Computer Science Journal of Moldova, vol.1, no.3(3), 1993

The model of the stochastic generator of the test programs and a method of its analyse

S.Filimonov

Abstract

A method of analyse of the main properties of the stochastic generator of the test programs using its mathematical model in the form of regular stochastic source is proposed.

The test programs produced by stochastic generators are used for the functional testing of the processors. The problem of the synthesis of efficient stochastic generator can be solved by developing of the generator's mathematical model and of the methods of analyse of its main properties.

This kind of generator must produse for the processor under test the sequences of the instructions, which must be informationaly connected and match the conditions of the processor's faults detection [1].

The next mathematical model of the stochastic generator is worked out, having the form of regular stochastic source:

$$\Gamma =< G, f \in A^D, S, P(S) > \tag{1}$$

where G is the generating stochastic multigraph with marked edges:

$$G =< V, D, P(D) > \tag{2}$$

V is the set of vertices, D is a set of edges, $P(D) = \{P_i(D_i)\}$, $P_i(D_i)$ is the distribution of the probabilities, defined on the subset of edges D_i , exiting from the vertex $v_i \in V$; f is a function, which make a correspondence between the edges of the graph G and the symbols of the vocabular $A : f(d_t) = a_t$, $a_t \in A$; $S = \{s_i(v_I, v_A)\}$ is the set of pathes in the graph which connect selected start vertex v_I with the selected end vertex v_A; $P(S)$ is the set of the probabilities, recalculated from the sets of edges of the graph to the set of its pathes.

The source (1) produce the stochastic regular language:

$$L(\Gamma) = \{< z_i, p_i >: [\exists s_i \in S : f(s_i) = z_i]\} \quad (3)$$

and the strings z_i of which correspond to the functionaly full programs of the processor under test [2].

Let to represent the generation graph (2) by a pair a matrices having dimention $|V| \times |V|$. The first matrix is defined as

$$M_G = \|\alpha_{ij}\| \quad (4)$$

where α_{ij} is a regular expesion of the next form: $\alpha_{ij} = [\sum_t^\Lambda(\alpha_t)]_{ij}$, $\sum^\Lambda$ designates addition operation, defined on the set of regular expresions; a_t is a regular expresion, corresponding to the event of generation of the symbol $a_t = f(d_t)$, $d \in D_{ij}$, D_{ij} is a set of parallel edges, which interconnect vertex v_i with vertex v_j, $t = 1, m_{ij}$, $m_{ij} = |D_{ij}|$.

The second matrix is defined as

$$M_G(p) = \|p_{ij}\|, \quad (5)$$

where $p = (p_1, p_2, \ldots, p_t, \ldots, p_m)$ is a vector of probabilities defined on D_{ij}.

Correctly established $P(D)$ must match next relation

$$\sum_j^{|V|} \sum_t^m (p_t)_{ij} = 1 \quad (6)$$

The matrices (4) and (5) together equivalently represents the graph (2) and can be used as base for researching its properties [3].

Let us denote by S^n the set of pathes of length n in graph (2) by S_{ij} the set of pathes from v_i to v_j. It can be mentioned that the intersections $S \cap S^n$ and $S \cap S_{ij}$ can be empty.

Let us to define the operation of catenation on the set of matrices (4): $\|\alpha_{ij}\| * \|\beta_{ij}\| = \|\gamma_{ij}\|$ where $\gamma = \sum_k^\Lambda(\alpha_{ik} * \beta_{kj})$.

Proposition 1 *The element γ_{ij} of the matrix $[M_G]^n$, which is the result of n-th catenation of the matrix (4) corresponds to the set of strings z_{ij}, generated by pathing through the ways from S^n_{ij} of the graph (2).*

Proof. The result of the catenation of the row i with the column j of the matrix (4) corresponds to the set of strings $z_{ij} = f(S^2_{ij})$. Therefore, the result of the catenanion of all rows of the matrix (4) with all its columns represents a $[M_G]^2$ matrix, the elements γ_{ij} of which corresponds to all possible sets z_{ij} of strings of length of 2.

The proof of the proposition will be given by continuing the above ponders on n-th catenation of the matrix (4).

Consequence 1 *The element γ_{IA} of the matrix $[M_G]^n$ corresponds to the language $L^n \subset L(\Gamma)$, which contains n-length strings.*
Let to denote the characteristical matrix of the symbol a_t as

$$M_G(a_t) = \|h_{ij}\|, \tag{7}$$

where $h_{ij} = 1$ when $a_t \in f(D_{ij})$, and $h_{ij} = 0$ otherwise when $a_t \notin f(D_{ij})$. Let to define the conjunction operation $\|h'_{ij}\|\&\|h''_{ij}\| = \|h'''_{ij}\|$ on the set the characteristical matrix of the form (7), where $h'''_{ij} = \underset{k}{\#}(h'_{ik} \wedge h''_{kj})$, $\#$ denote disjunction, and $\wedge$ denote conjunction of elements of matrices.

Proposition 2 *The sequense $z = a_1a_2 \dots a_t \dots a_n$ belongs to the language (3) if, and only if next relation is true:*

$$[M_G(a_1)]\&[M_G(a_2)]\& \dots \&[M_G(a_t)]\& \dots \&[M_G(a_n)] \neq \|0\|,$$

where $\|0\|$ is a matrix of zeroes.

Proof. The h_{ij} is an indicator function of generation of the symbol a_t pathing through edge $d_t \in D_{ij}$, by definition. The conjuction of the element h_{ik} of i-th row of the matrix $M_G(a_1)$ with the element h_{kj} of the j-th column of the matrix $M_G(a_2)$ produces an indicator function of generation of string a_1a_2 pathing through the way $s_k \in S^2_{ij}$. Moreover,

$h'''_{ij} = \underset{k}{\#}(h'_{ik} \wedge h''_{kj})$ is an indicator function of generation of the string $a_1 a_2$ pathing through all the ways of S^2_{ij}.

Therefore, if $[M_G(a_1)] \& [M_G(a_2)] \neq \|0\|$, then there is at least one path $s_k \in S^2$, for which next relation is true: $f(s_k) = a_1 a_2$.

Continuing this thouts for all the symbols from the string z the proof of the proposition will bi done.

Let to define the operation of the calculation of the minimum on the set of probabilities:

$$\triangle(p_1, p_2) = \begin{cases} \min(p_1, p_2), & \text{if } p_1 \neq 0 \text{ and } p_2 \neq 0; \\ p_1, & \text{if } p_1 = 0; \\ p_2, & \text{if } p_2 = 0; \\ 0, & \text{if } p_1 = p_2 = 0 \end{cases}$$

Let to define a matrix of the minimal probabilities the matrix of the next form

$$M_G(p_{min}) = \|p_{min\,ij}\|, \tag{8}$$

which is given from the matrix (5) using next rule:

$$p_{min\,ij} = \underset{t}{\triangle}(p_t)_{ij}.$$

Let to define a matrix of the maximal probabilities the matrix of the next form

$$M_G(p_{max}) = \|p_{max\,ij}\|, \tag{9}$$

which is given from the matrix (5) using next rule:

$$p_{max\,ij} = \max_t (p_t)_{ij}.$$

We can now define the extremal multiplying operation on the set of matrices of the form (8) and (9) $\|p'_{ext\,ij}\| * \|p''_{ext\,ij}\| = \|p'''_{ext\,ij}\|$, where $\|p'''_{ext\,ij}\| = \underset{k}{\triangle}(p'_{min\,ik} \times p_{min\,kj})$ for the matrix of the form (8), and $p'''_{ext\,ij} = \max_k(p'_{max\,ik} \times p_{max\,kj})$ for the matrix of the form (9).

Observation 1 *The operation of the extremal multiplying is associative due to associativity of the minimum calculation operation.*

Let to define the matrix $[M_G(p_{ext})]^n$ as n-th external power of the matrix $M_G(p_{ext})$.

Proposition 3 *The minimal (maximal) value of the probability of generation of the string $z_{ij} = f(S_{ij}^n)$ pathing through ways of graph (2) from S_{ij}^n is equal to the value of the element $p_{ext\,ij}$ of the matrix $[M_G(p_{ext})]^n$, where $M_G(p_{ext})$ is the matrix (8) (matrix (9)).*

The justice of this proposition follows from the properties of the given operations.

Consequence 2 *The minimal (maximal) value of the probability of generation of the string of lengh n of language (3) is equal to the value of the element $p_{ext\,IA}$ of the matrix $[M_G(p_{ext})]^n$, where $M_G(p_{ext})$ is the matrix of form (8) (form (9)), given from the matrix (5) of the graph (2).*

Let to define as a matrix of the quantity of the strings the matrix of the next form

$$M_G(m) = \|m_{ij}\|, \tag{10}$$

where $m_{ij} = |D_{ij}|$.

Proposition 4 *The number of the strings, generated when pathing through the subset S_{ij} of ways of graph (2) is equal to the value of the element m_{ij} of the matrix $[M_G(m)]^n$, which is the n-th power of the matrix (10).*

The justice of this proposition is obvious.

Consequence 3 *The number of the strings of length n belonging to the language (3) is equal to $\sum_i \sum_j m_{ij}$, where m_{ij} is the element of the matrix $[M_G(m)]^n$.*

Observation 2 *The number of the loops around the vertices of graph (2) is equal to $Sp\,[M_G(m)]$.*

Consequence 4 *The number of the loops of length n of the graph (2) is equal to $Sp\,[M_G(m)]$.*

The proposed method is quite effective for the purposes of researching of the properties of the stochastic generators, represented by the model of the form (1).

References

[1] Gulyaev V.A. About selection of the mathematical models for generation of the test programs for complex automated data processing systems. Precisity and Reliability of the cybernetical Systems. Kiev: Naukova dumka, no.5, 1977, p.61-64. (Russian)

[2] Klistorin I.F., Borshevich V.I., Filimonov S.N. Structural and lingvistic approach – the perspective direction of the development of the methods and tools used for checking of the microprocessors. Measurement, Checking, Automation: Scientifical and technical proceedings. M: VNIIIE, no.3(75), 1990, p.38-43. (Russian)

[3] Borshevich V.I., Filimonov S.N., Jdanov V.D., Sidorenco V.V. The estimation of the rightness of the stochastic testing of the microprocessors. Electronical Simulation, no.1, v.11, p.55-59. (Russian)

Sergey Filimonov
Moldavian Technical University,
168 Ştefan cel Mare str,
Kishinev, Moldova,
phone: (373+2)23-75-05

Received March 24, 1993