

# Determination of the Image Complexity Feature in Pattern Recognition

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## Abstract

The new image complexity informative feature is proposed. The experimental estimation of the image complexity is carried out. There are elaborated two optical-electronic processors for image complexity calculation. The determination of the necessary number of the image's digitization elements depending on the image complexity was carried out. The accuracy of the image complexity feature calculation was made.

**Keywords:** image, complexity feature, Fourier spectrum, processor

## 1 Introduction

In the of optical pattern recognition systems (OPR) of the last generation one of the very important parameter is the image complexity feature [1,2]. This parameter is used for system's architectures control, for comparing the different recognition algorithms performances etc.

It was shown that image complexity feature may be characterized either with the complexity of the objects in the image and their number. The object's complexity is often determined through the complexity of their forms and their structure peculiarities. The peculiarities of image details are known to have a great influence upon forming its higher frequency Fourier spectrum. These frequencies contain data necessary for objects recognition.

The new image complexity informative feature (ICIF) is proposed (Sec.2) which is based on the Fourier spectrum analysis. The suggested

measure of complexity may be obtained rather easily and fast by optical means at very high speed, which is very important in the real time mode functioning systems.

The experimental estimation of the image complexity was carried out (Sec.3) which showed that the proposed measure of ICIF is close to the subjective estimation of the image complexity and can be used in the practical utilization.

There are elaborated two optical-electronic processors for image complexity calculation (Sec.4) which are based on the utilization of the photo-receivers with circular and lines electrodes, and are characterized by different speed and accuracy.

The determination of the necessary number of the image's digitization elements DE depending on the image complexity was carried out (Sec.5). The importance of this stage consists in the fact that the knowledge of the exact value of the DE parameter determines the ulterior organization of computer processes, the structure of the systems, and influences the processing time and the results accuracy.

The accuracy of the image complexity feature calculation was carried out (sec.6) which permits to formulate the technical requirements at the stage of the processors design.

## 2 Images' complexity informative feature formation

The images' complexity informative feature may be characterized either with the complexity of objects in the image and their number. The object complexity is often determined through the complexity of their forms and their structure peculiarities. The peculiarities of image details are known to have a great influence upon forming its higher frequency Fourier spectrum. These frequencies contain data for object recognition.

The new image complexity informative feature is proposed which is based on the determination of the data from Fourier spectrum of an image and can be calculated as  $IC = f_m(k_c)^{1/2}$ , where  $f_m$  – is the

maximal frequency of Fourier spectrum of an image,  $k_c$  – the coefficient of the Fourier-spectrum image form.

The suggested measure of complexity may be obtained rather easily and fast by optical means, which is very important in realization of the real time mode systems.

The two methods of the parameters  $IC$ ,  $f_m$ ,  $k_c$  estimations are elaborated. According to the first method (fig.1), a collimated coherent illumination is modulated by the intensity of the input image  $P(x,y)$  and the two dimensional optical Fourier transformation is fulfilled:

$$P(u, v) = \iint P(x, y) \exp\{-j2\pi(xu + yv)\} dx dy.$$

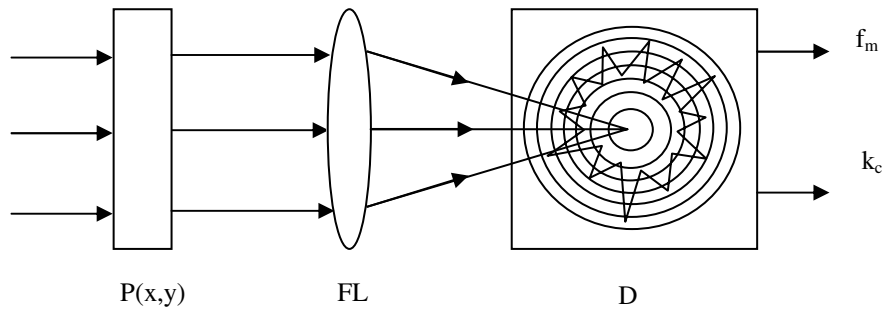


Figure 1. The method of the image complexity information feature formation

Then the Fourier spectrum of the function  $P(u, v)$  is formed, which is circular scanned by different radius. The highest frequency  $f_m$  of Fourier spectrum and the integral intensity  $I_{sp}$  of Fourier spectrum are determined at this frequency. Here the value  $f_m$  corresponds to the value of the maximal scanning radius, where the threshold signal excision has occurred. The image complexity informative feature is determined by the formula:

$$IC = f_m (I_{sp}/I_o)^{1/2}, \quad (1)$$

where  $I_o$  – is a reference value of the Fourier spectrum intensity at the corresponding radius.

In the formula (1) the relation  $I_{sp}/I_o$  characterizes the coefficient of the external figure, which describes Fourier spectrum of the input image. Such a figure can be a circle, rectangle, rhombus etc (fig. 2.a). The value  $I_o$  corresponds to the case when Fourier-spectrum is described by a circle.

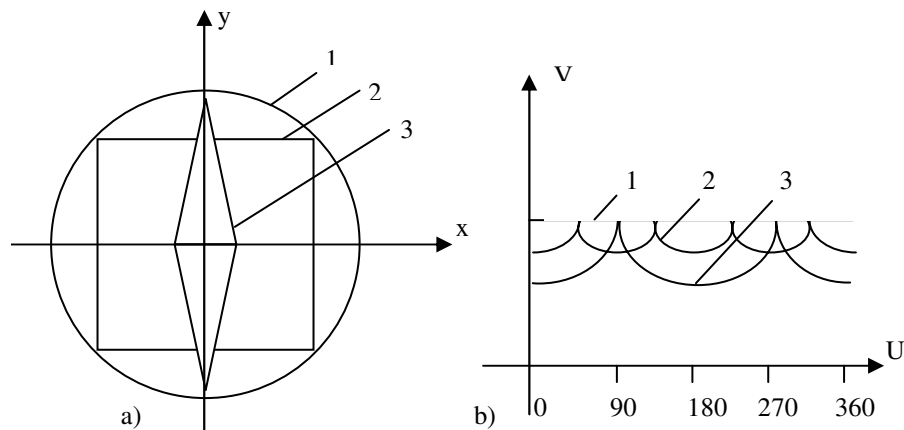


Figure 2. The figures, which describe the Fourier-spectrums in Cartesian (a) and polar (b) systems of coordinates

The peculiarity of the second method consists in the Fourier spectrum image transformation from a Cartesian system of coordinates  $(x, y)$  to a polar one  $(u, v)$  and its following linear scanning (fig. 2.b). Thus, the second method of the image complexity estimation requires one additional operation, but it allows to determine the  $IC$  parameter value more exactly. This is because the density of a linear scanning is higher than the density of circular scanning at the same sizes of the scanning zones.

### 3 Experimental estimation of image complexity

The experimental estimation of the image complexity of two classes of objects – aircrafts and ships (fig. 3) – was carried out. The experiments were made in the following way.

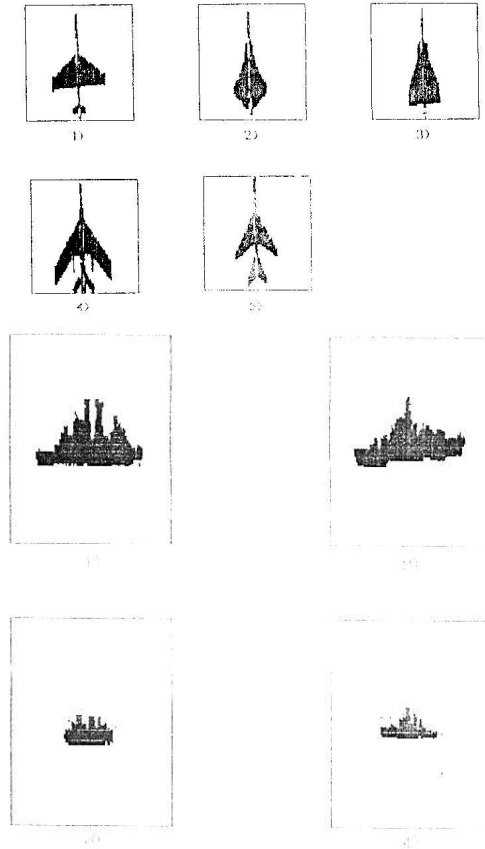


Figure 3. The objects for experimental image complexity feature calculation

- 1 The images of objects  $R_i(x, y)$  were introduced into computer by TV camera.

- 2 After images' preprocesses (noises' removal) their Fourier spectrums were formed using FFT algorithm. Later they were segmented and transformed into the binary images:

$$R_i(x, y) \rightarrow R_{bi}(x_1, y_1).$$

- 3 The images  $R_{bi}(x_1, y_1)$  were transformed into a polar system of coordinates:

$$R_{bi}(x_1, y_1) \rightarrow R_{bi}(x_2, y_2),$$

where  $x_2 = \arctg(y_1/x_1)$ ,  $y_2 = (x_1^2 + y_1^2)^{1/2}$ .

- 4 There were calculated the  $y_s$  coordinates of the lines, maximally distanced from the bottom, which consist of non zero values of the images pixels. The  $y_s$  value corresponds to the maximal frequency of the Fourier-spectrum image.
- 5 There were determined the coefficients of the Fourier-spectrum form in the following way:  $k_s = S_{in}/S_{cr}$ , where  $S_{in}$  – the area of the figure, covered by the image spectrum  $R_i(x_1, y_1)$ ,  $S_{cr}$  – the area of the circle which describes this spectrum. So we will have:

$$S_{in} = \sum_{y_2=y_s}^M \sum_{x_2=0}^N \{R_i(x_2, y_2)\}, \quad S_{cr} = \sum_{y_2=y_s}^M \sum_{x_2=0}^N \{A(x_2, y_2)\},$$

where  $N$ ,  $M$  – the number of the pixels in the rows, column  $N = M = 128$ ;  $A(x_2, y_2) = 1$ .

- 6 There was calculated the image complexity according to the following formula:

$$IC = y_s h_v (k_s)^{1/2} / 128.$$

Here the value  $h_v$  corresponds to the highness of the scanning zone of the TV camera.

The results of the IC parameters estimation for different images are presented in the table 1.

Table 1. Images objects complexity

Class No	OBJECTS	IC, $mm^{-1}$
1	Aircrafts	54.88
	1. U-24	37.37
	2. SU-7	24.05
	3. TU-144	25.03
	4. SU-11	21.66
	5. -50	
2	Ships	30.25
	1. SH1	25.17
	2. SH2	14.25
	3. SH3	14.18
	4. SH4	

The results show that the most complex object between the objects of the first class is the aircraft TU-24 ( $IC=54.88mm^{-1}$ ). This object really includes a bigger number of details in comparison with another objects. The complexities of the aircrafts TU-144 and SU-11 are the similar ( $IC=24.05mm^{-1}$ ,  $25.03mm^{-1}$ ) that corresponds to reality.

Between the objects of the second class the most complex one is the image of the ship SH1 ( $SL=30.25mm^{-1}$ ). The images of the ships SH3 and SH4 are approximately of the same complexity ( $IC=14.25mm^{-1}$ ,  $14.18mm^{-1}$ ).

As it is obvious from the presented data, the experimental results are close to the subjective estimation of the image complexity. This fact confirms the possibility of the practical utilization of the proposed image complexity informative feature.

## 4 Optical-electronic processors for ICIF calculation

There were elaborated two optical-electronic processors for ICIF calculation, which realize the methods, presented in Sec.2.

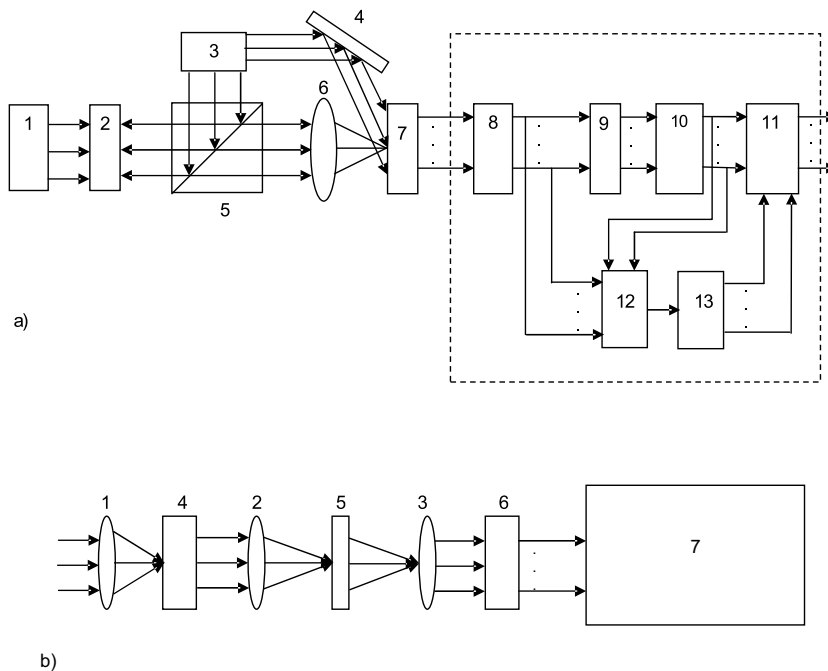


Figure 4. Optical-electronic processors on the basis of the photo-receiver with circle electrodes (a) and on the basis of the photo-receiver with linear electrodes (b)

The first processor (fig.4.a) contains the electronic tube 1, the optical spatial light modulator (SLM) 2, the laser 3, the reflective mirror 4, the semi-transparent mirror 5, the Fourier-lens 6, the semi-conductor photo-receiver with circular electrodes 7, the amplifier block 8, the



threshold elements block 9, the decoder 10, ROM 11, the analogous switch 12, the analog to digital converter (ADC) 13. The photo-receiver with circular electrodes is presented on the fig.5.a.

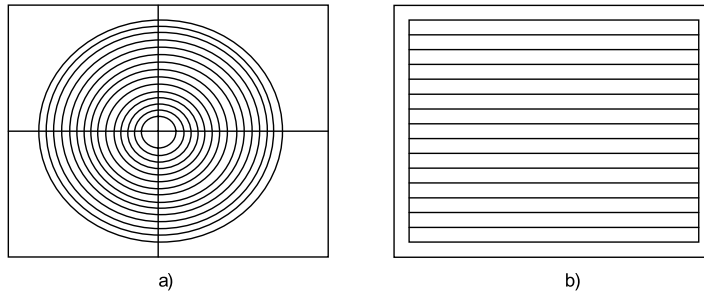


Figure 5. The photo-receivers with circle electrodes (a) and with linear electrodes (b)

The processor functions as follows. The non-coherent image from the electronic tube 1 is recorded on the SLM 2. The image from the SLM 2 is read out by the optical beam from the laser 3 through mirror 5 and is transformed into a coherent one. With the help of the lens 6 the bi-dimensional Fourier transformation is carried out. The photo-receiver 7 situated in the back focal plan of the lens 6, transforms the optical Fourier-spectrum into a set of electrical signals. The electrical signals are formed on that exits of the photo-receiver, the electrodes of which, connected with these exits are illuminated by optical light of the threshold intensity.

From the exit of the photo-receiver 7 the electrical signals pass in parallel through the amplifier block 8 at the inputs of the threshold elements block 9 and of the analogous switch 12. At the exit of the block 9 a binary code is formed which is introduced into the decoder 10. At the output of the decoder 10 a unitary binary code is formed, in which the position of the higher unit characterizes a higher significant unit in the binary code. From the output of the decoder 10 the binary

code passes to the first input of ROM 11 and to the controlling inputs of the switch 12, which commutate the analogous signal at the respective input. With the help of ADC 13 the analogous signal is transformed into a binary code which passes to the second input of the ROM 11.

So, in the ROM 11 the codes are introduced which contain the information about the number of the illuminated electrode maximally distanced from the center of photo-receiver 7 and about the illumination brightness of this electrode (the information about the parameter  $I_{sp}$ ). At the output of ROM 11 the information about the IC,  $f_m$ ,  $k_c$  is formed.

In fig. 4.b the structure of the optical-electronic processor based on the photo-receiver with linear electrodes is presented. The spatial light modulator 4 is used for the recording optical Fourier spectrum of the input image. By the collimated coherent beam the image of the Fourier spectrum is read out from the SLM 4 and is transformed into a polar system of coordinates with the help of optical Fourier lenses 2, 3 and holographic elements 5.

The image of the Fourier spectrum presented in the polar system of coordinates is projected on the photo-receiver which contains the set of horizontal linear electrodes (see fig.5.b). This photo-receiver is placed in the back focal plan of the Fourier lens 3. In the other, the component and functioning of this processor correspond to the processor on the basis of the photo-receiver with circular electrodes.

The estimated time expenditures in the processors are:  $T_{p1} = 4.2\mu s$  for the first one, and  $T_{p2} = 1004.2\mu s$  for the second one.

So, the processor which realizes the first method of ICIF formation is characterized by the higher speed. The second processor provides the higher precision of ICIF calculation, using photo-receiver with linear electrodes.

## 5 Determination of the necessary number of image's digitization elements

The determination of the necessary number of image's digitization elements DE represents one of the important stages in images' processing. The importance of this stage consists in the fact that the knowledge of the exact value of the DE parameter determines the ulterior organization of computer processes, the structure of the systems, influences the processing time and the results accuracy. It was proposed to determine the value of DE on the basis of the image complexity determination.

Let the input image  $P(x, y)$  include a group of objects and it is described by the function:

$$P(x, y) = \sum_{j=1}^{M_0} R_j(x, y), \quad (2)$$

where  $M_0$  is the number of the objects in image.

At the stage of digitization, the initial image is reflected on a reception area with dimension of  $N \times M$  elements and let  $L_1 = \max\{\max|e_{3i}|, \max|e_{4i}|\} = z^* D_m$ ,  $L_2 = \{\max(e_{1i})\} D_m = e_1 D_m$ ,  $L_3 = v_r D_m$ , where  $e_1$  is the parameter of the object's scale change;  $e_3, e_4$  are the parameters of the object's translation on coordinates  $x$  and  $y$  respectively;  $D_m$  is the maximal object's dimension; the  $v_r$  parameter describes the distance from the border of the reception area up to the closer border of the reception elements.

Considering this fact, in the analyzed image the area can be determined which will become subject to the effective processing, which has the following dimensions:

$$D_x D_y = S_n = \{2[L_1 + L_2/2 + L_3]\}^2 = \{D_m(e_1 + 2z + 2v_r)\}, \quad (3)$$

where  $D_x, D_y$  are the linear dimensions of the area of analysis;  $S_n$  – the surface of the analysis area.

From another part, the surface of analysis area can be determined by the parameters of the reception area  $q_r, d_r, b_r$  and the number of its columns  $N$  in the following way (at  $N = M$ ):

$$S_n = [(N - 1)q_r + d_r + 2b_r]^2, \quad (4)$$

where  $q_r, b_r$  – the distances between the centers and the closest borders of the two neighboring reception elements;  $d_r$  – the diameter of the reception element.

Equating the right parts of the expressions (3), (4) and accomplishing a set of mathematical operations, there will result the expression of the appreciation of the necessary number DE of digitization elements of the image:

$$DE = (N'_n)^2 = \{[D_m(e_1 + 2z + 2v_r) + d_r]/q_r - 1\}^2. \quad (5)$$

A more exact appreciation of DE value can be achieved, if in the stage of appreciation of the image's complexity, there will be considered the form of its Fourier spectrum. From the theory of digitization it is known [3] that its measure of efficiency is represented by the  $a_D = p_r S_{sp} 4\pi^2$ , where  $p_r$  – the surface of one pixel;  $S_{sp}$  – the surface of the figure which describes the image's spectrum;  $\pi = 3.14$ . At  $p_r = (q_r)^2$ , the value  $a_D = (q_r)^2 S_{sp} 4\pi^2$ . From the last equation we can determine the  $q_r$  value:

$$q_r = [D/S_{sp}]^{1/2}/2\pi. \quad (6)$$

The external form of the Fourier-spectrum can be interpolated by different figures – by the circle, by the rectangle, by the rhombus etc. It is possible to show that for more complex images their external Fourier-spectrum's form will be close to the form of the circle.

Really, as it was discussed earlier, one of the aspects of the image's complexity is the number of the objects presented in the image. The Fourier-spectrum of the image described by (2) will be:

$$|FT\{P(x, y)\}|^2 = |FT\{\sum_{j=1}^{M_0} R_j(x, y)\}|^2 = \sum_{j=1}^{M_0} |FT\{R_j(x, y)\}|^2,$$

where  $FT$  is the operation of Fourier transformation.

Considering that the objects in the input image can be oriented arbitrary and taking into account the fact that the Fourier spectrum of the rotated images is characterized by the same angle of rotation that the initial images, it is possible to conclude that at the increase of  $M_o$ , the form of the image  $P(x, y)$  spectrum will get closer to the circle's form.

Let  $k_c = S_{in}/S_{cr}$ , where  $S_{in}$  is the surface of the analyzed image's spectrum,  $S_{cr}$  – the spectrum's surface described by form of the circle. The value  $S_{cr} = \pi X_m^2$ , where  $X_m$  is the maximal spatial coordinate of the binarized non-zero value of the Fourier-spectrum in the frequency plan,  $X_m = f_m/2\pi$ .

Then  $S_{cr} = (f_m)^2/4\pi$ , and the expression (6) can be presented in the following form:

$$q_r = 2\pi(a_D/k_c S_{cr})^{1/2} = 1/f_m(k_c\pi/a_D)^{1/2}. \quad (7)$$

In this case, the formula for DE evaluation will be

$$DE = \{f_m(k_c\pi/a_D)^{1/2}[D_m(e_1 + 2z + 2v_r) + d_r] - 1\}^2.$$

Considering that  $IC = f_m(k_c)^{1/2}$ , and introducing parameter  $Q = e_1 + 2z$ , it will be obtained:

$$DE = \{IC(\pi/a_D)^{1/2}[D_m(Q + 2v_r) + d_r] - 1\}^2. \quad (8)$$

So, the expression (8) allows establishing the necessary number of the digitization elements in the input image in dependence of input image complexity and of objects' parameters.

On the basis of the expression (8) there were made calculations of the necessary numbers of the digitization's elements DE in dependence of the parameters  $IC$ ,  $Q$  at  $a_D = \pi/4$ ,  $D_m = 1.5\text{mm}$ ,  $d_r = 0.006\text{mm}$ ,  $v_r = 0.025$ .

The results of the calculations show that the images with  $IC < 70$  lines/mm can be processed without distortions only at  $Q \leq 0.5$  (for images of 128/128 pixels). In other cases, such images will be distorted at the processing or it is recommended the processing of more simple

images. So, at  $Q \leq 1.0$  the images with  $IC \leq 40.7$  lines/mm can be processed without distortions, at  $Q \leq 4.0$  – only the images with  $IC \leq 10.6$  lines/mm. At digitization of the images on 256x256 pixels and  $Q \leq 4.0$ , the images with  $IC \leq 21.3$  lines/mm can be processed without distortions, and at the digitization on 512x512 pixels – the same can be done with the images with  $IC \leq 42.18$  lines/mm.

The analysis of the obtained data show that the preliminary determination of the parameters  $IC$ ,  $Q$  allows to increase the accuracy of the image processing. This is explained by the possibility of the image processing with a higher value of  $IC$ ,  $f_m$  at the same number of the image's digitization elements. From another part, the knowledge of the exact values of the indicated parameters offers the possibility to increase the speed of the image processing by the decrease of the necessary number of the image's pixels, which have to be introduced and processed in the system.

## 6 Investigation of the image complexity accuracy calculation

The accuracy of the image complexity feature calculation was carried out. This parameter influences on the accuracy calculation of the DE – the necessary number of the image digitization elements. From the correctness of the parameter DE calculation the accuracy and the reliability of the data processing in the system, and the time of the tasks' resolving depend.

Let's write the expression (8) in the following mode:

$$DE = \{IC(\pi/a_D)^{1/2}[D_m(Q' + 2v_r) + d_r] - 1\}^2 = \{IC \cdot A - 1\}^2, \quad (9)$$

Taking into account that component  $IC \cdot A \gg 1$ , the formula (8) can be written as:

$$DE = N \cdot N = \{IC \cdot A - 1\}^2 = \{IC \cdot A\}^2. \quad (10)$$

where  $N$  is the number of the image digitization elements in the row (column).

Suppose  $\Delta_S$  is the absolute error of the  $IC$  parameter calculation. In this case, two values of  $IC$  can be calculated as:

- 1)  $IC' = IC + \Delta_S$ ;
- 2)  $IC'' = IC - \Delta_S$ .

The first case will bring to the increase of the DE parameter and consequently to the increase of the image processing time, but not to the decrease of the calculation results accuracy. The next case, inversely, can bring to the decrease of the tasks' resolving time and to the increase of the calculation volume. In connection with the fact that at present the computer means are characterized by considerable increasing capacity of the calculation speed, let's analyze the second case – the influence of the parameter DE value decrease relatively to the requested value of the tasks' resolving accuracy.

Suppose  $\Delta_S = \delta_S \cdot IC$ . In this case  $IC'' = IC - \Delta_S = IC(1 - \delta_S)$ , and the value  $N'' = IC'' \cdot A = IC(1 - \delta_S)A$ . The relation  $N/N''$  will be:

$$N/N'' = 1/(1 - \delta_S) = K_D, \quad (11)$$

where the parameter  $K_D$  is the image's digitization steps coefficient;  $\delta_S < 1$ .

On the basis of the expression (10), the value of  $\delta_S$  will be determined as:

$$\delta_S = (K_D - 1)/K_D, \quad (12)$$

And the absolute error  $\Delta_S$  will be:

$$\Delta_S = \delta_S \cdot IC = (K_D - 1) \cdot IC/K_D. \quad (13)$$

Let's analyze the error of the IC parameter calculation in the optical-electronic processors. In accordance with the formula  $IC = f_m(k_c)^{1/2}$ , at  $k_c = 1$ , the accuracy of  $IC$  parameter calculation will be determined by  $f_m$  accuracy calculation. In the elaborated processors the value  $f_m$  is determined in the following way:

$$f_m = 2\pi n_{me} d_c / \lambda h, \quad (14)$$

where  $n_{me}$  is the number of electrodes, maximally distanced from the photo-receiver center;  $d_c$  – the step between the two neighbor electrodes,  $\lambda$  – the coherent wave’s length;  $h$  – the Fourier lens’ focus distance.

Taking into account the expression (8), the absolute error of the  $f_m$  calculation will be determined as:

$$\Delta_f = \Delta_{sp} = 2\pi(n_{me} + 1)d_c/\lambda h - 2\pi n_{me}d_c/\lambda h = 2\pi d_c/\lambda h. \quad (15)$$

On the basis of the expression (15), the technical requirements can be formulated in the stage of the processor design, with purpose of obtaining the required absolute error  $\Delta_{sp}$ . For example at  $d_c = 50\mu\text{m}$ ,  $\lambda = 1\mu\text{m}$ ,  $h = 100\text{mm}$ , the value  $\Delta_{sp} = 3\text{mm}^{-1}$ .

## 7 Conclusion

1. It is proposed the new image complexity informative feature (ICIF), which is based on the Fourier spectrum maximal frequencies determination and its form’s taking into consideration. The suggested measure of complexity may be obtained rather easily and fast by optical and optical electronic means, which is very important in realization of the real time mode functioning systems.

2. There are elaborated two methods of the ICIF. The first method is based on the input image Fourier spectrum formation, which is then circular scanned by different radius. The peculiarity of the second method consists in the Fourier-spectrum image transformation from a Cartesian system of coordinates to a polar one and its following linear scanning. Thus, the second method of the image complexity estimation requires one additional operation, but it allows to determine the ICIF parameter value more exactly.

3. The estimation of the image complexity of two classes of objects – aircrafts and ships – was carried out which shows that the experimental results are close to the subjective estimation of the image complexity. This fact confirms the possibility of the practical utilization of the proposed image complexity informative feature.

4. There were elaborated two processors which realize the proposed methods of ICIF formation. The first processor is characterized by the



higher speed. The second processor provides the higher precision of ICIF calculation.

5. The determination of the necessary number of the image's digitization elements DE was carried out on the basis of the image complexity parameter. The analysis shows that the preliminary determination of the image complexity (IC) allows to increase the accuracy of the image processing. This is explained by the possibility of the image processing with a higher value of IC at the same number of the image's digitization elements. From another part, the knowledge of the exact values of the IC offers the possibility to increase the speed of the image processing by the decrease of the necessary number of the image's pixels, which have to be introduced and processed in the system.

6. The accuracy of the image complexity feature calculation was carried out which permitted to formulate the technical requirements in the stage of the processors' design.

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