BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Numbers 3(106), 2024, Pages 63–70
ISSN 1024–7696, E-ISSN 2587–4322

Integrated MILP model for job and crew scheduling to minimize job earliness and tardiness while balancing employee workload

Radu Buzatu

Abstract. This study aims to propose an original and flexible integrated mixed integer linear programming (MILP) model for scheduling jobs and crews with job earliness and tardiness, employee workload balancing, relationships between jobs, and multi-skilled crews. The objective is to minimize the total costs related to assigning jobs to crews, job earliness and tardiness, and employee workload heterogeneity.

Mathematics subject classification: 90C11, 90B35, 90C27.
Keywords and phrases: scheduling theory, job earliness and tardiness, workload balancing, relationships between jobs, mixed integer linear programming.

1 Introduction

Scheduling of jobs and crews is essential in manufacturing and the service industry. Effectively assigning jobs to crews can significantly reduce costs and increase company productivity. On the other hand, improper job and crew scheduling can make a company uncompetitive and even lead to failure.

Scheduling problems are known to be NP-hard [1], meaning they cannot be solved in polynomial time. As a result, most papers on scheduling problems focus on models for finding exact solutions and on heuristic algorithms to obtain near-optimal solutions.

The paper [2] provides a thorough comparative analysis of mixed integer programming models (MILP) for the classical job shop scheduling problem. A significant area of research in scheduling focuses on crew and manpower scheduling [3],[4]. Various scheduling models are designed to minimize job earliness and tardiness [5],[6]. Some studies prioritize balancing employee workload as a primary objective [7],[8]. Mathematical models incorporating multi-skilled crews and relationships between jobs are developed less frequently [9],[10].

The last decades have seen a substantial move forward regarding the development of integrated job and crew scheduling models in various industrial areas [11],[12], [13]. Considering the continuous improvement in IT and the existence of high-performance MILP solvers, it is now possible to formulate a more comprehensive version of the integrated problem. This version can include job and crew scheduling,

DOI: https://doi.org/10.56415/basm.y2024.i3.p63

job earliness and tardiness, balancing employee workload, relationships between jobs, and multi-skilled crews.

As a consequence, this article presents just such a rich and flexible model. The objective is to minimize the total costs of assigning jobs to crews, job earliness and tardiness, and employee workload heterogeneity.

The remainder of the paper is organized in four sections. Section 2 describes the problem, Section 3 develops the MILP model, Section 4 proposes a solution approach, and the final section concludes the findings.

2 Problem description

We consider time discrete, and the entire planning horizon is divided into time intervals of the same duration $d > 0$. A time interval is the smallest unit of time. In job scheduling, a time interval is either fully included in the duration of the job or not. Each time interval is assigned a corresponding time index associated with the interval's start time. We denote a set of time indexes $T = \{0, 1, ..., |T| - 1\}$ for referring to time intervals of the planning horizon. Here, 0 and $|T| - 1$ are the planning horizon's first and last indexes, and 0 and $|T| \cdot d$ are the planning horizon's start and end times. For $t \in T$, $t * d$ is the start time and $(t + 1) * d$ is the end time of the corresponding time interval. Further, when we say time, we consider the time index of the time interval.

The basic assumptions that describe the proposed problem are as follow:

1. There is a set of jobs to be performed by a set of crews.
2. Each job is assigned to exactly one crew at one time. Each job can be performed by any crew.
3. Each crew can perform many jobs at a time such that the number of crew employees working at a time can be at most the total number of crew employees.
4. There is a set of options for performing each job by each crew. An option is associated with the number of crew employees allocated to perform the assigned job for the job duration. The number of employees assigned to the job remains the same throughout performing the job.
5. Each crew's working timetable, including working and non-working time internals, is known in advance.
6. The performing duration (the number of time intervals) of a job by a crew according to each possible option and starting time is known in advance. The crew performs the assigned job without interruptions in accordance with the option. This means that all crew working time intervals that fall into job duration according to the option are used to perform the job by the allocated number of crew employees.
7. Each job's earliest starting time and latest completion time are known. Nevertheless, the earliness of job starting and the tardiness of job completion are

admissible but penalized. Job earliness appears when the job starts before the job's earliest starting time, and job tardiness appears when the job is completed later than the job's latest completion time.

8. All jobs must be completed within the planning horizon.
9. The cost of assigning a job to a crew reflects the crew's skill level. The higher the crew's skill level in performing the job, the lower the cost of assigning the job to the crew.
10. Employee workload balance is measured by the difference between the highest workload per employee and the lowest workload per employee among all crews.
11. Some traditional relationships can be defined for a pair of jobs. For more flexibility, lag times are defined in relationships between jobs.
12. The manpower of all crews is assumed to be sufficient to complete all jobs within the planning horizon.

As for job relationships, there are four traditional types of relationships between any two jobs:

- Finish-to-start (FS) – a relationship in which the completion time of the first job is coordinated with the start times of the second job.
- Start-to-start (SS) – a relationship in which the start time of the first job is coordinated with the start time of the second job.
- Finish-to-finish (FF) – a relationship in which the completion time of the first job is coordinated with the completion time of the second job.
- Start-to-finish (SF) – a relationship in which the start time of the first job is coordinated with the completion time of the second job.

More details about these types of relationships can be found in [9].

The objective of the problem is to minimize the total costs, consisting of assigning jobs to crews costs, job earliness and tardiness costs, and employee workload heterogeneity costs.

3 MILP model

In order to formulate an integrated MILP model for job and crew scheduling problem, the notations used in the model are given.

Sets and parameters:

J – set of jobs.

C – set of crews.

$O_{j,c}$ – set of options for performing each job j by crew c. An option is associated with the employees number of crew c allocated to perform job j.

T – set of time indexes of the planning horizon.

T_c^{crew} – set of time indexes associated to working time intervals of crew c according to it's work timetable, $T_c^{crew} \subseteq T$.

e_j – the earliest starting time of job j.

l_j – the latest completion time of job j.

n_c^{crew} – total number of employees in crew c.

$n_{j,c,o}^{work}$ – number of employees required to perform job j by crew c in line with option o.

$d_{j,c,o,t}^{all}$ – number of all time intervals (working and non-working) during which job j is performed by crew c in conformity to option o when the job starts at time t.

$d_{j,c,o,t}^{work}$ – number of working time intervals during which job j is performed by crew c in conformity to option o when the job starts at time t.

$r_{j,j'}^{FS}$ – FS relationship indicator between jobs j and j'. $r_{j,j'}^{FS} = 1$ if the completion time of job j is coordinated with the start times of job j', and $r_{j,j'}^{FS} = 0$ otherwise.

$r_{j,j'}^{SS}$ – SS relationship indicator between jobs j and j'. $r_{j,j'}^{SS} = 1$ if the start time of job j is coordinated with the start times of job j', and $r_{j,j'}^{SS} = 0$ otherwise.

$r_{j,j'}^{FF}$ – FF relationship indicator between jobs j and j'. $r_{j,j'}^{FF} = 1$ if the completion time of job j is coordinated with the completion times of job j', and $r_{j,j'}^{FF} = 0$ otherwise.

$r_{j,j'}^{SF}$ – SF relationship indicator between jobs j and j'. $r_{j,j'}^{SF} = 1$ if the start time of job j is coordinated with the completion times of job j', and $r_{j,j'}^{SF} = 0$ otherwise.

$g_{j,j'}^{FS}, g_{j,j'}^{SS}, g_{j,j'}^{FF}, g_{j,j'}^{SF}$ – lag times (number of time intervals) between jobs j and j' related to FS, SS, FF and SF relationships, respectively.

$\alpha_{j,c}$ – cost of assigning job j to crew c. This cost expresses the skill of crew c in performing job j.

β_j^{early} – earliness cost of job j.

β_j^{tard} – tardiness cost of job j.

γ – employee workload heterogeneity costs.

Decision variables:

$x_{j,c,o,t}$ – binary variable that is equal to 1 if job j is performed by crew c in conformity to option o and starting at time t, and 0 otherwise.

y_j^{early}, y_j^{tard} – non-negative integer variables indicating earliness and tardiness of job j as a number of time intervals.

z^{low}, z^{high} – non-negative real variables indicating the lowest and, respectively, the highest workload per employee among all crews.

Now, the job and crew scheduling problem is formulated as a mixed integer linear programming (MILP) model as follows:

Objective function:

$$\sum_{j \in J} \sum_{c \in C} \alpha_{j,c} \cdot \sum_{o \in O_{j,c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d_{j,c,o,t}^{all}}} x_{j,c,o,t} + \\ + \sum_{j \in J} (\beta_j^{early} \cdot y_j^{early} + \beta_j^{tard} \cdot y_j^{tard}) + \gamma \cdot (z^{high} - z^{low}) \rightarrow min \quad (1)$$

Subject to:

$$\sum_{c \in C} \sum_{o \in O_{j,c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d_{j,c,o,t}^{all}}} x_{j,c,o,t} = 1 \quad \forall j \in J \quad (2)$$

$$\sum_{j \in J} \sum_{o \in O_{j,c}} n_{j,c,o}^{work} \cdot \sum_{\substack{t' \in T_c^{crew} \\ t - d_{j,c,o,t'}^{all} + 1 \leq t' \leq t}} x_{j,c,o,t'} \leq n_c^{crew} \quad \forall c \in C, \forall t \in T_c^{crew} \quad (3)$$

$$\sum_{c \in C} \sum_{o \in O_{j,c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d_{j,c,o,t}^{all}}} (t + d_{j,c,o,t}^{all}) \cdot x_{j,c,o,t} + g_{j,j'}^{FS} \leq \\ \leq \sum_{c \in C} \sum_{o \in O_{j',c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d_{j',c,o,t}^{all}}} t \cdot x_{j',c,o,t} \quad \forall j, j' \in J, j \neq j', r_{j,j'}^{FS} = 1 \quad (4)$$

$$\sum_{c \in C} \sum_{o \in O_{j,c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d_{j,c,o,t}^{all}}} t \cdot x_{j,c,o,t} + g_{j,j'}^{SS} \leq \\ \leq \sum_{c \in C} \sum_{o \in O_{j',c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d_{j',c,o,t}^{all}}} t \cdot x_{j',c,o,t} \quad \forall j, j' \in J, j \neq j', r_{j,j'}^{SS} = 1 \quad (5)$$

$$\sum_{c \in C} \sum_{o \in O_{j,c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d_{j,c,o,t}^{all}}} (t + d_{j,c,o,t}^{all}) \cdot x_{j,c,o,t} + g_{j,j'}^{FF} \leq \\ \leq \sum_{c \in C} \sum_{o \in O_{j',c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d_{j',c,o,t}^{all}}} (t + d_{j',c,o,t}^{all}) \cdot x_{j',c,o,t} \quad \forall j, j' \in J, j \neq j', r_{j,j'}^{FF} = 1 \quad (6)$$

$$\sum_{c \in C} \sum_{o \in O_{j,c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d^{all}_{j,c,o,t}}} t \cdot x_{j,c,o,t} + g^{SF}_{j,j'} \leq$$
$$\leq \sum_{c \in C} \sum_{o \in O_{j',c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d^{all}_{j',c,o,t}}} (t + d^{all}_{j',c,o,t}) \cdot x_{j',c,o,t} \quad \forall j, j' \in J, j \neq j', r^{SF}_{j,j'} = 1 \tag{7}$$

$$e_j - \sum_{c \in C} \sum_{o \in O_{j,c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d^{all}_{j,c,o,t}}} t \cdot x_{j,c,o,t} \leq y^{early}_j \quad \forall j \in J \tag{8}$$

$$\sum_{c \in C} \sum_{o \in O_{j,c}} \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d^{all}_{j,c,o,t}}} (t + d^{all}_{j,c,o,t}) \cdot x_{j,c,o,t} - l_j - 1 \leq y^{tard}_j \quad \forall j \in J \tag{9}$$

$$\sum_{j \in J} \sum_{o \in O_{j,c}} d^{work}_{j,c,o} \cdot n^{work}_{j,c,o} \cdot \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d^{all}_{j,c,o,t}}} x_{j,c,o,t} \leq n^{crew}_c \cdot z^{high} \quad \forall c \in C \tag{10}$$

$$\sum_{j \in J} \sum_{o \in O_{j,c}} d^{work}_{j,c,o} \cdot n^{work}_{j,c,o} \cdot \sum_{\substack{t \in T_c^{crew} \\ t \leq |T| - d^{all}_{j,c,o,t}}} x_{j,c,o,t} \geq n^{crew}_c \cdot z^{low} \quad \forall c \in C \tag{11}$$

$$x_{j,c,o,t} \in \{0,1\} \quad \forall j \in J, \forall c \in C, \forall o \in O_{j,c}, \forall t \in T_c^{crew} \tag{12}$$

$$y^{early}_j, y^{tard}_j \in \mathbb{N} \quad \forall j \in J \tag{13}$$

$$z^{high}, z^{low} \in \mathbb{R}_+ \tag{14}$$

The objective function (1) minimizes the overall costs while fulfilling the set of constraints (2)-(14). It includes the costs of assigning jobs to crews, the cost of job earliness and tardiness, and the cost of workload heterogeneity per employee among all crews. Constraints (2) ensure that a unique crew, work option, and start time are chosen for each job in the final solution. Constraints (3) guarantee that the number of crew employees working at a time can be at most the total number of crew employees. Constraints (4), (5), (6) and (7) assure satisfying relationships FS, SS, FF and SF, respectively. Constraints (8) and (9) are used to calculate job earliness and tardiness. Constraints (10) and (11), in turn, are used to calculate the highest and the lowest workload per employee among all crews. Finally, constraints (12)-(14) define the types of decision variables.

4 Solution approach

The problem can be solved using the branch and cut algorithm, which is widely used to solve MILP problems. It combines a branch and bound algorithm with a cutting plane method. The branch and bound algorithm starts with a linear programming relaxation of the MILP. If some variables of the solution are not integer, the algorithm "branches" on a variable to create two new subproblems: one constraining the variable to be less than or equal to its floor value and the other to be greater than or equal to its ceiling value. The cutting plane method adds linear constraints to the linear programming relaxation to eliminate areas of the feasible region containing fractional solutions. By iteratively using the branch and bound algorithm and adding cuts, the algorithm explores different branches of the solution tree until it finds an optimal integer solution or proves that no feasible solution exists. For a detailed description see, for example, [14]

So, modern commercial high-performance MILP solvers such as CPLEX and Gurobi, which use the branch and cut algorithm, are capable of solving medium size instances of the proposed MILP model. For smaller instances of the problem, even open-source MILP solvers like SCIP or HiGHS can be utilized. However, large real industrial instances of the problem involve a vast number of variables, and in such cases, only near-optimal heuristic algorithms remain effective.

5 Conclusion

We have developed a precise MILP model for the flexible scheduling of jobs and multi-skilled crews. This model takes into account job earliness and tardiness, as well as workload balancing for employees. It also considers realistic job relationships, such as finish-to-start, start-to-start, finish-to-finish, and start-to-finish.

In our upcoming research, our primary goal is to create an efficient heuristic algorithm for solving the described problem. We also plan to compare the MILP model and future heuristic with other currently used solution methods.

References

[1] Lenstra J.K., Rinnooy Kan A.H.G. *Computational complexity of discrete optimization problems.* Annals of Discrete Mathematics, vol. **4**, 1979, pp. 121–140.

[2] Ku W.Y., Beck J.C. *Mixed integer programming models for job shop scheduling: A computational analysis.* Computers & Operations Research, **73**, 2016, pp. 165–173.

[3] Chen M., Niu H. *A model for bus crew scheduling problem with multiple duty types.* Discrete Dynamics in Nature and Society, vol. **2012**, no. **1**, 2012, pp. 1–11.

[4] Pan Q.K., Suganthan P.N., Chua T.J., Cai T.X. *Solving manpower scheduling problem in manufacturing using mixed-integer programming with a two-stage heuristic algorithm.* The International Journal of Advanced Manufacturing Technology, vol. **46**, 2010, pp. 1229–1237.

[5] Janiak A., Janiak W.A., Krysiak T., Kwiatkowski T. *A survey on scheduling problems with due windows.* European Journal of Operational Research, vol. **242**, no. **2**, 2015, pp. 347–357.

[6] Ronconi D.P., Birgin E.G. *Mixed-integer programming models for flowshop scheduling problems minimizing the total earliness and tardiness.* Just-in-Time systems, 2012, pp. 91–105.

[7] Hertz A., Lahrichi N., Widmer M. *A flexible MILP model for multiple-shift workforce planning under annualized hours.* European Journal of Operational Research, vol. **200**, no. **3**, 2010, pp. 860–873.

[8] Lin D., Tsai M. *Integrated crew scheduling and roster problem for trainmasters of passenger railway transportation.* IEEE Access, vol. **7**, 2019, pp. 27362–27375.

[9] García-Nieves J.D., Ponz-Tienda J.L., Ospina-Alvarado A., Bonilla-Palacios M. *Multipurpose linear programming optimization model for repetitive activities scheduling in construction projects.* Automation in construction, vol. **105**, 2019, pp. 1–11.

[10] Karam A., Attia E.A., Duquenne P. *A MILP model for an integrated project scheduling and multi-skilled workforce allocation with flexible working hours.* IFAC-PapersOnLine, vol. **50**, no. **1**, 2017, pp. 13964–13969.

[11] Artigues C., Gendreau M., Rousseau L.M., Vergnaud A. *Solving an integrated employee timetabling and job-shop scheduling problem via hybrid branch-and-bound.* Computers & Operations Research, vol. **36**, no. **8**, 2009, pp. 2330–2340.

[12] Mesquita M., Paias A. *Set partitioning/covering-based approaches for the integrated vehicle and crew scheduling problem.* Computers & Operations Research, vol. **35**, no. **5**, 2008, pp. 1562–1575.

[13] Ge L., Kliewer N., Nourmohammadzadeh A., Voss S., Xie L. *Revisiting the richness of integrated vehicle and crew scheduling.* Public Transport, 2022, pp. 1–27.

[14] Mitchell J.E. *Branch-and-cut algorithms for combinatorial optimization problems.* Handbook of applied optimization, vol. **1**, no. **1**, 2002, pp. 65–77.

Radu Buzatu
Vladimir Andrunachievici Institute of Mathematics and
Computer Science, Moldova State University
E-mail: *radubuzatu@gmail.com, radu.buzatu@usm.md*

Received August 29, 2024