, and the output is determined by complicated nonlinear interaction of the mixed capacities of link bandwidth and routers performance. Namely, the critical point is when the average traffic demand balances the network's effective capacity. Thus, roughly speaking the reliability of the network is expected to be highest at the critical point. In fact, an Ethernet simulation showed that throughput rapidly decreases above the critical point due to large-scale congestion [9].

For this reason we conjecture that buffer management algorithms, including packet scheduling and dropping policy, or end-to-end flow control algorithms, should try to keep the network state at the critical point. It is obvious in typical end-to-end control schemes like TCP that the source host cannot recognize the congestion state properly from information about both microscopic RTT values and dropped packets though the algorithm uses mean value and standard deviation.

There are two main reasons for this discrepancy:

- the larger values of RTT does not always indicate a highly congested state, and

- there are short-duration congestion periods even in the non-congested period.

We propose that a better control scheme should take into account the difference in macroscopic distribution of congestion duration length, or the value of critical flow density. The same discussion also applies to queueing algorithms, so it is important to know the current congestion level from a statistical packet drop algorithm like RED [6], which uses the current queue length and the averaged queue length to decide which packets to discard. We believe that our statistical approach is also useful for this discard policy.

5 Conclusion

We showed evidence for the existence of the local phase transition phenomenon by analyzing wide-area network traffic measurements, in which all packets passing through the border gateway were collected.

The phase transition phenomenon is characterized by drastic changes around the non-trivial critical point, and the self-similar properties appear only at the critical point. We proposed an empirical traffic model in order to represent such phase transition behavior. The model is defined by the simplest autoregressive map with random noise, and when a parameter is tuned the model can reproduce the basic properties of phase transition. We believe that this model is useful for a simple background traffic model for simulation or a real traffic generator. To apply the current results to control complex real traffic, we need to study further how to judge the critical flow density more simply in generalized traffic and analyze the queueing performance of the map model.

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