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Perceived Fuzzy Subsets of Finite Sets

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Abstract

The paper is addressed to the problem of determination of perceived fuzzy sets. It is assumed that a person is able to indicate two specific objects, such that to one of them a given vague notion is applied perfectly, and the other this term is not applicable at all. The ordering of applicability is obtained by projecting all the given object (represented as points in n-dimensional space) into the axis joining two indicated objects. The numerical representation of the ordering yields the grades of membership.

1 Intuitive idea

Fuzzy sets are invented mainly to capture the meaning, which a person intends to give to words or terms, particularly to vague terms. There are different senses of the word "meaning". One of them is the so-called denotative meaning. This means that the meaning of a term consists of the class of all objects to which the term may be applied.

If the term is applied to more than a single object, then it is called a general term, or a general name or occasionally it is also called a class term.

If the general term may correctly be applied to an object, then it is called to be denoted by this term. Hence the term denotational meaning. For example term "person" denotes all persons.

To give another, more complete example, let us consider the following set of figures

$$\Omega = \bigcirc, \Delta, \Box, \infty.$$

It is easy to see that the term "rounded" is applied to $\bigcirc$ and ∞, but neither to $\Box$ nor to Δ.

Denotation in this case can be formally treated as a function

$$X_{rounded} : \Omega \to 0, 1$$

defined as follows:

$$X_{rounded}(\bigcirc) = X_{rounded}(\infty) = 1,$$

$$X_{rounded}(\Delta) = X_{rounded}(\Box) = 0.$$

Let us consider now the term "nice".

It is not so easy to say which elements of Ω are denoted by this term.

Meaning of this term depends on a person, and on his subjective perception. On the other hand, even a concrete person e.g. You, may have a problem to decide whether or not is a given term applicable to a given object.

If there are elements where the applicability of the word is in doubt, then the term (or word) is called vague.

The denotational meaning of vague or imprecise terms can be identified with a generalised characteristic function, which takes on values in the unit interval. These values can be interpreted as grades of applicability.

The meaning of the mentioned above term "nice" can be defined by a function

$$\mu_{nice} : \Omega \to [0, 1],$$

for example in the following way:

$$\mu_{nice}(\bigcirc) = 1,\ \mu_{nice}(\infty) = 0.8,\ \mu_{nice}(\Box) = 0.5,\ \mu_{nice}(\Delta) = 0.$$

Functions of this type are known as fuzzy sets.

Formally, fuzzy subset X of a set Ω is defined as a mapping

$$\mu_X : \Omega \to [0, 1].$$

The value $\mu_X(\omega)$ is called a membership grade of an object $\omega \in \Omega$ in a set of objects to which term X is applicable.

It is useful to interpret this grade as a degree of proximity between an object ω and an ideal prototype of X.

Suppose that in the above example, the circle i.e. element $\bigcirc \in \Omega$ is considered as an ideal prototype of the term "nice". The perceived (say, psychological) similarity, conceived for example as a reciprocal of a distance between square and this ideal element, has been evaluated to be equal to 0.5.

2 General solution

The general setting of the problem considered in this paper is as follows.

There is given a set $\Omega = \{\omega_1, \omega_2, ..., \omega_N\}$ and some term (a general name), denoted here by X.

The problem is to define a perceived meaning of this word X by means of fuzzy subset

$$\mu_X : \Omega \to [0, 1].$$

It is supposed that a person, this may be You, for example, who perceives a meaning of the term X, is able to indicate two particular elements in Ω.

One of them, denoted as ω_{best}, is considered as an ideal prototype of the name X ; and the second one, denoted as ω_{worst}, is considered to be most far away from the ideal element.

For these two elements one puts

$$\mu_X(\omega_{worst}) = 0, \mu_X(\omega_{best}) = 1.$$

In order to determine the grades $\mu_X(\omega)$ for the remaining elements $\omega \in \Omega$ it is taken here the following basic assumption.

Namely, it is assumed that each element $\omega \in \Omega$ is characterised by n numerically valued features $(x_1, x_2, ..., x_n)$. This means that the set Ω can be interpreted as a set of N points
$x^i = (x^i_1, x^i_2, ..., x^i_n)$, $i = 1, 2, ..., N$ in the n dimensional space R^n.

Suppose that two indicated elements ω_{worst} and ω_{best}, correspond the following two points:

$$x^w = (x^w_1, x^w_2, ..., x^w_n),$$

$$x^b = (x^b_1, x^b_2, ..., x^b_n).$$

These two points indicate the direction from "the worst" to "the best" object with respect to the applicability of the considered term X.

Formally this direction is defined as a straight line in R^n joining the given two poles x^w and x^b.

Furthermore, it is assumed that the far away a point x^i from the ideal one x^b, the less is the grade $\mu_X(x^i)$ of applicability of word X to this point.

Symbolically

$$d(x^b, x^i) < d(x^b, x^j) \quad \Rightarrow \quad \mu_X(x^j) < \mu_X(x^i),$$

where a distance d is measured along the axis worst-best.

In order to make possible such a measurement, all points are orthogonal projected on the line joining two points x^w and x^b.

Points projected on this line are denoted as $z^i = (z^i_1, z^i_2, ..., z^i_n)$, $i = 1, 2, ..., N$.

The coordinates z^i_j are given by the following formula;

$$z^i_j = x^w_j + \frac{a_j(\sum_{l=1}^n a_l x^i_l - \sum_{l=1}^n a_l x^w_l)}{\sum_{l=1}^n a_l^2}, \ i = 1, 2, ..., N$$

where

$$a_j = x^b_j - x^w_j, \ j = 1, 2, ..., n.$$

All the points, which after this projection will be placed on the left side of the point x^w are assumed to be equivalent to x^w, with respect of the applicability of the term X. This means that all these points are assigned the grades of applicability equal to zero.

Similarly, all the points, which are placed to the right of the ideal prototype are assigned the grades equal to one. The grades for the reaming points are calculated according to the formula:

$$\mu_X(z) = \frac{d(z^w, z)}{d(z^w, z^b)},$$

where d is some distance measure.

3 Example

Let $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, and ω_i be characterised by $x^i = (x_1^i, x_2^i)$, $i = 1, 2, 3, 4$.

Suppose that $x^1 = (1, 2)$, $x^2 = (2, 4)$, $x^3 = (5, 4)$, $x^4 = (5, 1)$.

These points are presented graphically on Fig 1

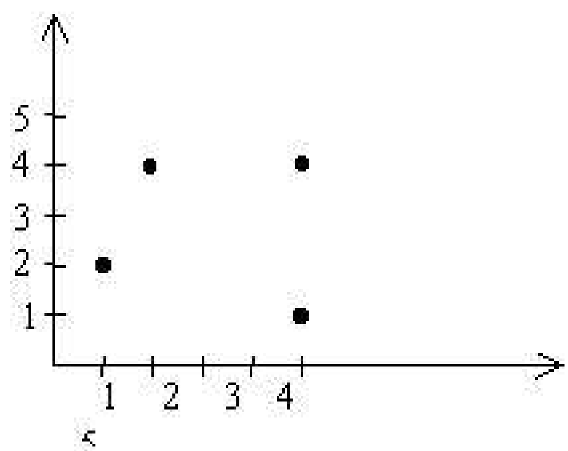

Figure 1. Four points in R^2.

Four points in R^2.

Suppose that x^3 is the ideal prototype of some term X, say for example, "nice", and x^1 is the worst element to be "nice".

From this follows that

$$a = (a_1, a_2) = (5 - -1, 4 - -2) = (4, 2).$$

Points projected on the line joining x^1 and x^3 have the following coordinates:

$$z^1 = x^1 = (1, 2),$$

$$z^2 = (2.6, 2.8),$$

$$z^3 = x^3 = (5, 4),$$

$$z^4 = (3.8, 3.4).$$

These points are shown on Fig 2.

Taking, for example, the city metric

$$d(x, y) = \sum_{i=1}^{n} |x_i - y_i|,$$

one obtains the following grades:

$$\mu_{nice}(z^1) = \frac{d(z^w, z^1)}{d(z^w, z^b)} = \frac{0}{|1-5| + |2-4|} = \frac{0}{6} = 0$$

$$\mu_{nice}(z^2) = \frac{|1-2.6| + |2-2.8|}{6} = 0.4$$

$$\mu \quad {}_{nice}(z^3) = 1$$

$$\mu \quad {}_{nice}(z^4) = \frac{4.2}{6} = 0.7$$

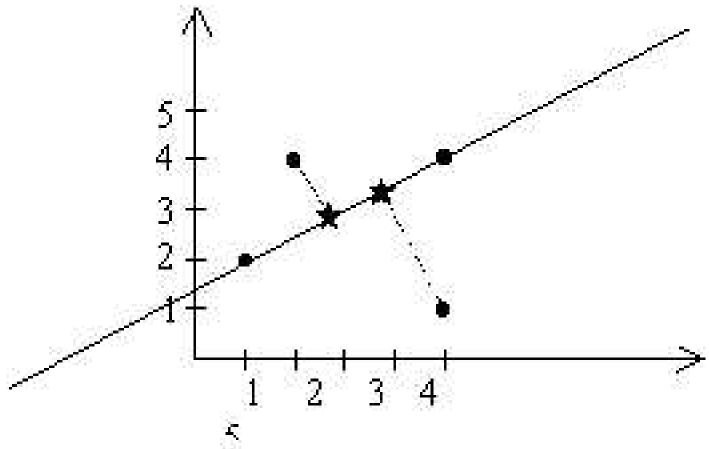

Figure 2. Projections

The obtained fuzzy subset is to be considered as coherent with a perceived meaning of the term "nice" applying to objects ω_1, ω_2, ω_3, and ω_4.

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