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Quasi-stability of a vector trajectorial problem with non-linear partial criteria *

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Abstract

Multi-objective (vector) combinatorial problem of finding the Pareto set with four kinds of non-linear partial criteria is considered. Necessary and sufficient conditions of that kind of stability of the problem (quasi-stability) are obtained. The problem is a discrete analogue of the lower semicontinuity by Hausdorff of the optimal mapping.

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The quasi-stability criterion of a vector trajectorial problem with linear and non-linear (MINMAX and MINMIN) partial criteria is obtained in [1]. Then it is shown in [2] that this criterion holds if all the partial criteria are in the form of MINMAX MODUL. In the article this result is extended to the case where the vector criterion has other kinds of non-linear partial criteria apart from the one mentioned above.

1 Basic definitions and concepts

Let, as usual $[1-6]$, vector criterion

$$f(t,A) = (\ f_1(t,A_1),\ f_2(t,A_2),\ ...,\ f_n(t,A_n)\ ) \to \min_{t\in T}$$

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be given on the set of trajectories $T \subseteq 2^E \setminus \{\emptyset\}$. Here $n \geq 1$, $m \geq 2$, A_i – the i-th row of matrix $A = [a_{ij}]_{n\times m} \in \mathbf{R}^{nm}$, $|T| > 1$, $E = \{ e_1, e_2, ..., e_m \}$.

We consider the case where the components of $f(t, A)$ are non-linear functionals of four kinds

$$f_i(t, A_i) = \max_{j \in N(t)} a_{ij}, \tag{1}$$

$$f_i(t, A_i) = \max_{j \in N(t)} |a_{ij}|, \tag{2}$$

$$f_i(t, A_i) = \min_{j \in N(t)} a_{ij}, \tag{3}$$

$$f_i(t, A_i) = \min_{j \in N(t)} |a_{ij}| \tag{4}$$

in an arbitrary combination. Here $i \in N_n = \{ 1, 2, ..., n \}$, $N(t) = \{ j \in N_m : e_j \in t \}$.

By I_{max}, I_{maxmod}, I_{min} and I_{minmod} we denote the sets of those indices from N_n, by which the criteria $(1) - (4)$ respectively are numbered ($I_{max} \cup I_{maxmod} \cup I_{min} \cup I_{minmod} = N_n$).

Under vector (n-criteria) problem $Z^n(A)$, $n \geq 1$, we understand the problem of finding the Pareto set consisting of all efficient trajectories

$$P^n(A) = \{ t \in T : P^n(t, A) = \emptyset \},$$

where

$$P^n(t, A) = \{ t' \in T : f(t, A) \geq f(t', A), \quad f(t, A) \neq f(t', A) \}.$$

It is obvious that $P^1(A)$ ($A -$ m-dimensional vector) is the set of all optimal solutions of scalar trajectorial problem $Z^1(A)$.

As usual $[1] - [6]$, we do perturbation of the parameters of vector criterion $f(t, A)$ by adding matrix $A \in \mathbf{R}^{nm}$ with the matrices of the set

$$\mathcal{B}(\varepsilon) = \{ B \in \mathbf{R}^{nm} : ||B|| < \varepsilon \},$$

where $\varepsilon > 0$, $|| \cdot ||$ - norm l_∞ in space $\mathbf{R}^{nm}$, i.e.

$$||B|| = \max\{ |b_{ij}| : (i, j) \in N_n \times N_m \}, \quad B = [b_{ij}]_{n\times m}.$$

Problem $Z^n(A+B)$ obtained from initial problem $Z^n(A)$ by addition of matrices A and $B \in \mathcal{B}(\varepsilon)$ is called perturbed; matrix B is called perturbing.

As usual [1], [2],[5-7], we assume quasi-stability of problem $Z^n(A)$ to be a discrete analogue of the lower semicontinuity in the sense of Hausdorff of the set-valued (multi-valued) mapping. The mapping establishes the choice function, i.e. the property of keeping the efficiency of trajectories for "small" independent perturbations of the parameters of matrix A. So problem $Z^n(A)$ is quasi-stable if and only if the condition

$$\exists\, \varepsilon > 0 \quad \forall\, B \in \mathcal{B}(\varepsilon) \quad (P^n(A) \subseteq P^n(A+B))$$

holds. For any index $i \in N_n$ put

$$\gamma_i(t, t', A_i) := -\frac{g_i(t, t', A_i)}{2},$$

$$g_i(t, t', A) := f_i(t, A_i) - f_i(t', A_i).$$

2 Lemmas

Lemma 1 *For any index $i \in N_n$ the following statement is true : if $0 < \varphi \leq \gamma_i(t, t', A_i)$, $t, t' \in T$, then the inequality*

$$g_i(t, t', A_i + B_i) < 0 \tag{5}$$

holds for any matrix $B \in \mathcal{B}(\varphi)$.

Proof. Let $B \in \mathcal{B}(\varphi)$. Then following the condition of the lemma we have

$$||B|| < \varphi \leq \gamma := \gamma_i(t, t', A_i). \tag{6}$$

Two cases are possible.

Case 1. $i \in I_{max} \cup I_{min}$. Then the inequalities

$$f_i(t^0, A_i) - ||B|| \leq f_i(t^0, A_i + B_i) \leq f_i(t^0, A_i) + ||B|| \tag{7}$$

are evident for any trajectory $t^0 \in T$. Therefore taking into account (6) we obtain

$$g_i(t,t',A_i+B_i) = f_i(t,A_i+B_i) - f_i(t',A_i+B_i) \leq$$

$$\leq g_i(t,t',A_i) + 2||B|| < g_i(t,t',A_i) + 2\gamma = 0.$$

Case 2. $i \in I_{maxmod} \cup I_{minmod}$. Using the evident inequalities

$$||a+b|| \leq ||a|| + ||b||,$$

$$||a|| - ||b|| \leq ||a-b||,$$

which are correct for any vectors $a, b \in \mathbf{R}^n$, we easily make certain that inequalities (7) are valid. Hence inequalities (5) are correct too. Lemma 1 has been proved.

For any two different trajectories $t, t' \in T$ and an index $i \in N_n$ put

$$\zeta_i(t,t',A_i) := \begin{cases} \gamma_i(t \setminus t', t', A_i) & if \quad i \in I_{max} \cup I_{maxmod}, \\ \gamma_i(t, t' \setminus t, A_i) & if \quad i \in I_{min} \cup I_{minmod}. \end{cases} \tag{8}$$

Here and henceforth we assume that

$$f_i(\emptyset, A_i) = \begin{cases} -\infty & if \quad i \in I_{max} \cup I_{maxmod}, \\ +\infty & if \quad i \in I_{min} \cup I_{minmod}. \end{cases} \tag{9}$$

It is easy to see that $\zeta_i(t,t',A_i) = +\infty$ only in two cases:

$$i \in I_{max} \cup I_{maxmod} \quad and \quad t \setminus t' = \emptyset;$$

$$i \in I_{min} \cup I_{minmod} \quad and \quad t' \setminus t = \emptyset.$$

It is obvious that

$$\forall\, i \in N_n \quad \forall\, t \in T \quad \forall\, t' \in T \quad (\zeta_i(t,t',A_i) \geq \gamma_i(t,t',A_i)), \tag{10}$$

and the inequality

$$\zeta_i(t,t',A_i) > \gamma_i(t,t',A_i)$$

is valid only in the following cases:

1^0 $\quad t \setminus t' \neq \emptyset, \quad i \in I_{max} \cup I_{maxmod} \quad (if \quad f_i(t \setminus t', A_i) < f_i(t, A_i)\ ),$

$2^0 \quad t' \setminus t \neq \emptyset, \quad i \in I_{min} \cup I_{minmod} \ \ (if \quad f_i(t' \setminus t, A_i) > f_i(t', A_i)\ ),$
$3^0 \quad t \setminus t' = \emptyset, \quad i \in I_{max} \cup I_{maxmod},$
$4^0 \quad t' \setminus t = \emptyset, \quad i \in I_{min} \cup I_{minmod}.$

Lemma 2 *Let the trajectories $t, t' \in T$, $t \neq t'$, be such that the inequality*

$$0 < \varphi \leq \zeta_i(t, t', A_i)$$

holds for any index $i \in N_n$. Then for any matrix $B \in \mathcal{B}(\varphi)$ the inequality

$$g_i(t, t', A_i + B_i) \leq 0$$

is correct.

Proof. In the case where $\zeta_i(t, t', A_i) = \gamma_i(t, t', A_i)$ the statement of the lemma is valid on account of lemma 1. Therefore according to (10) we only have to consider the case when $\zeta_i(t, t', A_i) > \gamma_i(t, t', A_i)$. Hence one of conditions $1^0 - 4^0$ mentioned above is fulfilled.

Case 1^0. By the condition of the lemma we have $\zeta_i(t, t', A_i) > 0$. Taking into account lemma 1 and inclusion $i \in I_{max} \cup I_{maxmod}$ for any number φ such that $0 < \varphi \leq \gamma_i(t \setminus t', t', A_i)$ we see that the inequality

$$g_i(t \setminus t', t', A_i + B_i) < 0 \tag{11}$$

holds for any matrix $B \in \mathcal{B}(\varphi)$. Two subcases are possible.

$1^0.1 \quad f_i(t, A_i + B_i) = f_i(t \setminus t', A_i + B_i)$. Then in view of (11) we obtain $g_i(t, t', A_i + B_i) < 0$.

$1^0.2 \quad f_i(t, A_i + B_i) = f_i(t \cap t', A_i + B_i), \quad t \cap t' \neq \emptyset$. The relationships

$$g_i(t, t', A_i + B_i) = f_i(t \cap t', A_i + B_i) - f_i(t', A_i + B_i) \leq 0$$

are evident in this subcase.

Case 2^0. By the condition of the lemma $\zeta_i(t, t', A_i) > 0$. Taking into account lemma 1 and inclusion $i \in I_{min} \cup I_{minmod}$ for any number φ such that $0 < \varphi \leq \gamma_i(t \setminus t', t', A_i)$ we obtain the inequality

$$g_i(t, t' \setminus t, A_i + B_i) < 0 \tag{12}$$

for any matrix $B \in \mathcal{B}(\varphi)$. Two subcases are possible.

$2^0.1$ $\quad f_i(t', A_i + B_i) = f_i(t' \setminus t, A_i + B_i)$. Then in view of (12) we obtain $g_i(t, t', A_i + B_i) < 0$.

$2^0.2$ $\quad f_i(t', A_i + B_i) = f_i(t \cap t', A_i + B_i), \quad t \cap t' \neq \emptyset$. Then

$$g_i(t, t', A_i + B_i) = f_i(t, A_i + B_i) - f_i(t \cap t', A_i + B_i) \leq 0.$$

Case 3^0. $t \setminus t' = \emptyset$ and $t \neq t'$. Then $t' \setminus t \neq \emptyset$. Therefore the relationships

$$g_i(t, t', A_i + B_i) =$$

$$= f_i(t, A_i + B_i) - \max\{\ f_i(t, A_i + B_i),\ f_i(t' \setminus t, A_i + B_i)\ \} \leq 0$$

are correct for any matrix $B \in \mathbf{R}^{nm}$ and any index $i \in I_{max} \cup I_{maxmod}$.

Case 4^0. Taking into account $t \setminus t' \neq \emptyset$ we see that the relationships

$$g_i(t, t', A_i + B_i) =$$

$$= \min\{\ f_i(t', A_i + B_i),\ f_i(t \setminus t', A_i + B_i)\ \} - f_i(t', A_i + B_i) \leq 0$$

are evident for any matrix $B \in \mathbf{R}^{nm}$.

Lemma 2 has been proved.

Let us assign the set of equivalent efficient trajectories to an arbitrary trajectory $t \in P^n(A)$

$$Q^n(t, A) := \{\ t' \in T \setminus \{t\}\ :\ f(t', A) = f(t, A)\ \}.$$

Lemma 3 *Let* $p \in I_{minmod}, \quad t \in P^n(A), \quad t' \in Q^n(t, A), \quad t' \setminus t \neq \emptyset, \quad f_p(t' \setminus t, A_p) = 0.$ *If* $\tau > 0$ *and* $C = [c_{ij}]_{n \times m}$ *is a perturbing matrix with elements*

$$c_{ij} = \begin{cases} \tau & if \quad i = p, \quad j \in N(t), \quad a_{pj} \geq 0, \\ -\tau & if \quad i = p, \quad j \in N(t), \quad a_{pj} < 0, \\ 0 & otherwise \end{cases}$$

then

$$g_i(t, t', A_i + C_i) = \begin{cases} \tau & if \quad i = p, \\ 0 & if \quad i \in N_n \setminus \{p\}. \end{cases} \tag{13}$$

Proof. According to condition of the lemma we have $f_p(t' \setminus t, A_p) = 0$. Since $t' \in Q^n(t, A)$ one can obtain $f_p(t', A_p) = f_p(t, A_p) = 0$. Taking into account the structure of the row C_p we have

$$f_p(t, A_p + C_p) = f_p(t, A_p) + \tau = \tau.$$

Therefore in view of the equality $f_p(t' \setminus t, A_p) = 0$ we obtain

$$g_p(t, t', A_p + C_p) = f_p(t, A_p + C_p) - f_p(t', A_p + C_p) =$$

$$= \tau - \min\{\ f_p(t' \setminus t, A_p + C_p),\ f_p(t \cap t', A_p + C_p)\ \} =$$

$$= \tau - \min\{\ f_p(t' \setminus t, A_p),\ f_p(t \cap t', A_p + C_p)\ \} = \tau.$$

Besides that for any index $i \in N_n \setminus \{p\}$ the inequalities

$$g_i(t, t', A_i + C_i) = f_i(t, A_i) - f_i(t', A_i) = 0$$

are evident.

Lemma 3 has been proved.

3 Quasi-stability criterion

Theorem 1 *Vector trajectorial problem* $Z^n(A), n \geq 1,$ *with the partial criteria of kinds* $(1) - (4)$ *is quasi-stable if and only if the condition*

$$\forall\ t \in P^n(A)\ \ (\ Q^n(t, A) \neq \emptyset\ \Rightarrow$$

$$\Rightarrow\ \forall\ t' \in Q^n(t, A)\ \ \forall\ i \in N_n \quad (\zeta_i(t, t', A_i) > 0)\ )$$

holds.

Proof. *Sufficiency.* Let $t \in P^n(A)$, $t' \in T \setminus \{t\}$. Then the following two cases are possible.

Case 1. $t' \in T \setminus Q^n(t, A)$. Then $f(t, A) \neq f(t', A)$ and in view of $t \in P^n(A)$ there is an index $s \in N_n$ such that $g_s(t, t', A_s) < 0$. Therefore according to the continuity of any of the partial criteria $(1) - (4)$ on the set of the matrices $\mathbf{R}^{nm}$ there exists a number $\varepsilon = \varepsilon(t') > 0$ such that the inequality

$$g_s(t, t', A_s + B_s) < 0$$

holds for any perturbing matrix $B \in \mathcal{B}(\varepsilon)$. Consequently there are no trajectories from $T \setminus Q^n(t, A)$ belonging to set $P^n(t, A + B)$ for any matrix $B \in \mathcal{B}(\varepsilon_1)$, where

$$\varepsilon_1 = \min\{\ \varepsilon(t')\ :\ t' \in T \setminus Q^n(t, A)\ \}.$$

Case 2. $t' \in Q^n(t, A)$. Then by the condition of theorem the inequality $\zeta_i(t, t', A_i) > 0$ is valid for any index $i \in N_n$. In view of lemma 2 we obtain

$$\forall\ i \in N_n\ \ \forall\ B \in \mathcal{B}(\varepsilon(t'))\quad (g_i(t, t', A_i + B_i) \leq 0),$$

where

$$\varepsilon(t') = \min\{\ \zeta_i(t, t', A_i)\ :\ i \in N_n\ \}.$$

Thus none of the trajectories from $Q^n(t, A)$ belong to set $P^n(t, A+B)$ for any matrix $B \in \mathcal{B}(\varepsilon_2)$, where

$$\varepsilon_2 = \min\{\ \varepsilon(t')\ :\ t' \in Q(t, A)\ \}.$$

So we conclude that $t \in P^n(A + B)$ for any perturbing matrix $B \in \mathcal{B}(\varepsilon)$, where

$$\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}.$$

Hence problem $Z^n(A)$ is quasi-stable.

We will prove *necessity* by contradiction. Let $t \in P^n(A)$ and there exist such $t' \in Q^n(t, A)$ and $p \in N_n$ that

$$\zeta_p(t, t', A_p) \leq 0. \tag{14}$$

Then for any $p \in I_{max} \cup I_{maxmod}$ according to (8) we have

$$f_p(t \setminus t', A_p) \geq f_p(t', A_p), \tag{15}$$

at that $t \setminus t' \neq \emptyset$ (see (9)). And for any $p \in I_{min} \cup I_{minmod}$ (see (8)) we have

$$f_p(t, A_p) \geq f_p(t' \setminus t, A_p), \tag{16}$$

at that $t' \setminus t \neq \emptyset$ (see (9)).

Let $0 < \beta < \varepsilon$, $B^* = B(t, t', p, \beta) = [b^*_{ij}]_{n\times m}$ is a perturbing matrix with elements

$$b^*_{ij} = \begin{cases} \beta & if \;\; i = p \in I_{max}, \;\; j \in N(t \setminus t'), \\ \beta & if \;\; i = p \in I_{maxmod}, \;\; j \in N(t \setminus t'), \;\; a_{pj} \geq 0, \\ -\beta & if \;\; i = p \in I_{maxmod}, \;\; j \in N(t \setminus t'), \;\; a_{pj} < 0, \\ -\beta & if \;\; i = p \in I_{min}, \;\; j \in N(t' \setminus t), \\ \beta & if \;\; i = p \in I_{minmod}, \;\; j \in N(t' \setminus t), \;\; a_{pj} < 0, \\ -\beta & if \;\; i = p \in I_{minmod}, \;\; j \in N(t' \setminus t), \;\; a_{pj} \geq 0, \\ 0 & otherwise. \end{cases} \tag{17}$$

It is evident that $||B^*|| = \beta$. Let us show that the equality

$$g_p(t, t', A_p + B^*_p) = \beta \tag{18}$$

holds. We will consider four possible cases.

Case 1. $p \in I_{max}$. Then inequality (15) is true. Thus the inequality $f_p(t \setminus t', A_p) \geq f_p(t \cap t', A_p)$ is true in view of the obvious equation $f_p(t', A_p) \geq f_p(t \cap t', A_p)$ and from here – $f_p(t \setminus t', A_p) = f_p(t, A_p)$. Therefore considering the equality

$$f_p(t, A_p) = f_p(t', A_p) \tag{19}$$

and taking into account the definition of matrix B^* we derive

$$g_p(t, t', A_p + B^*_p) = \max\{\; f_p(t \setminus t', A_p + B^*_p), \; f_p(t \cap t', A_p + B^*_p) \;\} -$$

$$- f_p(t', A_p + B^*_p) = \max\{\; f_p(t \setminus t', A_p) + \beta, \; f_p(t \cap t', A_p) \;\} - f_p(t', A_p) =$$

$$= f_p(t \setminus t', A_p) + \beta - f_p(t', A_p) = f_p(t, A_p) + \beta - f_p(t', A_p) = \beta,$$

i.e. equality (18) holds.

Case 2. $p \in I_{maxmod}$. Then inequality (15) is correct. Thus the inequality $f_p(t \setminus t', A_p) \geq f_p(t \cap t', A_p)$ is true in view of the obvious equation $f_p(t', A_p) \geq f_p(t \cap t', A_p)$ and from here – $f_p(t \setminus t', A_p) = f_p(t, A_p)$.

Therefore considering (19) and taking into account the definition of matrix B^* we obtain

$$g_p(t,t',A_p+B^*_p) = \max\{\ f_p(t\setminus t',A_p+B^*_p),\ f_p(t\cap t',A_p+B^*_p)\ \}-$$

$$-f_p(t',A_p+B^*_p) = \max\{\ f_p(t\setminus t',A_p)+\beta,\ f_p(t\cap t',A_p)\ \}-f_p(t',A_p) =$$

$$= f_p(t\setminus t',A_p)+\beta-f_p(t',A_p) = f_p(t,A_p)+\beta-f_p(t',A_p) = \beta.$$

We make sure once again that equality (18) holds.

Case 3. $p\in I_{min}$. Then inequality (16) is valid. Thus the inequality $f_p(t'\setminus t,A_p)\le f_p(t'\cap t,A_p)$ is true in view of the obvious equation $f_p(t,A_p)\le f_p(t\cap t',A_p)$ and from here – $f_p(t'\setminus t,A_p)=f_p(t',A_p)$. Therefore according to (19) and taking into account the definition of matrix B^* we derive

$$g_p(t,t',A_p+B^*_p) =$$

$$= f_p(t,A_p+B^*_p)-\min\{\ f_p(t'\setminus t,A_p+B^*_p),\ f_p(t'\cap t,A_p+B^*_p)\ \} =$$

$$= f_p(t,A_p)-\min\{\ f_p(t'\setminus t,A_p)-\beta,\ f_p(t'\cap t,A_p)\ \} =$$

$$= f_p(t,A_p)-(f_p(t'\setminus t,A_p)-\beta) = f_p(t,A_p)-(f_p(t',A_p)-\beta) = \beta,$$

i.e. (18) is true.

Case 4. $p\in I_{minmod}$. Then in view of (16) $t'\setminus t\ne\emptyset$. At first let us show that there are no zeros among numbers $a_{pj},\ \ j\in N(t'\setminus t)$. We assume that there exists an index $k\in N(t'\setminus t)$ such that $a_{pk}=0$, i.e. $f_p(t'\setminus t,A_p)=0$. Then by lemma 3 there exists a perturbing matrix $C\in\mathcal{B}(\varepsilon)$ such that $||C||=\tau$, where $0<\tau<\varepsilon$ and equalities (13) are correct. It means that trajectory $t\notin P^n(A+C)$, i.e. problem $Z^n(A)$ is not quasi-stable. Contradiction.

So it is shown that number $f_p(t'\setminus t,A_p)>0$. Therefore taking into account (16) and the definition of matrix B^* (see(17)) we easily derive

$$f_p(t'\setminus t,A_p+B^*_p) = f_p(t'\setminus t,A_p)-\beta,$$

where $0<\beta\le f_p(t'\setminus t,A_p)$ and

$$f_p(t'\cap t,A_p+B^*_p) = f_p(t'\cap t,A_p)\ge f_p(t',A_p)\ge f_p(t'\setminus t,A_p)$$

Since
$$f_p(t', A_p) \leq f_p(t' \setminus t, A_p)$$
and in view of (19) we have
$$f_p(t' \setminus t, A_p) = f_p(t, A_p) = f_p(t', A_p).$$
Summing up we derive
$$g_p(t, t', A_p + B_p^*) =$$
$$= f_p(t, A_p + B_p^*) - \min\{\ f_p(t' \setminus t, A_p + B_p^*),\ f_p(t' \cap t, A_p + B_p^*)\ \} =$$
$$= f_p(t, A_p) - \min\{\ f_p(t' \setminus t, A_p) - \beta,\ f_p(t' \cap t, A_p)\ \} =$$
$$= f_p(t, A_p) - (f_p(t' \setminus t, A_p) - \beta) = f_p(t, A_p) - (f_p(t', A_p) - \beta) = \beta.$$

So there exists such number $\beta > 0$ and such perturbing matrix B^* that the following relations hold
$$g_i(t, t', A_i + B_i^*) = \begin{cases} \beta & if \quad i = p, \\ 0 & if \quad i \in N_n \setminus \{p\}. \end{cases}$$

Thus we obtain
$$\forall\ \varepsilon > 0\ \ \exists\ B^* \in \mathcal{B}(\varepsilon)\ \ (t \notin P^n(A + B^*)).$$

Consequently in view of $t \in P^n(A)$ we conclude that problem $Z^n(A)$ is not quasi-stable. The obtained contradiction proves theorem 1.

It is evident that the partial case of theorem 1 is the criterion of the quasi-stability of vector l_∞-extreme trajectorial problem obtained in [2].

Let us define the Smale set, i.e. the set of strictly efficient trajectories:
$$S^n(A) := \{\ t \in P^n(A) :\ Q^n(t, A) = \emptyset\ \}.$$

It is obvious that $Q^n(t, A) \neq \emptyset$ if and only if $t \in P^n(A) \setminus S^n(A)$.

Defining the set

$$P^n_*(A) := \{\ t \in P^n(A) \setminus S^n(A)\ :$$

$$\forall\ t' \in Q^n(t, A)\ \ \forall\ i \in N_n\ \ (\zeta_i(t, t', A_i) > 0)\ \}$$

we see that the following equivalent definition of theorem 1 is true.

Theorem $1^{'}$ *Vector trajectorial problem* $Z^n(A), n \geq 1$, *with partial criteria of kinds* $(1) - (4)$ *is quasi-stable if and only if one of the conditions holds:*

- $P^n(A) = S^n(A)$,
- $\emptyset \neq P^n(A) \setminus S^n(A) = P^n_*(A)$.

Corollary 1 *If* $P^n(A) = S^n(A)$ *then problem* $Z^n(A)$, $n \geq 1$, *is quasi-stable.*

In particular it follows that problem $Z^n(A)$ is quasi-stable if $|P^n(A)| = 1$.

Corollary 2 *Scalar (single criterion) trajectorial problem* $Z^1(A)$ $(A \in \mathbf{R}^m)$ *with any of partial criteria* $(1) - (4)$ *is quasi-stable if and only if it has a unique optimal trajectory.*

Note that for linear scalar trajectorial problem (with partial criterion of kind MINSUM) this criterion was established in [8].

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