BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Number 1(98), 2022, Pages 99–110
ISSN 1024–7696, E-ISSN 2587–4322

A self–similar solution and the tanh–function method for the kinetic Carleman system

S. A. Dukhnovsky

Abstract. In this article, we consider the one–dimensional kinetic system of Carleman equations. The Carleman system is the kinetic Boltzmann equation. This system describes a monatomic rarefied gas consisting of two groups of particles. One particle from the first group, interacting with a particle of the first group, transforms into two particles of the second group. Similarly, two particles of the second group, interacting with themselves, transform into two particles of the first group, respectively. We found traveling wave solutions by using the tanh–function method for nonlinear partial differential system. The results of the work can be useful for mathematical modeling in various fields of science and technology: kinetic theory of gases, gas dynamics, autocatalysis. The obtained exact solutions are new.

Mathematics subject classification: 35L45, 35L60, 35Q20.
Keywords and phrases: Painlevé test, Carleman system, tanh–function method, traveling wave solutions.

1 Introduction

We consider the one–dimentional Carleman system [1, 16, 20, 21, 23]

$$\begin{aligned} &\partial_t u + \partial_x u = \frac{1}{\varepsilon}(w^2 - u^2), \quad x \in \mathbb{R},\ t > 0, \\ &\partial_t w - \partial_x w = -\frac{1}{\varepsilon}(w^2 - u^2). \end{aligned} \tag{1}$$

Here $u = u(x,t), w = w(x,t)$ are the densities of two groups of particles with velocities $c = 1, -1$, ε is the Knudsen parameter from the kinetic theory of gases. The Carleman system is a non-integrable system, i.e. the Painlevé test is not applicable [22]. Despite this, we will look for a solution for all values $u(x,t), w(x,t)$.

There are many known methods that allow to find exact solutions of PDEs such as the tanh–function method [8, 9], the extended tanh–function method [11], the Exp–function method [7, 12], the sine–cosine method [9, 10], Jacobi Elliptic function expansion method [13] et al. As noted in [7], the Exp–function method is more efficient and gives more different solutions than the tanh–function method. Few works were devoted to finding solutions for the Carleman system. In [3], the space–time Carleman system with Riemann–Liouville derivative is investigated and the exact solutions are obtained via the generalized Bernoulli Sub–ODE method. The

DOI: https://doi.org/10.56415/basm.y2022.i1.p99

asymptotic stability of the stationary solution of the kinetic models was proved in the works [16, 20, 21, 25]. In [15], Golse found the self–similar solutions of the one–dimentional Broadwell model. In [23], Ilyin found the stationary solution of (1). Nonstationary solutions of the Boltzmann models are found in [4–6, 17–19, 24]. In this article, we will seek, for the first time, new exact solutions of (1) by using the tanh–function method, the extended tanh–function method.

2 A self–similar solution of the kinetic Carleman model

We look for a self–similar solution of the system of equations (1) in the form [2, 18]:

$$u(x,t) = x^m U(\xi), w(x,t) = x^n W(\xi), \xi = tx^{-k}.$$

System (1) is scale invariant under

$$x = C\overline{x}, t = C^k\overline{t}, u = C^m\overline{u}, w = C^n\overline{w}, C > 0.$$

As a result, we obtain a reduction

$$u(x,t) = \frac{1}{x}U(\xi), w(x,t) = \frac{1}{x}W(\xi), \xi = \frac{t}{x}. \tag{2}$$

Substituting (2) into our original system, we obtain a system of ordinary differential equations

$$\begin{aligned} \frac{1}{x}U'_\xi\frac{1}{x} - \frac{1}{x^2}U - U'_\xi\frac{t}{x^2}\frac{1}{x} &= \frac{1}{\varepsilon}(\frac{1}{x^2}W^2 - \frac{1}{x^2}U^2), \\ \frac{1}{x}W'_\xi\frac{1}{x} + \frac{1}{x^2}W + \frac{1}{x}W'_\xi\frac{t}{x^2} &= -\frac{1}{\varepsilon}(\frac{1}{x^2}W^2 - \frac{1}{x^2}U^2). \end{aligned} \tag{3}$$

Hence

$$\begin{aligned} (1-\xi)U'_\xi &= U + \frac{1}{\varepsilon}(W^2 - U^2), \\ (1+\xi)W'_\xi &= -W - \frac{1}{\varepsilon}(W^2 - U^2). \end{aligned} \tag{4}$$

Let us perform the Painlevé test [14] for system (4). Then we look for a solution in the form

$$U = \sum_{j=0}^{\infty} U_j\varphi^{j-p}, W = \sum_{j=0}^{\infty} W_j\varphi^{j-\beta},$$

where $U_j = U_j(\xi), W_j = V_j(\xi)$. We find the leading terms for $j = 0$:

$$\begin{aligned} -(1-\xi)U_0\varphi' &= \frac{1}{\varepsilon}(W_0^2 - U_0^2), \\ -(1+\xi)W_0\varphi' &= -\frac{1}{\varepsilon}(W_0^2 - U_0^2). \end{aligned}$$

Then

$$U_0 = \frac{\varepsilon(1+\xi)^2(1-\xi)}{4\xi}\varphi', W_0 = -\frac{\varepsilon(1+\xi)(1-\xi)^2}{4\xi}\varphi'.$$

It follows that there are two resonances, $r = -1$ and $r = 1$. We have the truncated solution

$$U = U_0\varphi^{-1} + U_1, W = W_0\varphi^{-1} + W_1. \tag{5}$$

Substituting (5) into the system (1) and collecting terms with the same powers of φ, we have

$$\begin{aligned}
\varphi^{-2}\Big(-(1-\xi)\varphi' U_0 - \frac{1}{\varepsilon}(W_0^2 - U_0^2)\Big)& \\
+\varphi^0\Big((1-\xi)U_1' - U_1 - \frac{1}{\varepsilon}(W_1^2 - U_1^2)\Big)& \\
+\varphi^{-1}\Big((1-\xi)U_0' - U_0 - \frac{1}{\varepsilon}(2W_0W_1 - 2U_0U_1)\Big) &= 0, \\
\varphi^{-2}\Big(-(1+\xi)\varphi' W_0 + \frac{1}{\varepsilon}(W_0^2 - U_0^2)\Big)& \\
+\varphi^0\Big((1+\xi)W_1' + W_1 + \frac{1}{\varepsilon}(W_1^2 - U_1^2)\Big)& \\
+\varphi^{-1}\Big((1+\xi)W_0' + W_0 + \frac{1}{\varepsilon}(2W_0W_1 - 2U_0U_1)\Big) &= 0.
\end{aligned}$$

From here we obtain at the resonance $r = 1$ the equations

$$(1-\xi)U_0' - U_0 - \frac{1}{\varepsilon}(2W_0W_1 - 2U_0U_1) = 0, \tag{6}$$

$$(1+\xi)W_0' + W_0 + \frac{1}{\varepsilon}(2W_0W_1 - 2U_0U_1) = 0. \tag{7}$$

The compatibility condition holds for the resonance $r = 1$, since the two relations for $r = 1$ coincide. Hence, the system (4) passes the Painlevé test. Note that $U_1 = W_1 = 0$ is the trivial solution of (1). Substituting into one from these equations (6)–(7) yields

$$(1 + 3\xi^2)\varphi' + \varphi''\xi(\xi^2 - 1) = 0. \tag{8}$$

The solution of (8) has the form

$$\varphi(\xi) = -\frac{C_1}{2(\xi^2 - 1)} + C_2, \tag{9}$$

where $C_1, C_2 \in \mathbb{R}$ are constants of integration. By means of (9), the system solution is written in the form

$$u(x,t) = \frac{1}{x}\frac{\varepsilon C_1}{4(1-\xi)}\Big(-\frac{C_1}{2(\xi^2-1)} + C_2\Big)^{-1}, \xi = \frac{t}{x}, \tag{10}$$

$$w(x,t) = -\frac{1}{x}\frac{\varepsilon C_1}{4(1+\xi)}\Big(-\frac{C_1}{2(\xi^2-1)} + C_2\Big)^{-1}. \tag{11}$$

The solutions (10) and (11) are represented in Fig. 1–2. These solutions become infinite when $x = 0, t = \pm x$.

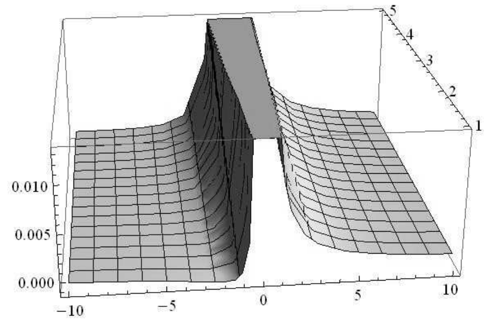

Figure 1. Figure of $u(x,t)$ for $C_1 = -10, \varepsilon = 0.01, C_2 = 0.01, -10 < x < 10, 1 < t < 5$ in (10)

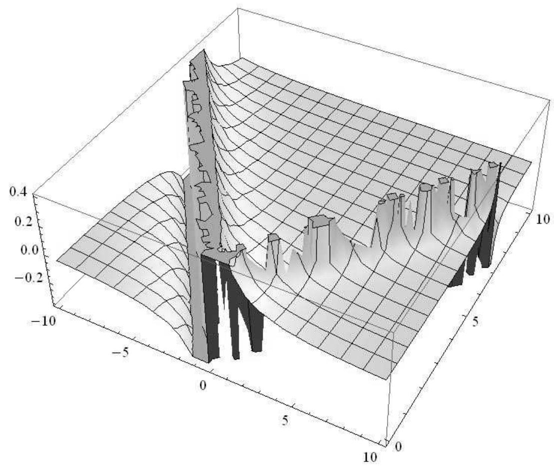

Figure 2. Figure of $w(x,t)$ for $C_1 = -10, \varepsilon = 1, C_2 = 10, -10 < x < 10, 0 < t < 6$ in (11)

3 The tanh–method for the Carleman system

We seek a solution in the following transformations

$$u = U(\xi), w = W(\xi), \xi = x - ct.$$

In this case we have

$$\begin{aligned} &U'(1-c) = \frac{1}{\varepsilon}(W^2 - U^2), \\ &-W'(1+c) = -\frac{1}{\varepsilon}(W^2 - U^2). \end{aligned} \tag{12}$$

The tanh method admits the use of finite series

$$U(\xi) = S(Y) = \sum_{m=0}^{M} a_m Y^m, W(\xi) = \bar{S}(Y) = \sum_{m=0}^{M_1} b_m Y^m, \tag{13}$$

where $Y = \tanh(\mu\xi)$, M and M_1 are positive integers. Substituting (13) into (12) yields

$$\begin{aligned} \mu(1-c)(1-Y^2)\frac{dS}{dY} &= \frac{1}{\varepsilon}(\bar{S}^2 - S^2), \\ -\mu(1+c)(1-Y^2)\frac{d\bar{S}}{dY} &= -\frac{1}{\varepsilon}(\bar{S}^2 - S^2). \end{aligned} \tag{14}$$

After substitution of Eq.(13) into Eq.(14), we balance the highest powers of Y. Then we have

$$2 + M - 1 = 2M_1 = 2M,$$

$$2 + M_1 - 1 = 2M_1 = 2M,$$

so that $M = M_1 = 1$. We get the truncated expansion

$$\begin{aligned} S(Y) &= a_0 + a_1 Y, \\ \bar{S}(Y) &= b_0 + b_1 Y. \end{aligned} \tag{15}$$

Substituting (15) into (14) and collecting the coefficients of Y, we have

$$\begin{aligned} -\mu a_1 + \mu c a_1 + \frac{a_1^2}{\varepsilon} - \frac{b_1^2}{\varepsilon} &= 0, \\ 2a_0 a_1 - 2b_0 b_1 &= 0, \\ \mu a_1 - \mu c a_1 + \frac{a_0^2}{\varepsilon} - \frac{b_0^2}{\varepsilon} &= 0 \end{aligned} \tag{16}$$

and

$$\begin{aligned} \mu b_1 + \mu c b_1 - \frac{a_1^2}{\varepsilon} + \frac{b_1^2}{\varepsilon} &= 0, \\ -2a_0 a_1 + 2b_0 b_1 &= 0, \\ -\mu b_1 - \mu c b_1 - \frac{a_0^2}{\varepsilon} + \frac{b_0^2}{\varepsilon} &= 0, \end{aligned} \tag{17}$$

Solving algebraic equations system with the aid of the Mathematica Package, we have the following solutions:

Case 1. $a_0 = -b_0, a_1 = b_1 = 0, b_0 \in \mathbb{R}$.

Case 2. $a_0 = b_0, a_1 = b_1 = 0, b_0 \in \mathbb{R}$.

Case 3.

$$\begin{aligned} a_0 &= \mu\frac{(c-1)^2(1+c)\varepsilon}{4c}, b_0 = -\mu\frac{(c-1)(1+c)^2\varepsilon}{4c}, \\ b_1 &= \mu\frac{(c-1)^2(1+c)\varepsilon}{4c}, a_1 = -\mu\frac{(c-1)(1+c)^2\varepsilon}{4c}, \mu \in \mathbb{R}. \end{aligned}$$

Case 4.

$$a_0 = -\mu\frac{(c-1)^2(1+c)\varepsilon}{4c}, b_0 = \mu\frac{(c-1)(1+c)^2\varepsilon}{4c},$$
$$b_1 = \mu\frac{(c-1)^2(1+c)\varepsilon}{4c}, a_1 = -\mu\frac{(c-1)(1+c)^2\varepsilon}{4c}, \mu \in \mathbb{R}.$$

For case 3 the solutions have the form

$$u(x,t) = \mu\frac{(c-1)^2(1+c)\varepsilon}{4c} - \mu\frac{(c-1)(1+c)^2\varepsilon}{4c}\tanh(\mu(x-ct)), \tag{18}$$
$$w(x,t) = -\mu\frac{(c-1)(1+c)^2\varepsilon}{4c} + \mu\frac{(c-1)^2(1+c)\varepsilon}{4c}\tanh(\mu(x-ct)). \tag{19}$$

For case 4 we have the solutions

$$u(x,t) = -\mu\frac{(c-1)^2(1+c)\varepsilon}{4c} - \mu\frac{(c-1)(1+c)^2\varepsilon}{4c}\tanh(\mu(x-ct)),$$
$$w(x,t) = \mu\frac{(c-1)(1+c)^2\varepsilon}{4c} + \mu\frac{(c-1)^2(1+c)\varepsilon}{4c}\tanh(\mu(x-ct)).$$

The solutions (18) and (19) are represented in Fig. 3–4.

4 The extended tanh–function method

The extended tanh method admits the use of finite series

$$\begin{aligned} U(\xi) = S(Y) &= \sum_{m=0}^{M} a_m Y^m + \sum_{l=1}^{L} b_l Y^{-l}, \\ W(\xi) = \bar{S}(Y) &= \sum_{p=0}^{P} c_p Y^p + \sum_{d=1}^{D} f_d Y^{-d}, \end{aligned} \tag{20}$$

where M, L, N, D are nonnegative integers. Substituting (20) into (14) and balancing the highest, lowest powers of Y, we obtain

$$2 + M - 1 = 2P = 2M,$$

$$2 + P - 1 = 2P = 2M$$

and

$$-L - 1 = -2D = -2L,$$

$$-D - 1 = -2D = -2L,$$

so that $M = P = N = D = 1$. Then we seek a solution of (12) in the form

$$\begin{aligned} U(\xi) = S(Y) &= a_0 + a_1 Y + b_1 Y^{-1}, \\ W(\xi) = \bar{S}(Y) &= c_0 + c_1 Y + f_1 Y^{-1}. \end{aligned} \tag{21}$$

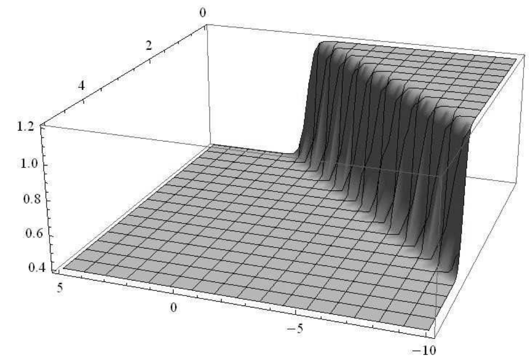

Figure 3. Figure of $u(x,t)$ for $c = -3, \varepsilon = 0.1, \mu = 3, -10 < x < 5, 0 < t < 5$ in (18)

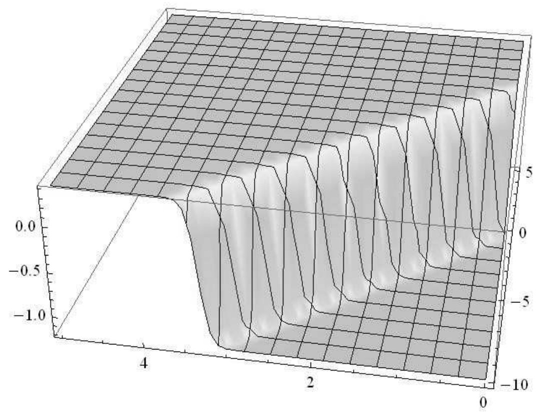

Figure 4. Figure of $w(x,t)$ for $c = -3, \varepsilon = 0.1, \mu = 3, -10 < x < 5, 0 < t < 5$ in (19)

Substituting (21) into (14) and collecting the coefficients of Y, we have

$$\begin{aligned}
&-\mu a_1 + \mu c a_1 + \frac{a_1^2}{\varepsilon} - \frac{c_1^2}{\varepsilon} = 0,\\
&2a_0a_1 - 2c_0c_1 = 0,\\
&2a_0b_1 - 2c_0f_1 = 0,\\
&-\mu b_1 + \mu c b_1 + \frac{b_1^2}{\varepsilon} - \frac{f_1^2}{\varepsilon} = 0,\\
&\frac{a_0^2}{\varepsilon} - \frac{c_0^2}{\varepsilon} + \frac{2a_1b_1}{\varepsilon} - \frac{2c_1f_1}{\varepsilon} + \mu a_1 + \mu b_1 - \mu c a_1 - \mu c b_1 = 0
\end{aligned} \tag{22}$$

and

$$\begin{aligned} &\mu c_1 + \mu c c_1 - \frac{a_1^2}{\varepsilon} + \frac{c_1^2}{\varepsilon} = 0, \\ &-2a_0 a_1 + 2c_0 c_1 = 0, \\ &-2a_0 b_1 + 2c_0 f_1 = 0, \\ &\mu f_1 + \mu c f_1 - \frac{b_1^2}{\varepsilon} + \frac{f_1^2}{\varepsilon} = 0, \\ &-\frac{a_0^2}{\varepsilon} + \frac{c_0^2}{\varepsilon} - \frac{2a_1 b_1}{\varepsilon} + \frac{2c_1 f_1}{\varepsilon} - \mu c_1 - \mu f_1 - \mu c c_1 - \mu c f_1 = 0. \end{aligned} \tag{23}$$

We obtain solutions:

Case 1.

$$c_0 = -\frac{(1+c)f_1}{c-1}, a_0 = f_1, a_1 = 0,$$
$$b_1 = -\frac{(1+c)f_1}{c-1}, c_1 = 0, \mu = \frac{4cf_1}{(c-1)^2(1+c)\varepsilon}.$$

Case 2.

$$c_0 = \frac{(1+c)f_1}{c-1}, a_0 = -f_1, a_1 = 0,$$
$$b_1 = -\frac{(1+c)f_1}{c-1}, c_1 = 0, \mu = \frac{4cf_1}{(c-1)^2(1+c)\varepsilon}.$$

Case 3.

$$c_0 = \frac{2(c+1)f_1}{c-1}, a_0 = -2f_1, a_1 = -\frac{(c+1)f_1}{c-1},$$
$$b_1 = -\frac{(c+1)f_1}{c-1}, c_1 = f_1, \mu = \frac{4cf_1}{(c-1)^2(1+c)\varepsilon}.$$

Case 4.

$$c_0 = -\frac{2(c+1)f_1}{c-1}, a_0 = 2f_1, a_1 = -\frac{(c+1)f_1}{c-1},$$
$$b_1 = -\frac{(c+1)f_1}{c-1}, c_1 = f_1, \mu = \frac{4cf_1}{(c-1)^2(1+c)\varepsilon}.$$

Here f_1 is any real number for cases 1–4. For case 1 we have

$$u(x,t) = f_1 - \frac{(1+c)f_1}{c-1}\coth\Big(\frac{4cf_1}{(c-1)^2(1+c)\varepsilon}(x-ct)\Big), \tag{24}$$

$$w(x,t) = -\frac{(1+c)f_1}{c-1} + f_1\coth\Big(\frac{4cf_1}{(c-1)^2(1+c)\varepsilon}(x-ct)\Big). \tag{25}$$

For case 3 we have

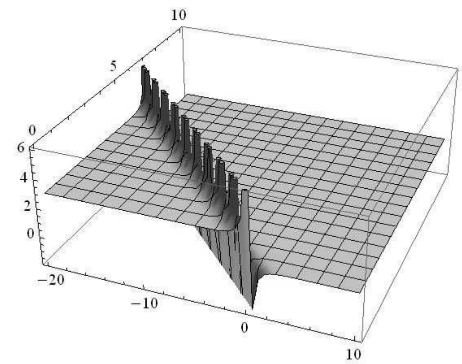

Figure 5. Figure of $u(x,t)$ for $c=-3, f_1=2, \varepsilon=1, -20<x<10, 0<t<10$ in (24)

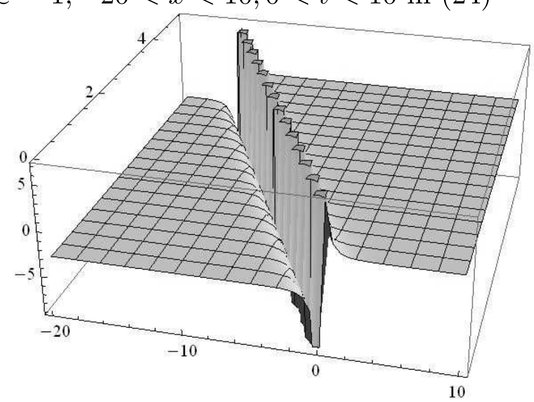

Figure 6. Figure of $w(x,t)$ for $c=-3, f_1=2, \varepsilon=1, -10<x<10, 0<t<5$ in (25)

$$
\begin{aligned}
u(x,t) &= -2f_1 - \frac{(c+1)f_1}{c-1}\tanh\left(\frac{4cf_1}{(c-1)^2(1+c)\varepsilon}(x-ct)\right) \\
&\quad -\frac{(c+1)f_1}{c-1}\coth\left(\frac{4cf_1}{(c-1)^2(1+c)\varepsilon}(x-ct)\right), \qquad (26)\\
w(x,t) &= \frac{2(c+1)f_1}{c-1} + f_1\tanh\left(\frac{4cf_1}{(c-1)^2(1+c)\varepsilon}(x-ct)\right) \\
&\quad +f_1\coth\left(\frac{4cf_1}{(c-1)^2(1+c)\varepsilon}(x-ct)\right). \qquad (27)
\end{aligned}
$$

The solutions (24)–(27) are represented in Fig. 5–8.

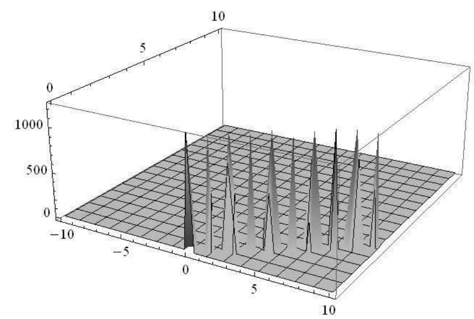

Figure 7. Figure of $u(x,t)$ for $c = 3, f_1 = 2, \varepsilon = 0.001, -10 < x < 10, 0 < t < 10$ in (26)

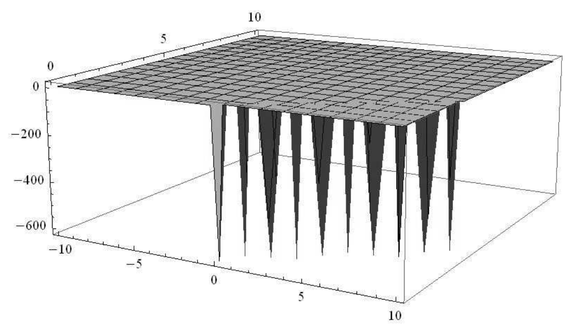

Figure 8. Figure of $w(x,t)$ for $c = 3, f_1 = 2, \varepsilon = 0.001, -10 < x < 10, 0 < t < 10$ in (27)

5 Conclusion

In this work, we have found the exact travelling wave solutions of the kinetic Carleman system by using the tanh–function method, the extended tanh–function method. All of the above solutions have been verified using the Mathematica package.

References

[1] Godunov S.K., Sultangazin U.M. *On discrete models of the kinetic Boltzmann equation*, Russian Math. Surveys, 1971, **26**, 1–56.

[2] Polyanin A. D., Zaitsev V. F. *Handbook of Nonlinear Partial Differential Equations*, Chapman & Hall/CRC Press, Boca Raton–London, 2004.

[3] Tchier F., Inc M. and Yusuf A. *Symmetry analysis, exact solutions and numerical approximations for the space–time Carleman equation in nonlinear dynamical systems*, Eur. Phys. J. Plus, 2019, **134**, 1–18.

[4] Kolodner I. I., Vindas J. *On the Carleman's model for the Boltzmann equation and its generalizations*, Ann. Mat. Pura Appl., 1963, **63**, 11–32.

[5] Currò C., Oliveri F. *Similarity analysis and exact solutions for a general discrete two-velocity model of Boltzmann equation*, Meccanica, 1987, **22**, 3–7.

[6] Lindblom O., Euler N. *Solutions of discrete-velocity Boltzmann equations via Bateman and Riccati equations*, Theoret. and Math. Phys., 2002, **131**, 595–608.

[7] Zhang W. *The Extended Tanh method and the Exp–function method to solve a kind of nonlinear heat equation*, Math. Probl. Eng., 2010, **2010**, 1–12.

[8] Wazwaz A. M. *The tanh method: exact solutions of the sine-Gordon and the sinh–Gordon equations*, Appl. Math. Comput., 2005, **167**, 1196–1210.

[9] Wazwaz A. M. *The tanh and the sine–cosine methods for the complex modified KdV and the generalized KdV equations*, Comput. Math. Appl., 2005, **49**, 1101–1112.

[10] Wazwaz A. M. *A sine–cosine method for handling nonlinear wave equations*, Math. Comput. Model., 2004, **40**, 499–508.

[11] Zhang W., Tian L. *An extended tanh–method and its application to the soliton breaking equation*, J. Phys.: Conf. Ser., 2018, **96**, 012069.

[12] Biazar J., Ayati Z. *Exp and modified Exp function methods for nonlinear Drinfeld–Sokolov system*, J. King Saud Univ. Sci., 2012, **24**, 315–318.

[13] Kumar V. S., Rezazadeh H., Eslami M. et al. *Jacobi elliptic function expansion method for solving KdV equation with conformable derivative and dual–power law nonlinearity*, Int. J. Appl. Comput. Math., 2019, **5**, 1–10.

[14] Weiss J., Tabor M., Carnevale G. *The Painlevé property for partial diferential equation*, J. Math. Phys., 1983, **24**, 522–526.

[15] Golse F. *On the self–similar solutions of the Broadwell model for a discrete velocity gas*, Commun. Part. Diff. Eq., 1987, **12**, 315–326.

[16] Dukhnovskii S. A. *Asymptotic stability of equilibrium states for Carleman and Godunov–Sultangazin systems of equations*, Moscow Univ. Math. Bull., 2019, **74**, 55–57.

[17] Dukhnovsky S. *On solutions of the kinetic McKean system*, Bul. Acad. Ştiinţe Repub. Mold. Mat., 2020, **94**, 3–11.

[18] Dukhnovsky S. A. *The tanh-function method and the (G'/G)-expansion method for the kinetic McKean system*, Differential Equations and Control Processes, 2021, No.2, 87–100.

[19] Dukhnovsky S. A. *New exact solutions for the time fractional Broadwell system*, Advanced Studies: Euro-Tbilisi Mathematical Journal, 2022, **15**, 53–66.

[20] Dukhnovkii S. A. *On a speed of solutions stabilization of the Cauchy problem for the Carleman equation with periodic initial data*, J. Samara State Tech. Univ., Ser. Phys. Math. Sci., 2017, **21**, 7–41.

[21] Vasil'eva O. A., Dukhnovskii S. A. and Radkevich E. V. *On the nature of local equilibrium in the Carleman and Godunov–Sultangazin equations*, J. Math. Sci., 2018, **235**, 393–453.

[22] Euler N., Steeb W.-H. *Painlevé Test and Discrete Boltzmann Equations*, Aust. J. Phys., 1989, **42**, 1–10.

[23] Ilyin O. V. *Existence and stability analysis for the Carleman kinetic system*, Comput. Math. Math. Phys., 2007, **47**, 1990–2001.

[24] Cornille H. *Exact (2+1)-dimensional solutions for two discrete velocity Boltzmann models with four independent densities*, J. Phys. A: Math. Gen., 1987, **20**, 1063–1067.

[25] Radkevich E. V. *On the large–time behavior of solutions to the Cauchy problem for a 2–dimensional discrete kinetic equation*, J. Math. Sci., 2014, **202**, 735–768.

Dukhnovsky S. A.
Moscow State University Of Civil Engineering (National Research University), 26, Yaroslavskoe shosse, Moscow, 129337, Russian Federation
E-mail: *sdukhnvskijj@gmail.com*

Received March 3, 2022