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About Shannon's problem for turing machines

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Abstract

Describe the universal turing machine with 3 states and 10 symbols and with 27 commands really used in the program.

Shannon [1] suggested to construct the simplest universal turing machine (a turing machine U is called universal if it can simulate each turing machine T). The measure of complexity the turing machine T using by Shannon is mn – number of command of machine T, where n – number of states of T and m – number of symbols of T. We don't consider Shannon's problem for variants of turing machines (see, for example, Lutz Priese [2] and K.Wagner[3]).

Lets UTM(n,m) – universal turing machine with n states and m symbols. It is known that UTM doesn't exist with 6 commands [4] and UTM(24,2), UTM(11,3), UTM(5,5), UTM(4,6), UTM(3,10) and UTM(2,21) exist [5]. All of these UTM use a construction similar to Minsky's UTM(7,4) using TAG-systems with deletion number $P = 2$ [6], but UTM(5,5) and UTM(4,6) simulate the special classes of TAG-systems. Robinson [7] constructed UTM(7,4) with convenient form of answer: a simpler procedure is used which preserves the output, and leaves the answer immediately to the right of the head of the turing machines. Recently author [8] constructed UTM(10,3) which simplifies the UTM(11,3) in [5]. From results obtained early we conclude that it hasn't been studied yet (if there is a universal turing machine?) 54 classes of turing machines.

Robinson [7] considers the number of commands really used in the program of UTM. According to this way the author [5] used 26 commands in UTM(7,4) (in Robinson's UTM(7,4) – 27 commands), in

UTM(5,5) – 23 commands and UTM(4,6) – 22 commands (minimum of known by author).

UTM(3,10) [5] used 28 commands, in this paper we suggest variant of this machine using 27 commands.

Definition (see, for example [6]). *TAG-system with* $P = 2$ *is a finite set of rules:*

$a_i \to \alpha_i \quad (i = 1, \ldots, n); \quad a_{n+1} \to STOP,$

where $\alpha_i = a_{i1}a_{i2}\ldots a_{im_i}$ *is a finite word in the alphabet* $A = \{a_k\}, \quad k = 1, \ldots, n+1 \quad (\alpha_i$ *can be empty).*

TAG-system T *with* $P = m$ *transforms the finite word* β *in the alphabet* A *like that: in initial word* β *one remembers the first letter from the left side (let* $a_l, l \in \{1, \ldots, n\}$*), after that* m *letters from the left side of the word* β *deletes and on the right side of the transforming word one concatenates the word* α_l*. The process continues and stops when the length of transforming word becomes less than* m *or the first letter of that word becomes* a_{n+1}.

Note. *We take into consideration the proof of existence universal TAG-system with* $P = 2$ *[6] and therefore propose that UTM simulates the TAG-system with* $P = 2$ *with such properties:*

(i) the work of TAG-system stop only by a_{n+1} *symbol;*

(ii) all α_i $(i = 1, \ldots, n)$ *are not empty.*

Common scheme of function of UTM which simulates TAG-system is considered in [5], [6] and [7]. Let to describe the UTM(3,10).

Symbols of UTM(3,10) are $-$ 0 (black symbol), 1, $\overleftarrow{1}$, $\overrightarrow{1}$, b, $\overleftarrow{b}$, $\overrightarrow{b}$, c, $\overleftarrow{c}$, $\overrightarrow{c}$, states are - q_1, q_2, q_3.

$N_1 = 1, \quad N_{k+1} = N_k + 2m_k + 2 \quad (k = 1, \ldots, n)$

Code the word $\alpha_i = a_{i1}a_{i2}\ldots a_{im_i} \quad (i = 1, \ldots, n)$ is:

$P_i = b1bbb1^{N_{im_i}}bb1^{N_{im_i-1}}\ldots bb1^{N_{i2}}bb1^{N_{i1}}$

$P_0 = b$

$P_{n+1} = \overrightarrow{c}\ b$

Code the word $\beta = a_r a_s a_t \ldots a_w$ of TAG-system is:

$S = 1^{N_r}c1^{N_s}c1^{N_t}\ldots c1^{N_w}c$

The program of UTM(3,10):

$q_1\ 0\ cLq_3$	$q_2\ 0\overrightarrow{1}\ Lq_2$	$q_3\ 0\ \ -$
$q_1\ b\overleftarrow{b}\ Rq_1$	$q_2\ b\overrightarrow{b}\ Lq_3$	$q_3\ b\overleftarrow{b}\ Rq_1$
$q_1\ \overleftarrow{b}b\ Lq_1$	$q_2\ \overleftarrow{b}\overrightarrow{b}\ Lq_2$	$q_3\ \overleftarrow{b}\overrightarrow{b}\ Lq_2$
$q_1\ \overrightarrow{b}\overleftarrow{b}\ Rq_1$	$q_2\ \overrightarrow{b}\overleftarrow{b}\ Rq_2$	$q_3\ \overrightarrow{b}b\ Rq_3$
$q_1\ 1\overrightarrow{1}\ Lq_1$	$q_2\ 1\overleftarrow{1}\ Rq_2$	$q_3\ 11\ Rq_3$
$q_1\ \overrightarrow{1}\overleftarrow{1}\ Rq_1$	$q_2\ \overrightarrow{1}\overleftarrow{1}\ Rq_2$	$q_3\ \overrightarrow{1}1\ Rq_3$
$q_1\ \overleftarrow{1}\overrightarrow{1}\ Lq_1$	$q_2\ \overleftarrow{1}\overrightarrow{1}\ Lq_2$	$q_3\ \overleftarrow{1}1\ Lq_3$
$q_1\ c\overrightarrow{1}\ Lq_2$	$q_2\ c\overleftarrow{c}\ Rq_2$	$q_3\ c1\ Rq_1$
$q_1\ \overleftarrow{c}\ \ -$	$q_2\ \overleftarrow{c}\overrightarrow{c}\ Lq_2$	$q_3\ \overleftarrow{c}c\ Lq_3$
$q_1\ \overrightarrow{c}\overleftarrow{c}\ Rq_1$	$q_2\ \overrightarrow{c}\overleftarrow{c}\ Rq_2$	$q_3\ \overrightarrow{c}\ \ -$

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